# MATHS SL

## Overall grade boundaries

| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range**: | 0 - 15 | 16 - 30 | 31 - 44 | 45 - 56 | 57 - 69 | 70 - 82 | 83 - 100 |

## Internal assessment

### Component grade boundaries

| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range**: | 0 - 7 | 8 - 13 | 14 - 19 | 20 - 23 | 24 - 28 | 29 - 33 | 34 - 40 |

## The range and suitability of the work submitted

Most teachers selected tasks from the Teacher Support Material (TSM). Some tasks taken from other sources, or designed by the teacher, did not allow for maximum achievement by candidates. A few schools are still using very old TSM tasks that do not match the current criteria well at all.

## Candidate performance against each criterion

**Criterion A - Notation and Terminology**

Most candidates were successful here, using the correct and appropriate notation/terminology throughout their work. Common errors were the use of calculator/computer notation and the inconsistent, or lack of, use of "approximately equals to" signs where needed. In the Stopping Distances task many candidates used the variable '$y$' for each of the three different functions. This is inappropriate as each model function relating to a different quantity should have its own variable.

**Criterion B - Communication**

Candidates generally communicated their work well, offering coherent mathematical writing that flowed smoothly. Some offered a set of distinct answers as if the task were a collection of homework exercises. These often required the reader to refer back to the task for clarification. Some referred the reader to graphs or data tables given as appendices. This breaks the flow of communication.

**Criterion C - Process - Type I**

Many candidates offered successful strategies that allowed for the development of a general statement. Not so many understood the notion of "validation", whereby the general statement must be tested against the observed pattern of mathematical behaviour. In some cases little data was presented as evidence before a conjecture was made.

**Criterion C - Process - Type II**

Candidates who presented a coherent mathematical analysis and considered how well their model fit the data performed well here. Some teachers are accepting regression models generated by calculator or computer as the primary "analysis". This does not reflect the intent of the criterion and can achieve a maximum of C2. Candidates offered various considerations of the data fit, from a thorough description of how well individual points fit to a cursory comment about how the model function fit "quite well".

**Criterion D - Results - Type I**

Many candidates were successful in achieving at least some kind of general statement. Where the correct ultimate general statement was obtained it was usually presented in appropriate mathematical notation. Some candidates gave full consideration to the scope of their general statement, while others looked at only the most obvious limitations. Few candidates offered cogent explanations for their conjecture.

**Criterion D - Results - Type II**

While the majority of candidates were able to produce some kind of model function they were often unable to interpret their models in the context of a car stopping, or a rising tide, etc. Most were satisfied to offer a purely mathematical interpretation, citing slopes and intercepts, etc. Very few were able to offer a critical discussion of the model`s application to the context or some extension of this.

**Criterion E - Use of Technology**

This criterion continues to be affected by the availability of computer and calculator resources. Work varied from multi-coloured graphs embedded in documents to hand-drawn versions copied from graphic calculators. Regardless of the ability to provide printed output, the resourceful use of technology was evident where the candidate had considered the true advantages of the technology available. This included offering multiple graphs on the same axes for comparison and presenting sequences of graphs of functions being transformed into a suitable model. Some offered only a few printed graphs that did not enhance the presentation much at all.

**Criterion F - Quality of Work**

Most candidates rightly achieved F1 here. Some candidates demonstrated superior understanding of the nuances of the task, or offered remarkable analyses or interpretations, to gain F2. Very few offered a totally inadequate effort.

## Recommendations for the teaching of future candidates

Candidates should have seen and worked with the proper notation or terminology unless the investigation is entirely new to them. In this case their own words/descriptions are acceptable. They should treat the work as an essay in mathematics; one that requires an opening and a conclusion as well as a smooth flow of thought throughout the body. The reader should not have to refer to the task for clarification, nor feel as if they are reading a set of study notes. Graphs and tables should be placed in the work as they arise in discussion. Investigation of concepts that are already known to candidates defeats the purpose of a Type I task. Sufficient data should be generated prior to making conjectures about general statements. The statement must be judged against the actual mathematical behaviour to be considered valid, and the scope of the statement should be explored as fully as the candidates` background knowledge permits. In modelling, the candidate must use their own analytical skills prior to using any regression features of calculators or computers. They should offer some substantive consideration of how well their function fits the data, and discuss how the model reflects the real circumstances of the task.

## Further comments

The assessment criteria should be distributed and explained to candidates. Concepts such as validation or scope and limitations should be clarified. Teachers must become aware of how a given task matches the criteria before they offer it. This means that they should work through the task and make notes as to how they might assess certain criteria within a given task. This will also help with the consistency of marking.

Teachers are reminded that new tasks are available from the IB for the 2009-10 sessions, and older TSM tasks will no longer be accepted for submission.

