MATHS SL

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 32	33 - 48	49 - 59	60 - 70	71 - 82	83 - 100

This was the second November examination session based on the revised program for mathematics SL.

Please remember that as announced in Diploma Programme Co-ordinator Notes, March 2006, the format of the examination papers is changing from May 2008. There are no changes to the syllabus or to the internal assessment requirements. Each paper will consist of two sections, each section worth 45 marks. Section A will consist of short questions, section B will consist of long questions. For Paper 1, **no calculators of any kind are allowed.** A graphic display calculator (GDC) will continue to be required for paper 2. This change is to enable the assessment of analytic skills to be more effective. It is not intended to assess arithmetic skills, and only simple arithmetic will be required.

Standard level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 13	14 - 19	20 - 23	24 - 28	29 - 33	34 - 40

The range and suitability of the work submitted

Most schools chose to offer tasks selected from the Teacher Support Material (TSM). Some teachers offered their own tasks, or tasks taken from other resources. The quality of these tasks varied, although many teachers are now taking extra care to ensure that their tasks meet the requirements and address the assessment criteria appropriately. Those teachers who design appropriate tasks are encouraged to share them through the Online Curriculum Centre so that others can offer constructive feedback, or make use of them in their classes. This professional cooperation is much appreciated.

There is still some concern regarding the selection of tasks. Some teachers chose old TSM tasks, some of which do not adequately meet the requirements as outlined in the Subject Guide. Particularly, they do not offer students full opportunity to succeed at every level of each criterion. It is critical that teachers work through any task they intend to set prior to assigning it to students, to ensure that their students can address each of the criteria levels. Otherwise students may be unintentionally penalized, as they might not be able attain the highest levels simply because the task does not provide for this.

Candidate performance against each criterion

Criteria A and B relate primarily to how students communicate their work, and how they use terminology and notation properly in doing so. The portfolio is also a teaching tool, so it is in the interests of all to be strict in assessing these criteria. Things such as the proper use of approximation notation, use of mathematical rather than calculator symbols, proper labelling of graphs, diagrams and tables, and the presence of complete, coherent explanations are important to the presentation of good work. Ideally, students will have a second or third opportunity to improve their work in this regard, and a strict interpretation and assessment right from the beginning will send the correct message.

Criteria C and D have different assessment descriptors for investigative tasks (Type I) and for modelling tasks (Type II). Type I tasks are intended to assess the students' abilities to work with mathematical patterns in numbers, expressions, shapes, etc. and to then generalize these patterns into a suitable mathematical statement. Aspects such as the validation of preliminary conjectures, exploration of scope and limitations of the variables, and some informal explanation as to why the statement is valid are also addressed. Type II tasks are intended to assess the students' abilities to analyze raw data to develop a model function, consider how well the model fits the data and modify it as appropriate, show how it can be applied to other situations, and to critically interpret in context how reasonable the model is, issues of accuracy, what limitations apply, and what modifications might be necessary to improve the model. It is important that students explicitly identify the variables, parameters, and constraints used in the model. Students need to know these things before they work at the tasks, and it is therefore incumbent upon teachers to share and discuss the criteria with them.

Teachers are reminded that the use of regression is not acceptable as the sole method of developing the model function in a type II task. Regression models may be used for comparison, but only after the model has been developed using the appropriate skills that are covered in the syllabus. This analytic work may involve matrix methods for polynomial functions, or systems of exponential functions. A graphical analysis using trial and error along with demonstrated knowledge of function transformation techniques is also acceptable.

Under criterion E, teachers are particularly advised to plan for the appropriate accommodation of technology in the tasks they use. They should consider how students can give evidence of the technology used, and how resourceful the use has been to the development and enhancement of the work presented. The presence of printed output does not in itself constitute resourceful use. It was pleasing to note that some teachers are making excellent use of the technology available, including a variety of graphing software and interactive packages such as Geometer's Sketchpad.

Criterion F offers the teacher an opportunity to assess holistically the quality of work presented. While there is no explicit link between performance on the other criteria and the mark awarded in criterion F, it is expected that only remarkable work, work that the teacher would stop and take admirable note of, should attain a mark of 2. On the other hand, it is expected that only a totally inadequate response would receive a mark of 0.

Recommendations for the teaching of future candidates

Teachers are reminded that the subject guide includes specific instructions regarding the assessment of portfolios, including annotations to the criteria to help explain their application. Further, a document titled "Internal assessment criteria and additional notes", available on the Online Curriculum Centre, provides teachers with additional clarification as to how to properly apply the assessment criteria.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 28	29 - 43	44 - 53	54 - 63	64 - 73	74 - 90

The areas of the programme and examination that appeared difficult for the candidates

- Finding the area under a curve below the x axis.
- Finding the equation of the normal to a curve.
- Sketching the graph of $y = g(-x)$, given that of $y = g(x)$.
- Properties of exponential growth.

The areas of the programme and examination in which candidates appeared well prepared

This varied widely and was partly dependant on the centre.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1 (Descriptive statistics)

The overwhelming majority got this opening question right—except for not always giving the answer to 3 s.f.

