November 2013 subject reports

MATHEMATICS SL

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 33	34 - 47	48 - 59	60 - 70	71 - 83	84 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 13	14 - 19	20 - 23	24 - 28	29 - 33	34 - 40

The range and suitability of the work submitted

All schools submitted tasks drawn from the current set provided by the IB. Some HL transfer candidates presented HL tasks. Teachers are reminded that this work should be re-assessed against the SL criteria.

Candidate performance against each criterion

Criterion A

Some work contained computer/calculator notation that was not penalized by teachers. Many candidates arrived at results that were approximate yet no approximation symbol was used nor the issue of accuracy addressed in any way. In modelling, candidates often used the same function name (usually ‘*y*’) for every function developed.

Criterion B

Candidates were generally better at providing coherent and complete communication. Some used a “question & answer” style which hinders the smooth flow of the work. Graphs, tables and explanations must be included in the body of the work, not as appendices.

Criterion C

Most candidates understood the process of searching for a pattern and were able to present sufficient data and supporting analysis. Some, however, used minimal evidence to draw conjectures. The most common mistake in testing the validity of the conjecture was to simply create new results using the general statement without testing these against the original patterns that produced the conjecture.

In Type II tasks there were many cases where candidates quickly went through the initial steps of modelling (the definition of variables and parameters) and jumped right into trying to develop a model function. Some candidates left the selection of a “best-fit” model to a calculator or computer and then they proceeded to use an analytical method to “find” the function. Candidates presenting good work were able to use their knowledge of functions to recognize which type of function would provide a good fit, and then use the proper analysis to determine the function. Comparisons of fit were often superficial with comments such as “it fit well” or similar. The best comparisons isolated intervals of good and bad fit over the domain of the function. While quantitative comparisons were effectively used it should be noted that only a qualitative comparison is expected at standard level. A common shortcoming was that candidates used a regression function to compare the fit and/or extended such functions to a further data set.

Criterion D

Candidates were generally able to arrive at some form of general statement in Type I tasks. Often, though, the consideration of scope and/or limitations was limited and missed key aspects. Very few candidates were able to provide satisfactory informal explanations for their generalizations.

Candidates sometimes appeared to get lost in the mathematics of their models and ignored the interpretation in the context of the task, or offered very limited and superficial interpretations. Accuracy was often ignored and the reasonableness of the model was rarely considered.

Criterion E

Many candidates achieved good marks here although often this was unsupported by evidence in the work. Teachers appeared to be generous in this criterion without explaining why they gave high marks. In some cases no technology use was apparent yet E2 was awarded. Even the presence of some graphs did not always enhance the presentation and should not have received E3. Many candidates are familiar with the features of graphing software and used this knowledge to good effect.

Criterion F

This criterion was well understood by the great majority of teachers. Levels F0 and F2 were awarded rarely and appropriately.

Recommendations and guidance for the teaching of future candidates

Teachers are reminded that from next year, there is a new model of internal assessment (the exploration). It will continue to be important that teachers and candidates come to know and understand the new criteria. It is essential that teachers explain the criteria levels to candidates so that they know what is expected of them. Further information is available on the Online Curriculum Centre.

Comments from teachers are highly encouraged as these help the moderator to understand why certain marks were awarded. Teachers should feel free to mark up the candidates’ work with both positive and constructive comments. Sharing the marked work with candidates can provide them with feedback on their efforts and understandings of the criteria. Only copies should be shared however, as the original work may be selected for moderation. No further editing is allowed for completed and marked tasks.

Short exercises presented in class can serve to address the important objectives in each criterion. For example, the process of generalizing an arithmetic or geometric sequence or series can highlight the process of developing a conjecture and checking its validity.

Teachers should help students better understand the importance of identifying approximate results, and in a modelling task, the need for addressing this aspect in the context of the task. Functions all labelled '*y*' can cause confusion. Some distinguishing notation such as subscripts or different variables should be used.

The portfolio is meant to be an exercise in mathematical writing (as is the exploration in 2014). As such, a "question & answer" approach diminishes this aspect. The response appears to be more of a set of answers to homework than a mathematical paper with coherence and completeness.

The concepts and processes of conjecture and modelling should be taught in class using examples that resemble the kinds of tasks that candidates will explore. Concepts such as validation of a conjecture and interpretation of a model in context are not well understood.

Teachers should model the effective use of technology in their lessons so that candidates can appreciate the potential of such technology. There are many good graphing software packages available for schools to purchase or for individual use on a trial basis. Above all teachers should think carefully about their own expectations for the use of technology, and then make this clear to the moderator in the background information provided with the sample.

