November 2014 subject reports

MATHEMATICS SL

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 36	37 - 49	50 - 61	62 - 73	74 - 85	86 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 17	18 - 20

The range and suitability of the work submitted

A wide range of appropriate and engaging topics with mixed quality was submitted, leading to a range of marks from 1 to 20. Explorations that were suitable tended to have more original aims that had clear personal relevance and foci. Many students still submit explorations with research questions similar to textbook problems, or they were not focused enough to be dealt with adequately within 10 to 12 pages. For instance, there were still many attempts on topics like the Golden Ratio, Monty Hall Problem, Pascal's Triangle, Handshake Problem and Koch Snowflake. These candidates generally produced work that was a summary of common facts and/or a general history of the topic. There were also many candidates who produced work that read like a common textbook example or explanation. In the Koch Snowflake, for example, student work mirrored the old IB Task, showing no personalization or extension. In both cases candidates tended to not score well.

The use of technology to develop regression functions in an attempt to model data was very common. Some schools included samples that were **all** modelling tasks and generally following the old portfolio style. Although there is nothing wrong with these tasks, per se, it would be disappointing if students felt limited to these or were specifically directed to do these by their teacher. In some cases these tasks were done effectively with suitable mathematical support. However there were cases where the regression model was simply created and applied via technology with very little understanding shown. It is hoped that students will be able to justify their choice of regression model and be able to reflect critically on their choice.

The students generally adhered to the suggestion that the exploration be between 6 and 12 pages long. However there were many that were very long. These were often found to be self-penalising.

Candidate performance against each criterion

Criterion A

Most students scored well on this criterion. Most of the work was well organized and systematically presented. They presented some sort of introduction, attempted an aim, made a conscious effort to organize the work and provided a conclusion at the end. Students who did poorly in this criterion usually did not have a focused aim and thus could not present a coherent development for the work. Also some students provided particularly contrived rationales or simply stated that they found the topic ‘interesting’, which is insufficient for a rationale. There were quite a few candidates that provided page after page of repetitive calculations that hurt the conciseness and flow of the paper. Students should provide only one or two sample calculations in the body and all other similar calculations should be summarized in a table. Coherence was generally good but at times the work flowed poorly, with missing explanations and poor linkage between subsections.

Criterion B

Presentation was generally done well. Most students had made a conscious effort to present their work appropriately and with a variety of mathematical presentations. They used an equation editor or other mathematical software to enter proper mathematical expressions. The use of appropriate diagrams with clear labelling was often a problem. It may be that tables and graphs are more easily generated by computer while diagrams take more effort. Many graphs and diagrams were cut and pasted from Internet sources and often these were without any real purpose. Graphs need a purpose and not just included to "use multiple forms of mathematical representation". Mathematical formulas and theorems just taken from the Internet were often included but did not always really add to the students’ work.

Criterion C

Many students made an effort to make the work their own by doing their own research, collecting their own data and providing convincing personal rationales for choosing the topics. On the other hand, there were quite a few students who did not make the exploration their own and only did descriptive work. Students who used textbook problems and basically cut-and-pasted from resources in the public domain often did poorly in this criterion. Similarly, there were still a good number of teachers awarding high marks for candidates who simply stated how much they enjoyed the topic or for the enthusiasm they demonstrated even though there was no evidence in the work of good personal engagement. It is important to note that this criterion cannot be used to penalise late submission of work.

Criterion D

Many students could produce some reflections and attempted to make these meaningful. They would at least consider the relevance of the mathematics they were using or investigating. Unfortunately, only a few were capable of producing critical and substantial reflections throughout their explorations. Nevertheless, this did not stop some teachers from awarding the top level for student work which simply summarized the results.

Criterion E

There was a wide variety of mathematics used in the explorations and a wide range of levels of understanding. The majority of the students were able to produce explorations that are commensurate with the mathematics SL syllabus and relevant to the tasks. However often they were not able to show that they understood the concepts well. For instance, the mathematics appeared to be regurgitated from textbooks or the internet and not really applied to the question in hand. Applying it to the student's own work needs to be encouraged. Only a few challenged themselves by going beyond the mathematics SL syllabus. The success rates of these attempts varied.

