MATHS SL

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 30	31 - 45	46 - 56	57 - 68	69 - 80	81 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 13	14 - 19	20 - 23	24 - 28	29 - 33	34 - 40

The range and suitability of the work submitted

The vast majority of schools selected tasks from the set of new tasks offered by the IB. The tasks on Binomial Matrices and Body Mass Index proved most popular. A few schools submitted older TSM tasks and were penalised for doing so. Schools are once again advised to stay informed of current developments and ensure that appropriate materials are being used for Internal Assessment work. The Online Curriculum Centre (OCC) and the Diploma Program Coordinator Notes are good sources for updates.

It was satisfying to note that there were very few incomplete portfolios. As teachers gain confidence with the tasks supplied by the IB, it is hoped that they will develop their own tasks. This can allow for teachers to better address the topics studied in their classrooms with their students, thus making the whole exercise more effective as both a pedagogic and an assessment tool.

Candidate performance against each criterion

It was surprising to see continued misuse of computer and calculator notation, despite numerous advice regarding this in past subject reports. With proper attention to notation, achieving A2 should be routine.

Communication has improved, especially with graphs. Students are making effective use of graphing software and are learning to properly label graphs. Overall the quality of communication has improved to a level expected from a mathematical presentation. There were few cases of treating the work as a set of homework answers in a "question & answer" style. It should be noted that this style is likely to prevent achieving the highest levels.

In the Type I tasks, marks of C3 and C4 were common, although attaining the highest level continues to prove difficult for students. Once a general statement is developed students must then use a number of examples of further data (e.g. further values of *n*) and show that the mathematical pattern of behaviour being considered continues to the same result as that predicted by the general statement. This is what is meant by validation in C5. In criterion D, Type I, the highest level is also elusive for many. At the D4 level, teachers should make clear to moderators what they expect in the way of scope and limitations. For example, should students consider M^{-1} as ${M^{-1}}^{n}$? For D5 an informal explanation with some mathematical basis must be presented. Teachers are reminded that a **formal** explanation is sufficient for C5, D4 and D5.

In the Type II tasks there were fewer cases of students using a regression tool to develop their model functions. However, the quality of analysis varied significantly. It is important that there is a clearly communicated development of the model function using appropriate mathematical methods. For C4 the proposed model must be compared to the actual data and modified as necessary to make for a better fit. The comparison should be substantive in nature (i.e. more than “it fits well”) but need not be quantitative. Applying the model to other data involves comparing the graph of the initial model to the new data and making appropriate modifications. The emphasis is on using the model developed rather than generating a new model from scratch, as many students chose to do.

Criterion D emphasises the interpretation of the model in context, with appropriate attention to accuracy and reasonableness. Often students were caught up in the mathematics and ignored or minimised the context.

The use of technology has improved as access to better and better resources improves. Nonetheless, students with limited access to technology should be able to attain E3 if they apply the technology in a resourceful manner that truly enhances the presentation of the work. Thus, more graphs do not make the work better but if those graphs are used effectively to demonstrate the evolution of a function to match a set of data then E3 may be appropriate. Students should be cautioned against using software and applications that they do not understand well. Even the use of χ^2 should be avoided unless the student understands its significance. As noted above, the comparison of fit need not be quantitative. Technology can also be used effectively to explore scope and limitations with non-integer and irrational values. However, students should learn that an “ERROR” message on their calculator is not necessarily the final word on the matter.

Most students were fairly successful with many parts of the tasks. Thus F1 was an appropriate mark for the overall quality of work. Teachers used F2 and F0 rarely and this was entirely appropriate.

Recommendations for the teaching of future candidates

Teachers should inform themselves fully as to the requirements and assessment of the portfolio tasks. They should work through each task before assigning it so as to identify potential areas of difficulty and their own expectations in terms of method, scope, contextual interpretation etc. Good information is available on the OCC and in the subject reports from past years.

Teachers and students should think carefully about the expectations for the highest levels of the assessment criteria. Practice tasks, perhaps using older TSM tasks, would provide a useful avenue to explore and discuss what each criterion level signifies. This is especially important for validation of a general statement, scope and limitations, explanations, critical interpretation of a model and methods of adapting a model to a new situation.

Encourage students to see the work as more of a mathematical essay, not just a set of questions. The communication can then be effectively organized in order to improve the readability of the work.

Some discussion of the balance between accuracy and reasonableness in modelling should take place. While technology may give us 8 or more significant figures of accuracy in coefficients, one should consider how much better this is compared to 2 or 3 significant figures, and how much more practical the function might be with simpler values.

Further comments

Teachers must be aware of which tasks are allowed for which sessions. This information is widely distributed through IB channels.