Coordinators should ensure that feedback and subject reports are read by teachers so that common or repetitive issues of concern are addressed. Coordinators and teachers should ensure that the appropriate supporting documents are completed properly and included in the sample.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 31	32 - 44	45 - 55	56 - 65	66 - 76	77 - 90

General comments

In general, the questions in this year's paper required good understanding of the concepts. In some of the questions, this conceptual understanding allowed for more efficient methods of solution. Candidates who did not understand the concepts often found themselves struggling with convoluted and unnecessary working which often did not lead them to an answer.

The areas of the programme and examination which appeared difficult for the candidates

- Vector geometry
- Infinite geometric series
- Transformations on sinusoidal graphs
- Integration of a known derivative
- Properties of integrals
- The difference between a gradient function and the gradient at a point
- Distinction between the minimum value of the function and the value of x at which this occurs

The areas of the programme and examination in which candidates appeared well prepared

It was pleasing to note that the large majority of candidates were able to make a good attempt on each question, and very few questions were left entirely blank. Time did not seem to be a factor, as it appeared that candidates were not rushing through the later questions. In general, candidates showed good preparation and knowledge in the following areas:

- Matrix algebra
- Reading information from a cumulative frequency curve.
- Composite and inverse functions
- Integration and differentiation of basic functions

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Vectors

It is worrying how many candidates do not have a basic understanding of the geometry of vectors. As this is not a particularly challenging question, it suggests that candidates in general are not being adequately prepared in this topic area.

In part (a), common errors were to find $\overrightarrow{\mathrm{PQ}}$ instead of $\overrightarrow{\mathrm{QP}}$, or give $\boldsymbol{p}+\boldsymbol{q}$ as the answer.

In part b), many candidates found a correct expression for $\overrightarrow{\mathrm{QT}}$ or $\overrightarrow{\mathrm{PT}}$ and used a valid approach to find $\overrightarrow{\mathrm{OT}}$ but left an unfinished final answer.

Question 2: Matrices

Many candidates showed clear knowledge of matrix multiplication, and many carried out the calculation for the determinant. Even those who did not multiply correctly in part (a) were often able to earn follow through marks in part (b).
Occasionally some candidates applied an incorrect rule: $\det(\boldsymbol{AB}+\boldsymbol{C})=\det(\boldsymbol{AB})+\det(\boldsymbol{C})$. Others found the inverse of $(\boldsymbol{AB}+\boldsymbol{C})$.

Question 3: Statistics: Cumulative frequency graphs

This question was the most well-done on the paper, with nearly all candidates earning full marks on all three parts of the question. Some candidates did not show their working in part (b) and thus earned less than full marks.

Question 4: Properties of definite integrals

This question was a good test of understanding of the notation and concepts of integration rather than routine processes. It is of concern that many lack this understanding.

Many answered part (a) correctly, but did not know how to approach part (b). Many simply added 2 to the value of the given integral, and gave an answer of 10, while others substituted 8 in for f(x) and then integrated getting 8 + 2x before substituting in their limits.

Question 5: Circular Functions

In part (a), while many candidates substituted $\left(\frac{\pi}{4},6\right)$ into the function, a surprising number did not know that $\sin\frac{\pi}{2}=1$ and so could not complete the calculation correctly, often using $\sin\frac{\pi}{2}=0$.

Most attempts for part (b) employed $f'(x)=0$, but few if any candidates could complete this approach. A common error was to consider that $x=\frac{\pi}{4}$ instead of $x=\frac{5\pi}{4}$. Not many candidates were able to say that the minimum value of $\sin\left(x+\frac{\pi}{4}\right)$ is -1, hence the minimum of the given function is $-1+5=4$. It appears as though students consider that they need to perform complicated working in order to answer the question.

In part (c), many found the correct value for q, but mistakenly said that $p=\frac{\pi}{4}$.

Question 6: Equation of a tangent to a curve

A good number of candidates recognized the need for the derivative in this question, yet fewer candidates attempted to calculate the gradient at $x=1$. As a result, an equation such as $y=2e^{2x}x-2e^{2x}+e^2$ was given as an answer for the tangent line. By giving the equation of a non-linear function, those candidates seem to show that they don't fully understand the concept of the tangent line. There seems to be confusion between the concept of a gradient function and the gradient of a function at a point.
Those who substituted the gradient and the coordinates of the given point often found the equation of the line successfully.

Question 7: Quadratic equations

Many candidates recognized the need for the discriminant in this question, and a good number even knew that it must be positive for the equation to have two different roots. Yet, only a handful could solve the resulting inequality to fully answer the question. Almost none of the candidates gave the correct final answer expressed in a correct form.
Other successful methods included analysing the graph of the resulting quadratic function to decide the values of *x* for which it was greater than 0. Others used the fact that any square is always positive or 0 to decide that $(k-2)^2>0$ when $k\neq 2$.