Students and teachers should be aware that just showing the correct answer is not the same as showing understanding, it must be demonstrated.

Recommendations and guidance for the teaching of future candidates

Teachers should ensure that they are familiar with all the relevant information in the guide and the Teacher Support Material (TSM), especially the teacher responsibilities in the TSM. In particular, the following points should be noted.

- Teachers should go over some of the exemplars in the TSM and mark them with their students so that the students will have a better idea on the expectations of each criterion.
- Teachers should encourage originality of work, particularly the idea of "making the work their own". They should stress the idea of applying the mathematics that students have discovered to their own work.
- Students should be guided to cite all resources used in the body of the work. These include images and data that are used. A bibliography is not sufficient because it does not inform the reader how and where these resources have been used in the exploration.
- Students should be guided to produce explorations that have clear and focused aims, with evidence supporting their personal engagement.
- Teachers need to follow the suggested procedures in the TSM which allows students to submit a first draft. This way, teachers can assess the suitability of the topic, check the general organization and coherence, orally test the students' knowledge of the mathematics and most importantly, ensure that the work is that of the student and not just a regurgitation of Wolfram, Wikipedia and other sites.
- Schools should be strongly discouraged from mandating a particular type of exploration. Rather students should be free to explore an area, where this leads to a

decent exploration, of their choice.

- It is extremely helpful to the moderation process if teachers annotate and comment on the student work as well as on the form 5/EXCS. Teachers must indicate that they have checked mathematical processes, and noted if they are correct or not.
- Where there is more than one teacher, it is essential that internal standardization between teachers takes place.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 46	47 - 56	57 - 66	67 - 76	77 - 90

The areas of the programme and examination which appeared difficult for the candidates

- Integration using substitution and/or inspection
- Expected value of a fair game
- Sketching functions, including important features of the graph
- Trigonometric ratios of obtuse angles
- Conditional and binomial probability
- Applying properties of logarithms
- Vectors and vector equation of a line

The areas of the programme and examination in which candidates appeared well prepared

- Applying formulas for terms and sums of an arithmetic sequence
- Sum of a probability distribution
- Integration and differentiation of polynomial functions
- Solving quadratic equations
- Simple probability and tree diagrams

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: quadratics

Parts (a) and (b) of this question were answered quite well by nearly all candidates, with only a few factoring errors in part (b). In part (c), although most candidates were familiar with the general parabolic shape of the graph, many placed the vertex at the *y*-intercept (0, -6), and very few candidates considered the endpoints of the function with the given domain.

Question 2: arithmetic sequences

All three parts of this question were very well done by the candidates. The occasional mistakes that were seen tended to be arithmetic errors which happened after the candidates had substituted correctly into the formulas given in the formula booklet.

Question 3: probability distribution and fair game

The large majority of candidates answered part (a) of the question correctly by summing the probabilities to 1. Part (b), however was not as well done. Many candidates seemed to be unfamiliar with the idea of a "fair game", despite this topic being listed in the syllabus. The most common error in part (b) was setting $\mathrm{E}(X)=1$ rather than $\mathrm{E}(X)=0$.

Question 4: properties of logarithms

Part (a) was answered correctly by a large number of candidates, though there were quite a few who applied the rules of logarithms in the wrong order. In part (b), many candidates knew to set their answer from part (a) equal to $-\ln x$, but then a good number incorrectly said that $\ln 2=-\ln x$ led to $2=-x$.

Question 5: rational functions

Parts (a) and (b) were generally well done. Some candidates incorrectly answered $q=-3$, rather than $q=3$, in part (a), but then were able to earn follow-through marks in part (b). Many candidates did not recognize the connection between parts (b) and (c) of this question, and many did a good deal of unnecessary work in part (c) before giving the correct answer. In part (c), many candidates did not write the equation of the asymptote, but just wrote the number.

