MATHEMATICS SL

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 34	35 - 46	47 - 58	59 - 70	71 - 82	83 - 100

General Comments

This was the first session November of the new course for mathematics SL. In general students seemed to be well prepared. However as detailed below there appeared to be some new areas of the syllabus with which candidates from some centres were unfamiliar. Details are also given below of ways in which the new requirements for Internal Assessment had not been fully implemented. Teachers should be sure they are working from the subject guide for mathematics SL for first examinations in 2006 that was sent to schools in 2004.

All teachers are encouraged to complete G2 examination feedback forms. These are all read by the senior examining team at the Grade Award meeting and consideration is given to issues raised. G2 forms are available from the IB diploma co-ordinator or online on the OCC.

In response to some comments made on G2 forms in this session teachers are asked to note the following points:

- A standard level course can have a maximum of three hours external assessment thus increasing paper one to 1.5 hours means paper two now has to be 1.5 hours.
- The suggested teaching hours provided in the subject guide will not necessarily be precisely reflected in the number of marks allocated to a particular topic in a particular session.
- For paper one, the change in format from boxes with answer spaces to lines is intended to reflect the change in the assessment model. Correct answers with no working may not receive full marks so the answer space has been removed to try to help candidates and to encourage them to show their working in a clear and organized way. Final answers should be written in the lined section and not against the question at the top.

Finally a couple of things that would make things easier for examiners:

- Students are required to write their answers in pen. If pencil is used it can be very difficult to read under artificial light.
- Please do not ask students to double over the green tags. It makes it extremely difficult to open out the papers for marking.

Standard level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 13	14 - 19	20 - 23	24 - 28	29 - 33	34 - 40

The November session schools are coming to terms with the new syllabus and making the adjustment to the new Internal Assessment rubrics. The requirements of the portfolio have changed significantly from the previous syllabus, and most schools have taken note of this. The majority of schools chose tasks from the Teacher Support Material document (TSM), and through these provided good opportunity for candidates to be successful. There are still schools, however, that have not fully adjusted to the changes. Some have submitted old tasks that are no longer appropriate to the criteria, or have missed the intent of the changes in assessment. The assessment of portfolios reflects the changes in the requirements, and schools must take note of these changes to ensure appropriate and consistent assessment, and to allow candidates the opportunity to be fully successful.

The range and suitability of the work submitted

The great majority of schools took a cautious approach and used tasks taken from the TSM. This is a wise choice at this point, given the understandable desire to implement the new IA requirements with success. It is hoped, however, that teachers will feel more confident in the years to come in offering tasks of their own design. In the meantime, it is recommended that teachers review the TSM tasks prior to assigning them and make revisions to best suit the needs and abilities of their candidates, while maintaining the integrity of the task. The only proviso for any task is that it be designed to offer candidates full opportunity to be successful within the new criteria. It is critically important that teachers receive appropriate training, and review the advice given here, on the OCC, and in the feedback to schools, so that they will be able to design and offer suitable tasks.

Teachers used some tasks taken from other resources. While these resources can provide excellent ideas for portfolio tasks, each task must be considered in light of the criteria. Teachers should identify in any task how a student might achieve each level of the assessment criteria. If a task does not allow for success, then it must be revised or rejected.

Tasks that include two or more distinct parts create significant difficulties in assessment. A candidate may achieve success on one part, but score poorly against the assessment criteria on another part. This makes an overall assessment problematic. Tasks submitted for the portfolio must be limited to a single, coherent investigation or modelling problem.

Candidate performance against each criterion

Criterion A

There is still fairly common use of calculator and computer notation. While a single incident may be forgiven, a penalty should generally apply where such incorrect notation is used. Appropriate notation for "approximately equals to" is seldom used. This is very important in situations where accuracy is an issue. Where model functions are developed, the most appropriate choice of variables is those that are representative of the measures used. Candidates generally favoured the traditional x and y, leading at times to multiple use of 'y' for different functions; clearly an inconsistent use of notation.

Criterion B

The issues that arise under this criterion are those same issues that have persisted since the inception of the portfolio. Graphs and tables must be clearly and appropriately labelled. These should appear within the work itself rather than attached as appendices.

The work should not be excessively detailed, nor should it be answered in a question-answer format. This is a piece of mathematical writing, and as such should read more like an essay, including a brief introduction as to the nature of the task, appropriate explanations and annotations. There should be a smooth "flow" to the whole piece. A guideline for assessing communication under the rubric can be that "attempts" means that, in the end, the message was not communicated. However, "adequate" means that, with some effort on the part of the reader, the message is there. "Clear and coherent" is generally obvious.