Question 8: Reciprocal functions

Parts (a) and (b) proved to be very accessible for a majority of candidates. Many found the y-intercept in (ci) without difficulty, although at times a candidate would set the y=0. While some successfully sketched the resulting graph, showing major features of shape and asymptotic behaviour, many candidates did not consider the given domain.
Many students found the inverse of *h* in order to answer parts (d) and (e). While correct and earning full marks, the more efficient and astute approach was often overlooked. The astute student used that fact that if $h(a)=b$, then $h^{-1}(b)=a$. Part (dii) was often left blank, further suggesting a disconnect between the graph of a function and its inverse.

Question 9: Geometric series

The algebraic nature of this question proved elusive for many candidates, as many were bogged down in unnecessary manipulations that often went nowhere. It was not uncommon for candidates to use a formulaic approach to showing the result in (aii), eg. $u_3 = u_1 r^2$. While not incorrect, few could carry through with the algebra completely.

For part (b), many candidates chose to solve by quadratic formula over factorization. While this is an entirely appropriate approach, some could not find the root of 169.
Part (c) showed that the vast majority of candidates who attempted this question do not understand the conditions upon which a finite sum for an infinite geometric series depends. It was common for students to justify the choice of *r* by stating it must be greater than 1, or that it must be positive. It seems that many candidates associate the word “finite” with the type of series rather than a sum that exists for an infinite series.

Question 10: Calculus

Even though most candidates knew they needed to apply either the chain rule, the quotient or product rule in part (a), some had difficulties in showing the result. Many candidates opted to apply the quotient rule, which if done correctly earns full marks. However, most candidates made some error in their quotient rule, often not recognizing that the derivative of a constant is zero. There were candidates who did not show enough work, while others made errors in the application or substitution into the chosen rule. Questions where candidates need to show a result prove to be demanding. It seems that they did not always realize that the focus is not just in obtaining a result but in showing how that result can be obtained.

In part (b), most candidates related the fact that there was a minimum with the derivative being equal to zero. However, some showed difficulties in solving the equation $\ln x = 0$. Common errors were answering $x = 0$ or $x = 10^0$.

A considerable number of candidates earned full marks in (c), even if they could not answer part (b) correctly. Many did not see the connexion between these two questions and solved each equation separately.

For part (d), many candidates were able to write the correct equation, but showed difficulties in the solution. Some could not solve $\ln x = 1$, others wrote $\ln x^2 = x$. A few candidates found $x = 10$.
Few attempted part (e), and even fewer made the link to part (a), that the integral of a derivative is the original function. Occasionally the candidate reversed the functions in the integration setup.

Recommendations and guidance for the teaching of future candidates

Candidates need to be familiar with the vocabulary and underlying concepts, rather than simply practicing standard routines and processes. There were many areas of the paper where misunderstanding and faulty reasoning led to marks not being awarded.

Successful completion of this examination depends on students receiving complete coverage of the syllabus. The topic of vectors seems undervalued and in need of greater attention.

Students should be taught to appreciate the underlying assumptions and conditions that allow for a particular mathematical idea to be applied. Too often students resort to formulaic algorithms without understanding underlying principles. Such is the case with the formula for the sum of an infinite geometric series. Students are all too willing to substitute whatever numbers are available into the formula without considering appropriateness.

Long questions ask students to sustain a line of reasoning within multiple parts, and often require the linking of one part to another. Students need formative experience with such questions before becoming confident and fluent in the approach.

Questions that ask to show a certain result also need to be considered for reflection. Students need to understand that the focus is not just in obtaining a result, but is justifying how that result can be obtained.

There are still many candidates who should work on basic algebraic skills. Candidates should be reminded not to leave their answers in unfinished form. Further information is available on the Online Curriculum Centre.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 28	29 - 40	41 - 51	52 - 62	63 - 73	74 - 90

The areas of the programme and examination which appeared difficult for the candidates

- Conversion to and working with fractional indices
- Recognising when to use a graphic display calculator (GDC) for complex calculations, particularly integration
- Algebraic manipulation
- Kinematics
- Vector applications

The areas of the programme and examination in which candidates appeared well prepared

- Matrices and their inverses
- Volumes of revolution
- Finding probabilities given mean and standard deviation of a normal distribution
- Trigonometry (sine and cosine rules) and radian measure
- Basic vectors and application of scalar product
- Basic probability – use of tree diagrams, recognizing conditional and binomial probabilities

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Inverse Matrices

This question was well done by the vast majority of candidates.

Question 2: Volume of revolution

Most candidates were able to find the intercepts and write a correct expression for the volume required. Many candidates unfortunately went on to find the definite integral algebraically, which although is possible, most often resulted in error, rather than using their GDC as expected.