Question 6: integration and area under a curve

Very few candidates earned full marks in this question. While most candidates knew to integrate, many seemed unfamiliar with integrating using substitution or inspection. This topic is part of the syllabus, but it did not occur to many candidates to use a substitution method. A large number of them tried to integrate the individual terms in the numerator and denominator as though this were a polynomial function. While there were some candidates who knew the integral would involve a natural log function and substituted 4 and 0 into their function, many ended up with undefined values such as $\ln 0$ or did not know what to do with expressions containing $\ln 1$.

Question 7: vectors and trigonometry

The large majority of candidates were able to find the correct expression for $\cos \mathrm{C\hat{A}B}$, but few recognized that an angle with a negative cosine will be obtuse, rather than acute, and many stated that $\mathrm{C\hat{A}B}=30°$. When substituting into the triangle area formula, a common error was to substitute $5\sqrt{3}$ rather than 10, as many did not understand the relationship between the magnitude of a vector and the length of a line segment in the triangle formula.

Some of the G2 comments from schools suggested that it might have been easier for their students if this question were split into two parts. While we do tend to provide more support on the earlier questions in the paper, questions 6 and 7 are usually presented with little or no scaffolding. On these later questions, the candidates are often required to use knowledge from different areas of the syllabus within a single question.

Question 8: probability

Parts (a) and (b) of this question were answered correctly by nearly all candidates, and the majority earned full marks on part (c), as well. Unfortunately, there were a number of candidates who made arithmetic errors when multiplying or adding fractions. Candidates were not as successful in parts (d) and (e) of this question. Although many knew that conditional probability was necessary in part (d), many did not know to use their values from parts (b) and (c), and started from scratch with brand new, and often incorrect, calculations for the numerator and denominator. A majority of candidates did not recognize that binomial probability was needed in part (e), not realizing that there were three ways for Adam to be "late exactly once". A very common incorrect solution to part (e) was $\frac{1}{4}\times\frac{3}{4}\times\frac{3}{4}=\frac{9}{64}$.

Question 9: calculus and transformation of polynomial functions

The majority of candidates approached part (a) correctly, and most recognized that only one solution was possible within the given domain. Nearly all candidates answered part (b) correctly, earning all the available marks for integrating the polynomial and solving for C. Part (c) proved to be much more difficult for candidates, who either did not know how to apply the transformations correctly, or who engaged in lengthy and unnecessary manipulations of the function, rather than simply finding the image of the local minimum point A.

Question 10: vector equation of a line and calculus

In part (a), most candidates correctly substituted 1 for x, although many of them did not earn full marks for their work here, as they wrote their vector equation using $L_1 =$, not understanding that L_1 is the name of the line, and not a vector. Very few candidates answered parts (b) and (c) correctly, often working backwards from the given answer, which is not appropriate in "show that" questions. In these types of questions, candidates are required to clearly show their working and reasoning, which will hopefully lead them to the given answer. Fortunately, a good number of candidates recognized the need to find the derivative of the given expression for d in part (d) of the question, and so were able to earn at least some of the available marks in the final part.

Recommendations and guidance for the teaching of future candidates

Candidates should practice working with different types of functions, and sketching these functions. They should be able to identify the important features of a graph, such as asymptotes, domain and range, local maxima and minima, and axial intercepts, and identify these features in their sketches. Candidates should also practice answering "show that" type questions, clearly showing all their working. Candidates should recognize that the answers to earlier parts of a question may be useful in later parts of a question, especially in Section B of the paper.

Teachers can greatly assist their students by being aware of all the components of the new Maths SL syllabus, with the first examinations in 2014. Topics such as expected value of a fair game and integration involving substitution seem to have been neglected in many schools. In addition, vectors are a topic that, although not new, seems to be challenging to many students each year. Teachers should help their students understand the geometry behind vectors, rather than just treating things like magnitude and vector equation of a line simply as formulas to be "plugged into".