Criterion C – Type I

Given the proper task, most candidates do a good job of the data creation, organization and analysis that prepares them to make a generalized statement. Thus marks in this criterion are generally 3 or above. Testing the validity of a generalized statement requires that the candidates use their statement to predict a result, then consider the pattern of behaviour to show that the result from this behaviour is the same as that from the statement. Many candidates simply substitute values into their general statements and arrive at an answer that fits. They do not consider whether that answer would have come from the process involved.

Criterion C – Type II

This criterion deals with the mathematical analysis necessary to formulate a model function, and with how suitable the model function is in terms of how it fits the data and how it transfers to other situations. The analysis requires correct and complete identification of the variables involved, along with any constraints and parameters. Many candidates simply fall back on x and y, with little thought as to what these represent.

Certainly the analysis must involve the mathematical skills of the student. While the use of regression features on a calculator, or in computer graphing software, is allowed for comparison, the primary model must come from the candidates' mathematical understanding. If regression is the sole technique for developing a model, then minimal analysis is present and a penalty will apply. Note that tasks that require candidates to work with a given model function do not allow for success in this criterion.

The application to other situations may be laid out in the task itself, or may require research to gather more or different data. While it is not expected that candidates will undertake another full analysis of this new situation, they should identify and make the necessary revisions to their first model.

Criterion D – Type I

There is a distinction between arriving at "a" general statement by making a reasonable analysis of incorrect data, and arriving at "the" correct and intended general statement that is expected from a correct analysis of the correct data. If candidates have not arrived at the correct statement in its proper mathematical form then they cannot score well here.

Criterion D – Type II

The emphasis here is that work be discussed and interpreted "in the context of the task". Discussions of the mathematical properties of the function and its variables do not address the reality of the situation. In order to critically interpret the model, candidates must discuss how it addresses the situation from which it arose.

Criterion E

Candidates have generally improved their ability to effectively use graphing software or graphing calculators. Multiple functions on the same set of axes, or examples of trial functions that fit better as parameters are changed are examples of good use of these graphing features. Critical distinction between scatter plots of data as opposed to the continuous functions that model them is important. Too often candidates use a regression feature to plot a best-fit function on top of the data points, without making clear the possible discrete nature of the process. This was especially evident in the Koch Snowflake task, where continuous functions were assumed throughout, despite the variables taking on only discrete values by the very nature of the process.

Criterion F

In this criterion the work is evaluated in a holistic manner. If the overall product is poor or incomplete, then a mark of 0 is appropriate. Most candidates will be awarded a mark of 1, given that they have reasonably addressed the majority of the assessment levels. Only work that is remarkable in its quality, work that makes one stand up and take notice, should receive a mark of 2.

Recommendations for the teaching of future candidates

There is a wide degree of variation in how teachers approach the use of notation. There are some who allow a few errors and some who are very strict or very tolerant. The intention of this criterion is to encourage candidates to use proper **mathematical** notation and terminology. Given the time available, there should be no problem in a candidate researching the appropriate notation or terminology, nor in proofreading the work to check for errors.

Candidates should treat the solution to a task much as an essay in mathematics. That is, there should be a structure to the work presented and a flow that allows the reader to follow the work without difficulty, and without reference to the task itself. Graphs must be labelled appropriately and placed in context, not added on as appendices. Explanations must be clear without being overly detailed. A list of calculator operations is not useful communication.

In both types of task there is an expectation that the candidate is using the mathematics they have learned to perform a suitable analysis on the information provided or generated. The use of regression techniques is allowed to compare functions, and the interpretation necessary for the task should be based upon the candidate's function, not the "best-fit" function found through regression.

The use of technology will always be dependent upon the minimum level of the technology available to the class. However, teachers and candidates must think carefully about how any use truly enhances the development of the task.