Question 3: Calculus

Working with fractional indices was a challenge for some candidates. Most understood the basic processes of differentiation and integration, however, careless errors such as omitting '+*c*' or forgetting to integrate $-\frac{1}{2}$ occurred.

Question 4: Probability

Candidates showed a good awareness of the meaning of mutually exclusive and independent events. In part (b) a correct expression for $\mathrm{P}(A \cap B)$ was often seen, as was an attempt to use the formula for $\mathrm{P}(A \cup B)$, but solving for $\mathrm{P}(B)$ proved challenging.

Question 5: Sketching and Kinematics

Graphs were not always as well sketched as they could have been, considering that a GDC was at hand. The maximum point and curve features such as passing through the origin and the asymptotic behaviour were often poorly sketched. Many candidates also forgot that the sketch required a restricted domain. In part (b), candidates were expected to use the GDC to evaluate the definite integral. In part(c) very few candidates made the connection between the maximum point on the graph of v and when the acceleration is zero. Candidates should be careful to ensure they are answering the question asked. i.e. the independent variable t tells you when the acceleration is zero , but the dependent variable v tells you what the velocity is. This question was a good example of linked sub-parts and efficient GDC methods available but often not utilized.

Question 6: Normal Distribution

Most candidates were able to successfully find the probability in part (a). In part (b) the most common error was that candidates used the probability given in the question as the z-value. Unfortunately, to one significant figure, this was the same as the z-value, and its use led to an almost correct answer. Showing evidence of subtracting 0.3 from their part (a) answer was necessary for full marks to be awarded.

Question 7: Problem Solving and Calculus Applications

Most candidates were able to find a correct expression for PQ, however, a significant number were unable to determine a correct expression for the area of the rectangle. In part (b)(i) many candidates found the derivative of the given function $f(x)$, rather than the derivative of the area. In part (b)(ii) only a small number of candidates considered the reasonableness of their answer for b in the context of the problem, instead stating a value of b that resulted in a negative area.

Question 8: Circular Functions and Trigonometry

This question was well tackled by the vast majority of candidates, in particular parts (a) and (b). The majority of candidates worked comfortably in radians. As is usual with these types of questions, there were a wide variety of geometric and trigonometric approaches to solving the problem. Common errors in part (c) were that candidates mistakenly assumed radii to the vertices of triangle ABC bisected the angles or they used the angles at the circumference instead of the central angles.

Question 9: Vectors

Part (a) and (b) posed few problems for most candidates. Part (c) however, was more challenging. Candidates who were successful in part (c) usually sketched a diagram enabling a visual clarification of the problem; especially to recognize that both points Q and R lie on L_1. Those who showed this typically completed the problem successfully. The majority of candidates who attempted this question substituted Q into line L_1 to find $s=-1$. At this point, few candidates attempted to gain a better understanding of the question.

Question 10: Probability

The wording in this question was not as clear as intended and candidates were interpreting the scenario in a couple of ways - a five-day or a seven-day week. The markscheme included solutions to both. Candidate scripts did not indicate any adverse effect.

Most candidates drew a tree diagram and used this correctly to solve part (a). Conditional and binomial probability was recognized by most in parts (b) and (c) respectively, although not always successfully calculated, especially the conditional probability.

Part (d) clearly discriminated with few candidates successfully solving for *n*. Many candidates did not recognize the need for the complement of ‘at least’ or if they did, did not proceed beyond this. Those who used trial and error did a good job demonstrating the cross-over values which determined the solution.

Recommendations and guidance for the teaching of future candidates

Paper 2 is a GDC required paper, not simply a GDC allowed paper. Candidates should be encouraged to consider whether use of the GDC is appropriate when answering any question on Paper 2. Although basic GDC skills are improving, there are still candidates who are opting for an analytical approach rather than a more efficient GDC approach particularly with definite integrals. This often leads to simple algebraic errors and consumes valuable time. It should be emphasized that once an equation is established, no algebraic working is needed to support an answer. Teachers should place greater emphasis on integrating the use of technology as a tool for learning and for better understanding key concepts as well as for solving problems by communicating solutions clearly.

Many candidates continue to struggle with what work to show when using technology. Working should be used to show any set up required before using the GDC, *eg* if asked to find an area under a curve, write the integral required, including the limits and function, then state the answer. Mathematical notation should be used, not calculator notation. Writing “used GDC” is not enough evidence of a valid approach.

Students should be taught not simply to transcribe graphs from their GDC without considering their intrinsic knowledge of key features and behaviours of functions. They should be encouraged to use the appropriate GDC tools to find and label key features of graphs.

Numerical values (including answers given correct to three significant figures) should be stored in the memory, and the more accurate “long” value used if needed in subsequent parts. Inaccurate values or premature rounding of values can lead to wrong final answers.