Teachers should take care that they understand the assessment criteria as these apply to each task. The development of a solution key, or a standardisation matrix addressing the demands of each criterion level for the task, will help in marking and moderating. It is required to send such a solution key or standardisation matrix along with the sample so it is in the teachers' interests to develop such things.

External assessment

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 37	38 - 48	49 - 58	59 - 69	70 - 90

The areas of the programme and examination that appeared difficult for the candidates

- Using the laws of logarithms and the laws of exponents to find the inverse of a function

- Solving a trigonometric equation involving a double angle substitution and a quadratic equation
- Finding the image of a point under a transformation
- Explaining the use of the second derivative test to show that a point is a maximum of a function
- Explaining the conditions that must be verified by a point of inflexion, in relation to the second derivative of the function
- Set up of area expressions and evaluation of definite integrals without technology and involving more than one function
- Finding the range of a function given by its formula when it involves vertical and/or horizontal asymptotes
- Determining the asymptote of a function given by its formula
- Solving problems on normal distribution using the symmetry of the curve
- Algebraic manipulation

The levels of knowledge, understanding and skill demonstrated.

Although the levels of knowledge and understanding varied widely, a large number of candidates seemed to be prepared for taking the Mathematics SL paper 1 without a calculator. Candidates were mostly capable when using basic formulas or solving straight-forward problems, in particular of scalar product, magnitude of vectors finding simple derivatives or the inverse of a polynomial function.

In most cases the candidates showed working to support their answers. There were very few outstanding papers and there were a number of questions unanswered on many scripts.

Candidates appeared to be familiar with all areas of the syllabus, although there were areas of weakness.

Without the aid of technology, weaknesses in algebraic manipulation become more apparent.

Candidates showed good basic knowledge of:

- inverse (of a polynomial function) and compound functions
- scalar product
- integration and differentiation of **simple** functions
- calculation of probabilities

- equation of the line normal to a graph

Candidates found difficulty with the following:

- giving arguments when the question asked for a justification; they not only made conceptual errors but also gave confused explanations
- in "show that" questions, they start working with the given answer, ie. reverse working
- solving a quadratic trigonometric equation
- differentiating $y = p / x^2$, composite functions and quotients
- algebraic manipulation, especially when logarithms and trigonometric expressions appear.

The strengths and weaknesses of the candidates in the treatment of individual questions

The following questions were done quite well by a number of candidates:

Q1, Q2, Q4a), Q8a)b)c), Q9 a)i), Q10b)

Question 1: Composite functions, inverse of a function

This question was generally done well, although some students consider e^0 to be 0, losing them a mark. A few candidates composed in the wrong order. Most found the formula of the inverse correctly, even if in some cases there were errors when trying to isolate *x* (or *y*). A common incorrect solution found was to find $y = \frac{\sqrt[3]{x-3}}{2}$.

Question 2: Scalar product, perpendicularity and magnitude

Most candidates knew to set the scalar product equal to zero. A pleasing number found both answers for *q*, although some often neglected to provide both solutions.

Question 3: Normal distribution

Not all candidates realised that the problem could be solved by only using the symmetry of the normal distribution curve and the information given. Some of them saw the need to use tables and others just left it blank.

Most candidates did very well on part (a), shading the area under the normal curve. They were only moderately successful on parts (b) and (c), which required understanding of the symmetry of the curve. Many candidates resorted to formulae or tables instead of reasoning through the question.

Question 4: Transformation of functions

Part (a) was generally solved correctly. Students had no trouble in deciding what transformation had to be done to the graph, although some confused $f(-x)$ with $-f(x)$.

Part (b) was generally poorly done. They could not "read" that the transformation shifted the curve 1 unit to the right and stretched it in the *y*-direction with a scale factor of ½. It was often seen that the shift was interpreted, but in the opposite direction. Also, the stretch was applied to both coordinates of the point. Those candidates who answered part (a) incorrectly often had trouble on (b) as well, indicating a difficulty with transformations in general. However, there were also candidates who solved part (a) correctly but could not interpret part (b). This would indicate that it is simpler for them to plot the transformation of an entire function than to find how a particular point is transformed.

Question 5: Maximum and derivatives

Candidates did well on (a). For (b), a great number of candidates substituted into the function instead of into the derivative.

The derivate of x^2 was calculated without difficulties, but there were numerous problems regarding the derivative of $\frac{p}{x}$. There were several candidates who considered both *p* and *x* as variables; some tried to use the quotient rule and had difficulties, others used negative exponents and were not successful.

Question 6: Trigonometric equation

This question was quite difficult for most candidates. A number of students earned some credit for manipulating the equation with identities, but many earned no further marks due to algebraic errors. Many did not substitute for $\cos 2x$; others did this substitution but then did nothing further.