Teachers and candidates must better inform themselves of the assessment criteria and the meaning of each level in the context of the task at hand. It is the teacher's responsibility to share their interpretation of the assessment levels with students. The teacher should seek out the proper training and advice on this interpretation. This can be had through this report, additional notes on moderation available on the OCC, and at IB teacher training workshops.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 31	32 - 42	43 - 54	55 - 65	66 - 77	78 - 90

General comments

G2 summaries

- **Comparison with last year's paper**

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
0	6	20	16	0

- **Suitability of question paper:**

	Too easy	Appropriate	Too Difficult
Level of difficulty	3	63	1

	Poor	Satisfactory	Good
Syllabus coverage	2	22	43
Clarity of wording	1	17	49
Presentation of paper	0	20	47

The areas of the programme and examination that appeared difficult for the candidates

Recognizing the need for the double angle formula and then solving the trigonometric equation proved difficult for many candidates. Extended-problem solving strategies were lacking, particularly in the use of the normal distribution (Question 14) and the volume of revolution (Question 15).

The areas of the programme and examination in which candidates appeared well prepared

Throughout the paper, candidates demonstrated a good level of skill carrying out standard procedures particularly in basic matrix calculations, simple probability, quadratic graphs, vector equation of a line, logarithms, and composite functions.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Matrices

Problems most often occurred in the final simplification of part (b). A few candidates squared matrix $\boldsymbol{A}$ rather than multiplying by 2.

Question 2: Logarithms

Some students substituted p and q before using the "log rules".

Question 3: Box and whisker plot

This seemed to be an all or nothing question. Most candidates who were familiar with box plots found it straightforward. Some did not appear to understand interquartile range and how it related to the box plot.

Question 4: Graphs of first and second derivatives

Overall, this problem was done quite well.

Question 5: Probability

Part (b) caused the most difficulty. Some candidates assumed incorrectly that the events were mutually exclusive and so omitted to subtract the intersection.

Question 6: Parabola

Some candidates simply guessed at the value of d or made an incomplete argument based on transformations. They did not seem to realize that they must substitute to get the value.

Question 7: Vectors

On the whole, this question was well done. The most common mistake occurred in part (b), where the incorrect direction vector was used.

Question 8: Exponential equations

Although many took the long way around, the question was well done. In some cases, the *a* was replaced by 10.

Question 9: Composite functions

Many errors occurred here, such as finding $g(h(x))$, replacing x with g(x) in the numerator but not in the denominator, not setting the numerator equal to 0, and giving an extra answer of $x = 2$ which came from setting the denominator equal to 0.

Question 10: Integration to find displacement

The major problem was the integration itself. A few candidates did not realize that integration was involved and simply substituted the values into the velocity equation.

Question 11: Vectors and scalar product

Part (a) was done quite well. Many students used the whole formula for finding the angle between two vectors. In part (b) some candidates did not use the scalar product but instead used the concept of negative reciprocal slopes.

Question 12: Binomial distribution

Candidates who realized that the binomial distribution was needed did very well. Those who didn't realize this struggled with tree diagrams or other diagrams and generally did not get far.

Question 13: Trigonometric identities

Very few candidates knew to use the trig identity to find part (a). Solving the equation proved difficult. In many cases not all solutions were given.

Question 14: Normal distribution

This was a difficult question for most candidates. Many struggled to set up the two equations, particularly the one involving the 8 and 90%. Arithmetic errors occurred quite often in the solving of the system of equations.

Question 15: Volume of revolution

This was done quite well. The most frequent errors were not squaring the function or doing so incorrectly.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 30	31 - 40	41 - 50	51 - 61	62 - 71	72 - 90

General comments

G2 Summaries

- **Comparison with last year's paper**

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
0	4	15	14	2

- **Suitability of question paper:**

	Too easy	Appropriate	Too Difficult
Level of difficulty	0	54	4
	Poor	**Satisfactory**	**Good**
Syllabus coverage	3	21	34
Clarity of wording	3	18	37
Presentation of paper	0	13	45

The areas of the programme and examination that appeared difficult for the candidates

- Few candidates were able to complete a probability distribution table and find the expected value.
- There was confusion in attempting to write three linear equations as a matrix equation.
- Using integration to find areas between curves also proved difficult.
- Providing the sample space appeared to be a source of difficulty for many candidates.
- Very few candidates were able to find the range of values of k for which $f(x) = k$ has two solutions. Many thought this had something to do with the discriminant even though it was not a quadratic equation.
- Candidates continue to have difficulty with "show that" questions.
- They also have difficulty with unstructured questions where they need to decide on a suitable strategy. (See especially Q4(e) and Q5(d))

The areas of the programme and examination in which candidates appeared well prepared

Candidates showed proficiency at working with linear equations, the cumulative frequency curve and applying differential calculus concepts. Candidates demonstrated skill in using the GDC to find intercepts, maximum point, point of inflexion and the graph of a function. Most candidates showed their working in a clear and organized way.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Cumulative frequency and Probability

Part A

This was answered very well. In part (a), some candidates lost some marks because they failed to show any working. Some had difficulty in finding the value of k in part (b); they found the value for *k* which went with 40% of the people spending **less** than *k* minutes rather than **longer** than *k* minutes.