Question 2 (Probability)

A large number managed to get between 1 and 3 marks in parts (a) and (b). Very few answered part (c) correctly, with many believing that the union of the three sets contained 106 rather than 76 members. A number of candidates answered “how many students” instead of “find the probability”.

Question 3 (Probability)

Most got the tree diagram right. Some interchanged multiplying and adding in (b).

Question 4 (Area under a curve)

A poorly answered question, generally, except for part (a). Many failed to recognise that the area of B is the absolute value of the integral. Those who used the calculator to compute the (easy) integrals in (b) and (c) often got into trouble.

Question 5 (Geometric sequences)

A question correctly answered by the vast majority of candidates. In (a) some assumed the sequence geometric and worked backwards, which is not acceptable.

Question 6 (Arithmetic sequences)

Part (a) was almost universally correct. However many candidates appeared not to fully understand the sigma notation in parts (b) and (c). In (c) some candidates who did understand the notation recognised that the first term was 63 but still used n=100.

Question 7 (The logarithmic function)

The majority of candidates were successful in part (a). Many sketches appeared to be attempts to reflect in the line $y = x$ but of these a significant number did not pass through (0,1) and/or were not asymptotic to the x-axis.

Question 8 (Equation of the normal)

Most candidates were successful in taking the derivative but many were not able to complete the process. Some did not find $f(1)$, and others did not know or even recognise the need to fine the gradient of the normal.

Question 9 (Rules for differentiation)

A large proportion of candidates showed an adequate to perfect knowledge of the various differentiation rules. Quite a few also revealed their lack of understanding of basic algebra by performing nonsensical “simplifications” on an already correct answer.

Question 10 (Trigonometric graphs)

The asymptotes were correctly drawn by all except a handful of candidates. About half of them got at least 1 mark in part (b). A surprisingly high number tried to answer (c) by solving a trigonometric equation (practically always without success) instead of looking at the graph.

Question 11 (Graph of a quadratic, transformations)

The values of *h* and *k* were almost always correctly found. Relatively few candidates earned the 4 remaining marks in parts (b) and (c), many not even attempting to sketch the graph.

Question 12 (Probability, expected value)

Most candidates got part (a) correct. Some didn’t go any further than (a), but those who did generally got the rest of the question right.

Question 13 (Binomial expansion)

Part (a) was often well done. Only about 10 per cent of candidates got (b) right. Hardly any candidates could see the connection between the two parts.

Question 14 (Exponential growth)

Typically, they got the first A1 in (a) and the first A1 in (b). Only one or two candidates used differentiation (i.e. $\frac{\mathrm{d}A}{\mathrm{d}t}$). Predictably, in (a), many used the conclusion $k = 0.2$ in their “proof”, sometimes arriving at a different value for *k*. About one candidate in four managed to do (b) correctly.

Question 15 (Relation between graphs and derivatives)

In some cases, marks were lost due to poor drawing skills, rather than from lack of understanding of the properties involved.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 43	44 - 54	55 - 64	65 - 75	76 - 90

The areas of the programme and examination that appeared difficult for the candidates

Candidates showed difficulty answering the questions on:

- Binomial probability
- Finding the position vector of the intersection of two lines
- Using the GDC to find a volume of a solid of revolution
- Solving a trigonometric equation on a given domain

Judicious use of the GDC proved challenging for many candidates.

Candidates also showed difficulty extracting appropriate mathematical relationships from a graphical or geometric situation.

Some candidates still do not know when to use radians and when to use degrees in their GDC.

It was disconcerting the number of candidates who were unable to answer the first question without incurring the accuracy penalty.

The areas of the programme and examination in which candidates appeared well prepared

There were a good number of excellent papers where the work was done with clarity and economy. Common topics for which many candidates showed success include:

- Normal probabilities
- Matrix algebra
- Vector equations of lines and the angle between two lines
- Basic differentiation

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Normal distribution, binomial probability and matrices

Part A

Well-prepared candidates clearly understood the requirements of the question and typically used their GDC to obtain correct results. There were some who found correct *z*-scores and left these as their answers. A significant number of candidates inappropriately used their GDC to “show” the probability in part (iv), which required candidates to demonstrate a correct method using their previously obtained values. Generally, “show that” questions should not be analyzed using a calculator. This is stated in the glossary of command terms.

Many candidates left part (b) completely blank, which suggests that binomial probability is in need of attention in many centres. Where candidates considered binomial probability, many completed the question well, although a good number of candidates misinterpreted what is meant by “at least 7” and found $\mathrm{P}(X > 7)$ instead. Some used a complement principle in their working, while others summed the individual probabilities from $\mathrm{P}(X = 7)$ to $\mathrm{P}(X = 10)$. The vast majority of candidates who completed this question used their GDC to find intermediate and final results, which is what was expected in the question.