## External assessment

This was the first November session with the new assessment model, where paper 1 allows no calculator and paper 2 requires use of a graphic display calculator (GDC). Students did not appear to encounter any undue difficulties working without the calculator on paper 1.

However it appears that many students are still not clear what “working” to write in the examination when using the GDC, so candidates often spent precious time writing analytic methods to problems most efficiently solved using the GDC. To “show working” does not mean to perform algebraic steps or manipulations. Rather, what is important is to show the mathematical thinking, the setup, before reaching for the GDC, and then to let the GDC do the work of calculation. Whatever supports the solution, making the problem “calculator-ready,” is what students need to show as working.

To help teachers and students to understand more clearly what this means in practice, model solutions for May 08 paper 2 were produced. These are available on the Online Curriculum Centre (OCC). When looking at the markscheme for paper 2, please bear in mind that any analytical approaches given there are to inform examiners how to award marks to such attempts. It is not intended to imply that these are the preferred or expected approaches.

# Paper one

### Component grade boundaries

| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range**: | 0 - 12 | 13 - 25 | 26 - 36 | 37 - 48 | 49 - 61 | 62 - 73 | 74 - 90 |

## The areas of the programme and examination that appeared difficult for the candidates

It was pleasing to see a great number of candidates demonstrate a comprehensive knowledge and understanding of the syllabus. The following areas continue to provide difficulties for some candidates:

- working from a graph such as in sketching an inverse function and justifying relationships between graphs and their derivatives
- justifying maxima/minima and points of and inflexion
- applying transformations of functions
- trigonometric identities
- determining a conditional probability without using a formula
- interpreting horizontal stretches (dilations)

## The levels of knowledge, understanding and skill demonstrated.

The candidates in this session generally showed a good level of knowledge and understanding. It is clear that certain topics are better understood than others.

For example, vector questions are often quite well done and candidates demonstrate a good knowledge and understanding of calculus. Understanding geometric approaches to inverse functions was surprisingly limited but most candidates were able to use an analytic approach to finding the inverse of a function. Candidates were by and large, well-informed of probability and statistical processes, although conditional probability is still somewhat problematic.

Candidates demonstrated a keen ability to apply analytic techniques, such as with matrices, inverse functions, probabilities of combined events and using trigonometric identities. Greater difficulty was encountered when a question was more conceptual in nature and when asked to give arguments.

## The strengths and weaknesses of the candidates in the treatment of individual questions

**Question 1:**

This question was well done by most candidates. There were a surprising number of candidates who lost a mark for not simplifying $\frac{3}{0.1}$ to $30$, and there were a few candidates who used the formula for the finite sum unsuccessfully.

**Question 2:**

This question was quite well done. Marks were lost when candidates found the vector $\overrightarrow{\mathrm{BA}}$ instead of $\overrightarrow{\mathrm{AB}}$ in part (a) and for not writing their vector equation as an equation. In part (b), a few candidates switched the position and velocity vectors or used the vectors $\overrightarrow{\mathrm{OA}}$ and $\overrightarrow{\mathrm{OB}}$ to incorrectly write the vector equation.

**Question 3:**

Candidates were quite successful with this question, most being able to find the product ***AB*** correctly. Candidates used two approaches equally well in part (b), the determinant approach being the most "inefficient".

**Question 4:**

There were a large number of candidates who were unaware of the geometric relationship between a function and its inverse. Those that had some idea of the shape of the graph often did not consider the specified domain. Many more students were able to use an analytical approach to finding the inverse of a function and had little problem using logarithms to solve for $y$. Candidates were clearly more comfortable with algebraic procedures than graphical interpretations.

**Question 5:**

This question was well done by most candidates. When errors were made, candidates confused the terms "independent" and "mutually exclusive" and did not subtract the intersection when finding $\mathrm{P}(A\cup B)$. Candidates should also be aware of the command term "hence" used in part (c) where they were expected to provide a reason that involved $\mathrm{P}(A\cap B)$ from their work in part (b). It seemed that many turned to the formula in the booklet instead of considering the conceptual meaning of the term.

**Question 6:**

The variation in successful and unsuccessful responses to this question was remarkable. Many candidates did not even attempt it. Candidates could often determine from the graph, the minimum and maximum values of the original function, but few could correctly use the graph to analyze and justify these results. Responses indicated that some candidates did not realize that they were looking at the graph of $f'$ and not the graph of $f$. In part (c), many candidates once more failed to respect the command term "show" and often provided an incomplete answer. Candidates should be encouraged to refer to the number of marks available for a particular part when deciding how much information should be given.

**Question 7:**

Not surprisingly, this question provided the greatest challenge in section A. In part (a), candidates were able to use the identity $\tan x = \frac{\sin x}{\cos x}$, but many could not proceed any further. Part (b) was generally well done by those candidates who attempted it, the major error arising when the negative sign "magically" appeared in the answer. Many candidates could find the value of $\cos x$ but failed to observe that cosine is negative in the given domain.