Part B

This proved difficult for candidates as a large number wrote down the number of elements in the sample space rather than the sample space itself. Some candidates also wrote the sums of scores as their sample space. They were able to compute the probability that two scores of 4 were obtained. Few students were able to complete the probability distribution table correctly. Some earned follow-through marks using incorrect probability distribution values to calculate the expected value.

Question 2: Function and Matrices

Most of the candidates were able to do parts (a), (b) and (c) correctly. A significant number had difficulty writing a correct matrix equation, often writing the matrices in reverse order or interchanging rows and columns. However, many were able to find the solutions to the system. The final part (e) of the question was challenging to the majority of candidates and they showed a variety of different approaches when answers to part d(ii) were achieved.

Question 3: Arithmetic Sequence

The vast majority of candidates correctly did part (a); some students summed $1+2+....+\ 20$ to obtain 210 cans. Many candidates continued on to find that 80 cans were needed in the bottom row in part (b). However, a good number of the weaker candidates were unable to make the necessary connections between the problem context and the concepts in arithmetic sequences. Candidates found part (c)(i) very difficult and many left it blank and others did not clearly arrive at the answer given. The final part (c)(ii) was not attempted by many candidates. Often those who were able to show that the equation in (c)(i) was correct were unable to do (ii) and explain why 2100 cans could not be organized in a pile. The candidates were expected say that there were no integer solutions to the equation obtained and that an integer was needed for the number of cans in the bottom row. Instead of the word “integer”, candidates used terms such us “ exact answer” , “number without comma” , etc.

Question 4: Calculus

The graphing was well performed although some graphs did not have any indications of scale on the axes. Students often did not consider the domain restriction on the problem, but many provided a good curve with the required labels. For part (b)(i), most candidates were able to use the product rule for the first derivative. However, many of them went to their GDC to find the coordinates of the maximum instead of finding the exact value as requested. Part (c) was poorly done with very few candidates realizing the connection between the equation $f(x) = k$, and the work they had previously done. There were many good responses to part (d) and these most commonly used the fact that “$f''(x)=0$ has only one solution, at $x=1.5$”. Others correctly graphed the second derivative and showed that it had only one *x*-intercept. There were few candidates who answered part (e) well, often the line [PR] was neglected entirely. In some cases, candidates tried to employ an analytic method of integration when the problem requires using the GDC to obtain a result.

Question 5: Trigonometry

In part (a) students were generally fine in explaining why the triangle was isosceles and they correctly set up the cosine rule. In part (a)(iii) some candidates made the error of assuming a right angle. Finding the area was generally well done. However, in some cases students included $\sin\frac{\sqrt{80}}{9}$ in their calculations rather than recognizing that the $\frac{\sqrt{80}}{9}$ is the sine of the angle. In part (b) there was considerable success with finding both the size of the angle and the area of the sector , although some candidates used the degree measure in the radian formula. It was pleasing to see a good number of candidates finding angle QOP in part (c). Very few candidates were able to correctly find the area of the shaded region and most did not know where to begin.

Recommendations and guidance for the teaching of future candidates

- Candidates need to have more practice on when and how to use the GDC and when analytic approaches are called of. If a graph is used to find the solution to an equation or a maximum or minimum, a sketch of the graph must be included. If exact answers are required, the GDC should not be used.
- Candidates need practice in giving explanations for results and in justifying their answers.
- Students need practice with questions that request they "Show that…" something is true. Each step must be shown, so generally, these types of questions cannot be done with the GDC. It is also important that students do not simply verify that the answer is correct by working in reverse.
- Candidates need to have more practice with basic algebra skills so that they can find and complete solutions correctly.
- Students must avoid premature rounding of answers which leads to inaccuracy in the final result.
- Candidates need to consider when it is appropriate to use degrees and when it is appropriate to use radians.
- Candidates need to use the given domain when they draw any graph.
- Students need more practice with longer, unstructured, questions where several different parts of the syllabus may be linked together.