Few candidates were able to get a correct equation in terms of $\cos x$ and many who did get the equation didn't know what to do with it. Candidates who correctly solved the resulting quadratic usually found the one correct value of *x*, earning full marks.

Question 7: Logarithms, exponents and functions

A very poorly done question. Most candidates attempted to find the inverse function for *f* and used that to answer parts (a) and (b). Few recognised that the explicit inverse function was not necessary to answer the question.

Although many candidates seem to know that they can find an inverse function by interchanging *x* and *y*, very few were able to actually get the correct inverse. Almost none recognized that if $f^{-1}(1)=8$, then $f(8)=1$. Many thought that the letters "log" could be simply "cancelled out", leaving the 2 and the 8.

Question 8: Calculation of probabilities

Overall, this question was very well done. There were some problems with the calculation of conditional probability, where a considerable amount of candidates tried to use a formula instead of using its concept and analysing the problem. It is the kind of question where it can be seen if the concept is not clear to candidates.

In part (c), candidates were generally able to explain in words why events were mutually exclusive, though many gave the wrong values for P(*A*) and P(*B*).

There was a great amount of confusion between the concepts of independent and mutually exclusive events. In part (d), the explanations often referred to mutually exclusive events.

It was evident that candidates need more practice with questions like (c) and (d).

Some students equated probabilities and number of elements, giving probabilities greater than 1.

Question 9: Functions, maximum, minimum, first and second derivatives

Almost all candidates earned the first two marks in part (a) (i), although fewer were able to apply the quotient rule correctly.

Many candidates were able to state how the second derivative can be used to identify maximum and inflection points, but fewer were actually able to demonstrate this with the given function. For example, in (b)(ii) candidates often simply said “the second derivative cannot equal 0” but did not justify or explain why this was true.

Not too many candidates could do part (c) correctly and in (d) even those who knew what the range was had difficulty expressing the inequalities correctly.

Question 10: Tangents, normals, area under a curve, volume of revolution

Parts (a) and (b) were well done by most candidates. While quite a few candidates understood that both functions must be used to find the area in part (c), very few were actually able to write a correct expression for this area and this was due to candidates not knowing that they needed to integrate from 0 to 4 and then from 4 to 4.5. On part (d), some candidates were able to earn follow through marks by setting up a volume expression, but most of these expressions were incorrect. If they did not get the expression for the area correct, there was little chance for them to get part (d) correct.

For those candidates who used their expression in part (c) for (d), there was a surprising amount of them who incorrectly applied distributive law of the exponent with respect to the addition or subtraction.

The type of assistance and guidance teachers should provide for future candidates.

As always, teachers need to be sure to cover the entire syllabus for this course. In addition, candidates should be expected go beyond simply using basic formulae. It is important that candidates' work is neat and clearly written, so that examiners are able to see what they have done.

There were some questions on this examination which required a real understanding of the concepts being covered. This is a good thing, as the candidates had to do more than just look up a formula in their information booklet. This, of course, requires dedicating teaching time to the discussion of concepts and not just to getting a correct answer to a problem.

Paper 1, as it does not allow the use of a GDC, has room for students to show their algebraic skills; they need to show and explain their working and this usually results in a difficulty.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 42	43 - 53	54 - 63	64 - 74	75 - 90

The areas of the programme and examination that appeared difficult for the candidates

Candidates showed difficulty answering questions on:

- Binomial probabilities
- Matrix algebra
- Independence in probability
- Areas between curves
- Vector equation of a line, including choosing the correct vectors when finding the angle between lines.
- Non-routine problems
- Giving explanations based on correct mathematical reasons
- More complex algebraic manipulation

The levels of knowledge, understanding and skill demonstrated

Candidates were comfortable with simple, one or two-step procedures that tested basic skills and substitutions into formulas, particularly for sequences/series and vectors. Most candidates seem familiar with the syllabus content covered in this paper. However, when asked to step beyond the formulaic aspects of the topic, candidates often could not approach the problem.

Use of the GDC varied. While many candidates were able to generate the graphs of functions and the inverse of a 3 x 3 matrix, others did not seem to approach the paper with the GDC in mind. Often long algebraic approaches were employed where the GDC would simplify the process. Using less efficient methods cost candidates valuable time.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Arithmetic sequence

Most candidates answered this question correctly. Those who chose to solve with a system of equations often did so algebraically, using a fair bit of time doing so and sometimes making a careless error in the process. Few candidates took advantage of the system solving features of the GDC.

Question 2: Derivatives

Almost all candidates earned at least some of the marks on this question. Some weaker students showed partial knowledge of the chain rule, forgetting to account for the coefficient of x in their derivatives. A few did not know how to use the product rule, even though it is in the information booklet.