**Question 8:**

This question was intended to be the most accessible in section B and it did not disappoint. The few errors observed in part (a) were the result of incorrect counting or not writing answers as probabilities. There were many more candidates who were unable to find the conditional probability in (a) (iii) correctly mainly because they reached for a formula in a booklet rather than using an intuitive counting approach.

Part (b) was generally well done. Most candidates could substitute the correct expressions into the formula for expected value but often lost marks for not appreciating that Elena **loses** a positive number of points.

**Question 9:**

Parts (a) and (b) of this question were generally well done. Problems arose in part (c) with many candidates not substituting $s(3)-s(0)$ correctly, leading to only a partially correct final answer. There were also a notable few who were not aware that $\cos 0 = 1$ in both parts (a) and (c). There were a variety of interesting answers about the motion of the particle, few being able to give both parts of the answer correctly.

**Question 10:**

This question was the most difficult on the paper. Where candidates attempted this question, part (a) was answered satisfactorily. Few answered part (b) correctly as most could not interpret the horizontal stretch. As a result, there were many who were unable to answer part (c) although follow through marks were often obtained from incorrect answers in both parts (a) and (b). The link between the answer in (b) and the value of *C* in part (c) was lost on all but the most attentive. In part (d), some candidates could name the transformations required, although only a handful provided the correct order of the transformations to return the graph to its original state.

## The type of assistance and guidance the teachers should provide for future candidates.

One feature of candidate performance was how often candidates reached for a formula instead of thinking through the requirements of a question. Formulas can be helpful when a calculation is required, but for questions that assess conceptual understanding, the formula approach often led candidates away from the goals of the problem.

Candidates seem more comfortable with analytic processes than with graphical interpretations. When preparing candidates for future examinations, emphasizing a graphical understanding in conjunction with analytical techniques may be helpful.

Another surprising outcome was the number of candidates who left their answers unsimplified. Results such as $\frac{30}{0.1}, \frac{0.2}{0.4},$ and $\frac{\ln x}{0.5}$ are surely best written as $30, \frac{1}{2},$ and $2\ln x$. This is very likely the result of candidates not having a calculator to complete their answers. They should be instructed, at the very least, not to leave decimals within fractions.

Teachers should continue to work with students to help them work problems without a calculator. One area that should be emphasized is the value of basic trigonometric functions such as the sine and cosine of zero.

Teachers should stress the meaning of the command terms and have students look at the number of marks allocated to each question part to determine how much "work" they should show.

Teachers should work with students on how to effectively answer questions that require reasoning and how to correctly work a "show" type question. Some students are still working backwards.

# Paper two

### Component grade boundaries

| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range**: | 0 - 14 | 15 - 28 | 29 - 41 | 42 - 52 | 53 - 64 | 65 - 75 | 76 - 90 |

## The areas of the programme and examination that appeared difficult for the candidates

Candidates showed difficulty answering questions on:

- Binomial probability
- Normal probability
- Integration to find areas between curves
- Using the GDC to find areas and volumes of revolution.
- Representing a linear system by a matrix equation

## The areas of the programme and examination in which candidates appeared well prepared

Overall, candidates demonstrated a wide range of knowledge and skill in this paper. Where candidates made attempts, most could earn at least some of the marks associated with each question.

The GDC was not used effectively by some candidates, yet knowing when to choose the GDC is an essential feature of this paper. Where a GDC approach was envisioned, analytic approaches were sometimes chosen, which mired the candidate in unnecessary algebra or made arithmetical errors more likely.

Candidate strengths include the following areas:

- Quadratic graphs
- Cumulative frequency curves and interquartile range
- Vectors
- differentiation

## The strengths and weaknesses of the candidates in the treatment of individual questions

**Question 1 (quadratic graphs)**

Many candidates answered this question with great ease. Still, some found themselves unable to correctly find the vertex algebraically, often mixing the signs of the $h$ and $k$ values. Using the GDC may have been a more fruitful approach. Some candidates did not write the axis of symmetry as an equation.

**Question 2 (binomial expansion)**

Where candidates recognized the binomial nature of the expression, many completed the expansion successfully, although some omitted the negative signs. Few recognized that only the multiplications that achieve an index of 3 are required in part (b) and distributed over the entire expression. Others did not recognize that two terms in the expansion must be combined.

**Question 3 (cumulative frequency curve)**

This question was answered successfully by a majority of candidates. A common error was to use values of 20 and 60 for the lower and upper quartiles. Some were careless when reading the graph scale and wrote incorrect answers as a result.