Teachers also need to be familiar with other aspects of the Maths SL guide, including the notation list and command terms, and share this information with their students. For example, the command term "sketch", while not requiring a perfect drawing of a graph of a function, does require the relevant features of the graph to be shown. With respect to the notation list, it is apparent from the G2 comments from schools that many are not aware that the PQ notation used in question 10 refers to the length of a line segment. Many of the G2 comments recommended that instead of using the notation PQ^2, this question should have used notation such as $(PQ)^2$. The notation (PQ) represents a line, and therefore is a geometric figure and not a numerical value which can be squared. The symbols in notation list will be used without explanation on the examination papers, and candidates are expected to be familiar with them.

Finally, teachers and students need to be aware of the fact that the candidate scripts are scanned, and then marked by examiners who download the images to their computers. Teachers need tell their students that when the scripts are scanned, everything the student has written becomes black-and-white. Common student comments such as "look at the blue line, not the black one" are not helpful to examiners, as all the lines on a scanned script look black. Candidates should make sure their responses are written clearly, and that symbols are not written on the dots of the answer lines – minuses and equals sign are often hard to distinguish. Very faint or incomplete erasures on the page scan as solid black working, so students are reminded to simply cross out any working they do not wish the examiners to consider.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 46	47 - 57	58 - 67	68 - 78	79 - 90

The areas of the programme and examination which appeared difficult for the candidates

- Sketching of graphs from the GDC.
- Volume of revolution
- Finding probabilities from tables of values, and in particular conditional probabilities.
- investigating unfamiliar situations, as in the sequence in question 9
- Solving equations and inequalities graphically
- Working with indices
- The terminology “standardised value”
- Interpreting/reasoning supported by mathematical results

The areas of the programme and examination in which candidates appeared well prepared

- Composite functions
- Using the GDC to find and use the equation of the regression line
- Circle geometry: arcs and sectors
- Use of sine and cosine rules
- Transformation geometry
- Cumulative frequency graphs
- Normal distribution

This was an accessible paper and one which most candidates were able to finish within the allotted time. Overall, there seems to have been an improvement in candidates’ preparedness to handle such papers, which shows an improvement of the teaching and learning in the Mathematics SL classrooms.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: composition of functions

Generally well done, though there were some careless errors with the substitution into *f* in part (ai) and rearranging the equation in part (b). Although candidates understood that they were supposed to solve the equation $2x^3+3=0$, many wrote $2x^3=3$ or $x=\sqrt{\frac{3}{2}}$. The majority of the candidates chose an algebraic method instead of using their GDC.

Question 2: linear regression

Parts (b) and (c) of this question were correctly answered by most candidates.

However, a few students did not recognize that this question involved linear regression. And for those who did, not all of them knew what the correlation coefficient was. Some of them left this part of the question blank, and others wrote the value of r^2.

A number of students tried to find the values of *a* and *b* by forming two linear equations with two points from the table and solving them.

Question 3: circle geometry: arcs and sectors

Parts (a) and (b) were well done, but it was not uncommon to see students finding area instead of perimeter in part (c). Most candidates recognized the need to use the cosine rule in part (b), and other candidates chose to use the sine rule to find the length of AB.

There are candidates who do not seem comfortable working with radians and transform the angles into degrees. Other candidates used an angle of 1.2π instead of 1.2, supposing that angles in radians always should have π.

Question 4: graph of a function, zeroes and volume of revolution

Despite being a straightforward question, and although most candidates had a roughly correct shape for their graph, their sketches were either out of scale or missed one of the endpoints. In part (b), a few did not give both answers despite going on to use 1.84 in part (c).

Part (c) proved difficult for most candidates, as only a small number could write the correct expression for the volume: some included the correct limits but did not square the function, whilst others squared the function but did not write the correct limits in the integral. Many did not find a volume, or found an incorrect volume. The latter included finding the integral from 0 to 2, or dividing the region into three parts, showing a lack of understanding of "enclosed".

Question 5: graphs of trigonometric functions

Many candidates found the correct value for the amplitude and vertical shift, but very few managed to find the correct value of the period and therefore of *q* in part (c). Some candidates substituted the coordinates of a point into the function but were not able to write a correct equation in terms of *q*. Many candidates who found the correct answer did not show sufficient work to gain all three marks. The rubrics stress the need to show working.

Question 6: binomial expansions.