Part B

The matrix algebra of this question proved to be very accessible with most candidates. Many used their GDC to find the required square and inverse matrices, and the matrix equation was easily solved to find the values of *p* and of *q*. Many candidates also used their GDC to find $\boldsymbol{A}^{-1}\boldsymbol{B}$, however some candidates failed to make the link between this answer and the solution of the matrix equation $\boldsymbol{AX} = \boldsymbol{B}$. Often candidates did not recognize that matrix multiplication is not commutative and evaluated $\boldsymbol{BA}^{-1}$ instead.

Question 2: Vector equations of lines

This vector question proved to be a strength for many candidates. A majority were able to find a vector from given coordinates and correctly write the equation of a line in vector form. Many candidates correctly chose the direction vectors when finding the angle between two lines, although some rounded intermediate values and thus did not obtain a final result accurate to three significant figures. In finding the position vector of D, many candidates equated the parametric equations of the two lines, although some candidates only used a single parameter and thus did not solve an appropriate system when finding a value for *s* or for *t*. A small number of candidates cleverly considered a geometric approach, such as finding the centre of the cuboid, which earned full marks provided that adequate reasoning accompanied their working.

Question 3: Function graphing and calculus

Many candidates provided a clear sketch of the function which included both branches of the curve, although a significant number of candidates continue to rely on their GDC screen without interpreting it. Many did not clearly indicate the vertical asymptote on their sketch or failed to give a scale on the x-axis. The equation of the asymptote was often given correctly. Deciding which area expression was inappropriate did not elude candidates, as many recognized the asymptote as a discontinuity of the function. A good number of candidates could write a completely correct integral expression for finding the volume of the region rotated about the x-axis, however, far fewer candidates could evaluate the expression correctly. Many candidates attempted a fruitless analytic approach, where the "write down" instruction clearly indicates such an approach is unnecessary. Those who thought to use the GDC often neglected to square the function or to include the pi in the calculation. Many were able to differentiate the function correctly, although few answered part (e) with any merit. It was not uncommon for candidates to attempt to find the minimum by setting their derivative to zero rather than using their GDC, as is often expected when given a "write down" instruction.

Question 4: Trigonometry and quadratics

Although many candidates correctly considered the discriminant in determining k, many neglected the negative value when solving their quadratic. Others considered the discriminant, but solved for k using an inequality giving incorrect answers such as $k > 4$ or $k < -4$. Many candidates correctly applied a double-angle formula to "show that" $f(\theta)$ can be quadratic. Answering part (c) proved more elusive as few candidates correctly recognized that there is **one** value of $\cos\theta$ that satisfies the equation, thus missing the link between part (c) and part (a). Even after solving for $\cos\theta = -0.5$, most candidates still gave the number of values for θ. Candidates who chose to graph the function often had no difficulty finding all 4 correct values of θ, whereas those who took an analytic approach to solving the equation rarely found all 4 values in the given domain. Those who graphed the function in part (c) found little problem in answering part (d). Those who took an analytic approach in part (c) often left part (d) blank.

Question 5: Trigonometry, differentiation and optimization

Many candidates were able to apply sector formulas and evaluate trigonometric expressions correctly in this question, although some used degrees where the given angles are in radians. Part (c) proved difficult for most as it required candidates to think beyond the formulas and consider the geometric nature of the question. A number of candidates attempted to use the rule of cosines to find a length opposite an angle of $\frac{2\pi}{3}$. This proved fruitful for some, but many others failed to use this value as the hypotenuse of a right triangle where one side is the required height.

There are a variety of approaches candidates used to find when C first reaches its highest point, but each required some justification of it being a maximum. This could be as simple as indicating the maximum point on a sketch of the function, or a verbal statement that the height is 30 meters at this point. Candidates who found a maximum on their graph were far more successful than those who chose an analytic approach to solving the trigonometric equation.

While finding the derivative in part (e) proved straightforward, many candidates found difficulty sketching a reasonable graph of $h'(t)$. Candidates often neglected to include any range of values on their sketch, sketched what was seen on the GDC screen with the standard window range, or sketched the curve with its maximum point on the *y*-axis. Although sketches need only be approximately correct, they must reasonably reflect the general features of the correct graph. Few candidates recognized that the maximum (or minimum) of the graph of $h'(t)$ served to answer the question of "changing most rapidly." When attempted, most candidates set the second derivative to zero and solved their equation, often correctly.

Recommendations and guidance for the teaching of future candidates

- Proficient use of the GDC was advantageous on this examination, as there were many opportunities for candidates to answer questions using a graphical approach. However, candidates in this session often chose analytic methods when graphical considerations might have proven more efficient. Teaching students to think graphically in combination with the relevant analytic skills will make for a more flexible approach to problem-solving, which may serve candidates well in deciding when to use the GDC and when to take a more analytic approach.

- Give students practice in giving explanations for results and be tough in the marking of such explanations, demanding accuracy and clarity.

- Much more work needs to be done on binomial probability.
- It is important for students to be attentive to the domains of functions. This can be helpful to candidates when deciding what viewing window to use in their GDC, deciding what part or parts of a graph to include in a sketch, choosing degrees or radians appropriately, or recognizing which and/or how many solutions answer a question.
- Make sure that **all** areas of the syllabus are covered if the candidates are going to do well on the examination.