**Question 4 (curve sketching, volume of revolution)**

Many candidates sketched a clear and smooth freehand curve with the local maximum, $x$-intercept and endpoints in approximately correct positions. Commonly, candidates sketched a graph across $[-3,3]$, which neglects the given domain of the function. There were some candidates who sketched a straight line through the origin, presumably from being in the degree mode of their GDC. A good number of candidates could set up the correct integral expression for volume, but surprisingly few were able to use their GDC to find the correct value. Some attempted to analytically integrate the square of this unusual function, expending valuable time in this effort. A small but significant number of candidates wrote a final answer as $1.88\pi$, which accrued the accuracy penalty.

**Question 5 (Binomial Probability)**

Candidates who recognized binomial probability answered this question very well, using their GDC to perform the final calculations. Some candidates misinterpreted the meaning of “at least two” in part (b), and instead found $\mathrm{P}(X > 2)$. Others wrote down a correct interpretation but accumulated to $"2"$ in their GDC (*e.g.* $binomcdf(7, 0.18, 2)$). Still, the number of candidates who either left this question blank or approached the question without binomial considerations suggests that this topic continues to be neglected in some centres.

**Question 6 (triangle trigonometry)**

A good number of candidates found this question very accessible, although some attempted to use the theorem of Pythagoras to find $\mathrm{AC}$. Often candidates correctly found $\mathrm{B\hat{A}C}$ in part (b), but few added the $30^\circ$ to obtain the required bearing. Some candidates calculated $\mathrm{B\hat{C}A}$, misinterpreting that the question required the course of the second ship.

**Question 7 (normal probability)**

Candidates who clearly understood the nature of normal probability answered this question cleanly. A common misunderstanding was to use the value of $0.8$ as a $z$-score when finding the standard deviation. Many correctly used their GDC to find the probability in part (b). Fewer used some aspect of the symmetry of the curve to find a value for *b*.

**Question 8 (vectors)**

Candidates performed very well in this question, showing a strong ability to work with the algebra and geometry of vectors. Some candidates were unable to find the scalar product in part (c), yet still managed to find the correct angle, able to use the formula in the information booklet without knowing that the scalar product is a part of that formula. Few candidates considered that the area of the parallelogram is twice the area of a triangle, which is conveniently found using $\mathrm{B\hat{A}D}$. In an effort to find $\text{base} \times \text{height}$, many candidates multiplied the magnitudes of $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AD}}$, missing that the height of a parallelogram is perpendicular to a base.

**Question 9 (differentiation, integration and area)**

A good number of candidates demonstrated the ability to apply the product and chain rules to obtain the given derivative. Where candidates recognized that the gradient of the tangent is the derivative, many went on to correctly find the equation of the normal. Few candidates showed the setup of the equation in part (c) before writing their answer from the GDC. Although a good number of candidates correctly expressed the integral to find the area between the curves, surprisingly few found a correct answer. Although this is a GDC paper, some candidates attempted to integrate this function analytically.

**Question 10 (Matrices)**

Many candidates used their GDC effectively to answer parts of this question, although few used their GDC throughout. Finding the inverse matrix of ***A*** was accomplished well, although some candidates attempted some process of reciprocating the determinant, such as is done for a $2 \times 2$ matrix. Finding matrix ***B*** was usually attempted without the GDC, which if done correctly earned full marks. However, these candidates often made some arithmetical error while working with so many elements. Finding det ***B*** can also be accomplished in the GDC, although many calculated this by hand, often with arithmetical errors. The "write down" instruction is meant to communicate that such an analytic process is not required. Most used their GDC to correctly find the solution to the system of equations in part (c), however a significant number of candidates incorrectly wrote $\boldsymbol{X} = \boldsymbol{C}\boldsymbol{B}^{-1}$, unaware of the non-commutativity of matrix multiplication. Few candidates appreciated the link between the matrix equation and the system of linear equations.

## Recommendations and guidance for the teaching of future candidates

Judging by the number of candidates who did not attempt or showed little appreciation for some topics, notably in probability and integration, it is clear more emphasis could be placed in these areas when preparing candidates.

In the GDC paper, it is advisable to teach students to consider the GDC as the primary approach when finding such answers as roots of functions, intersection values, numerical derivatives, areas and volumes, inverse matrices, and maximums and minimums of graphs.

However, the GDC does not satisfy the requirements of “show that” questions, where a full analysis is expected.

When finding answers in the GDC, students should still show the appropriate mathematical setup before writing down answers, such as writing the equation that is being solved for an intersection value, or writing a fully correct integral expression when finding a volume or area. This is not required in “write down” questions.

Few candidates escaped the accuracy penalty in this examination. In a two-year course, it may be helpful to emphasize the three significant figure rule from the onset and throughout the two years. When students get used to this rule in class they may be less likely to neglect it in their final examination.