Question 3: Binomial probability

Most candidates were able to find the mean by applying various methods. Although many recognised binomial probability, fewer were able to use the GDC effectively. Part (c) was problematic in some cases but most candidates recognized that either a sum of probabilities or the complement was required. Many misinterpreted “more than three” as inclusive of three, and so obtained incorrect answers. When adding individual probabilities, some candidates used three or fewer significant figures, which resulted in an incorrect final answer due to premature rounding.

Question 4: Matrix algebra

Candidates who knew how to work with matrices on their GDC almost always obtained the correct inverse with very few transcription errors. In part (b), one common error was to subtract correctly but then multiply by the inverse on the right rather than the left. Another frequently seen mistake was incorrectly multiplying by the inverse before subtracting. Some candidates attempted to perform calculations by hand or with systems of equations leading to time-consuming working that did not usually yield correct results.

Question 5: Finding function parameters using calculus

A good number of candidates were able to obtain an equation by substituting the point $1,3$ into the function's equation. Not as many knew how to find the other equation by using the derivative. Some candidates thought they needed to find the equation of the tangent line rather than recognising that the information about the tangent provided the gradient of the function at the point. While they were usually able to find this equation correctly, it was irrelevant to the question asked.

Question 6: Probability of independent events

Many candidates confused the concept of independence of events with mutual exclusivity, mistakenly trying to use the formula $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$. Those who did recognise that $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ were often able to find the correct equation, but many were unable to use their GDC to solve it. A few provided two answers without discarding the value greater than one.

Question 7: Finding minimum cost

Although this question was a rather straight-forward optimisation question, the lack of structure caused many candidates difficulty. Some were able to calculate cost values but were unable to create an algebraic cost function. Those who were able to create a cost function in two variables often could not use the area relationship to obtain a function in a single variable and so could make no further progress. Of those few who created a correct cost function, most set the derivative to zero to find that the minimum cost occurred at $x = 15$, leading to $420. Although this is a correct approach earning full marks, candidates seem not to recognise that the result can be obtained from the GDC, without the use of calculus.

Question 8: Sector and triangle

This question was generally quite well done, and it was pleasing to note that candidates could come up with multiple methods to arrive at the correct answers. Many candidates worked comfortably with the sine and cosine rules to find sides of triangles. Some candidates chose alternative right-angled triangle methods, often with success, although this proved a time-consuming approach. Some unnecessarily converted the radian values to degrees, which sometimes led to calculation errors that could have been avoided. A large number of candidates accrued the accuracy penalty in this question.

Question 9: Parabola and sinusoid

Graph sketches were much improved over previous sessions. Most candidates graphed the two functions correctly, but many ignored the domain restrictions. Many candidates found parts (b) and (c) accessible, although quite a few did not know how to find the period of the cosine function. Part (d) proved elusive to many candidates. Some used creative approaches that split the area into parts above and below the *x*-axis; while this leads to a correct result, few were able to achieve it. Many candidates were unable to use their GDCs effectively to find points of intersection and the subsequent area.

Question 10: Vector equation of a line

Most candidates answered part (a) easily. For part (b), a number of candidates stated that the vector was a “starting point,” which misses the idea that it is a position vector to some point on the line. Parts (c) and (d) proved accessible to many, while for part (e), a surprising number of candidates chose incorrect vectors. Few candidates seemed to have a good conceptual understanding of the vector equation of a line.

Recommendations and guidance for the teaching of future candidates

Time to complete the test seemed to be an issue for some candidates. On further scrutiny, however, it is often the case that candidates are taking long algebraic approaches where a faster, more efficient GDC approach is intended. Teachers would do well in preparing students to choose the GDC as the primary approach to solve equations and systems, calculate binomial probabilities, perform matrix arithmetic, calculate minimums, and find areas between curves. The more familiar with these practices, the better prepared are students to tackle the time constraint of a paper designed with the GDC in mind. Teachers may find the IB teacher support material for GDCs helpful.

As always, teachers must ensure to cover the entire syllabus for the course. Candidates need to know that even on Paper 2, they cannot rely solely on their calculators; they still have to demonstrate their own understanding, and they certainly need to check that their answers make sense.

When candidates embarked on algebraic solutions to problems best solved with technology, it became apparent that some have quite poor algebra skills. While complex algebraic manipulations are not assessed, teachers need to continue to emphasise basic proficiency.

Candidates should be as comfortable working in radians as they are in degrees. It is not the expectation of candidates to convert radian measures to degree measures prior to embarking on a question. This approach also often leads to accuracy penalties when candidates use rounded degree measures in their solutions.

Candidates should be aware that there is a gradation of problems within each section of the paper. For example, the last question of Section A is more challenging than the first question of Section B. Candidates should allocate their time accordingly.

Since working through unstructured problems proved to be difficult for many candidates, additional learning time should be spent with such problems. One option is to remove the structure from previous examination questions when they are used in a classroom setting.