Candidates tended to either do very well or very poorly in this question. Some had difficulty understanding what the constant term was, while others were unable to find the value of *r* that led to the constant term. Many algebraic errors were seen in the calculation of the term, mostly having to do with forgetting to square $\frac{1}{2}$. Some missed the negative solution for *p*, despite the fact that the question asked for the "values" of *p*.

Question 7: velocity, displacement and distance

For part (a), a large number of candidates chose the correct formula to find the distance but many got an incorrect value. A considerable number of candidates misread the function as $v(t) = \mathrm{e}^{\frac{1}{2}\cos t}$, losing a mark for this part.

Only a few candidates gained full marks in part (b). Although many mentioned the change of direction, very few supported their answer with a calculation of the distance travelled back or the displacement, thus showing poor understanding of the command term “explain”.

The periodic nature of the function confused many candidates, who used this fact to assure that the particle would pass through A again.

Question 8: statistics and probability

This question was well handled by most candidates. Except for miscalculations and incorrect readings from the cumulative frequency graph, the processes and concepts seemed to be well understood by the majority.

A number of students did not gain full marks in parts (bii) and (e), for not showing their process. In part (c), some candidates wrote things like “using GDC”, without showing relevant work, and so lost marks. Those who chose a formulaic approach to the conditional probability question in (dii) were often not as successful as those who could interpret the question in terms of the table values.

A large number of candidates could not find the mean value in (e). Some used the incorrect mid-interval values and others did not consider their use.

Question 9: sequences

Most candidates answered part (a) correctly. A surprising number assumed the second sequence to be geometric as well, and thus part (b) was confusing for many. It was quite common that students did not clearly show which work was relevant to part (i) and which to part (ii), thus often losing marks. Few students successfully completed part (c) as tried to solve algebraically instead of graphically. Those who used the table of values did not always show two sets of values and consequently lost marks.

Question 10: normal distribution

There was a wide range of ability shown by candidates in this question. While the majority knew how to find probabilities, very few understood the concepts behind the normal distribution, including the answer to the straightforward question (ai). Quite a few students did not yet recognize the instruction “write down”, spending considerable time trying to find the 0.5 answer in (ai) or the standardised value in (bi).

Many candidates did not understand question (bi), giving either a probability value as the z-value or finding the correct value later on in part (bii) in the calculation of the standard deviation (without recognising its significance). For many of those who did understand these concepts, the context of the question was not a real challenge and a number of candidates managed to answer the entire question correctly.

Recommendations and guidance for the teaching of future candidates

The main issues that came across this paper were the efficient use of GDC, showing relevant working and the conceptual understanding that supports mechanical applications of rules and formulae. It is essential that candidates get the chance to clearly show their working, give reasons for what they do and how they do it, and learn what constitutes an acceptable and complete explanation. This can only be achieved if it is considered a teaching point.

Teachers should continue to stress the importance of showing working at all times and remind candidates that full marks are not always awarded for a correct answer without working. In general if a part of a question is worth more than 2 marks it is advisable to show the process of obtaining the answer.

Students need to have a clear understanding of what constitutes “working” when they are reading a solution from a table of values (how many sets of results must be shown etc.)

It is important to continue to impress upon students what constitutes an “accurate” sketch of a function graphed on their GDC.

Students should be encouraged to give unrounded answers, before recording their final answer correct to 3 significant figures (sf). They should also be made aware of the consequences of giving answers to 1sf and even 2sf (it was not uncommon to see inaccurate rounding to 2sf with no indication of the unrounded answer). Early rounding can lead to incorrect answers.

It is important to insist that students accurately label their working, indicating which part of the question it pertains to. Many students lost marks as it was not always clear which work was for which part.

In the teaching of kinematics, emphasis needs to be placed on the understanding and interpretation of velocity/time, displacement/time graphs etc., and how graphing facilities on the GDC can be used to enhance this understanding.

A considerable number of candidates do not realize when it is necessary to use their GDC instead of trying an algebraic solution. It is essential to dedicate time in class to construct criteria that will help students decide when to try an analytic approach and when to use their calculators. As a rule, on paper 2, a GDC approach is expected.