

May 2017 subject reports

Mathematics SL – Time Zone 2

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2017 examination session the IB has produced time zone variants of Mathematics SL papers.

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0–18	19–37	38–50	51–61	62–72	73–82	83–100

Standard level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–2	3–5	6–8	9–11	12–14	15–17	18–20

The range and suitability of the work submitted

It is noted that the exploration has allowed candidates to make connections between mathematics and different subjects of the curriculum, be it the candidates other subjects or in a few cases TOK or CAS. The range of topics chosen continues to be interesting and demonstrates the wide use mathematics beyond the discipline itself. Candidates are collecting their own data, researching independently, conducting experiments, running simulations – all of which represents the true joy of learning.

There continue to be many explorations related to areas that personally interest the candidates and this is encouraging. Many of these were based on sports, computer and card games, music and arts. These real-life problems, using data generated by the candidate, help show personal engagement.

Regression continues to be a common area that frequently lacked understanding by using only technology generated models without even justifying the choice of model. Other explorations involved modelling the path of travel of an object that were usually not up to the level of good understanding or demonstrating mathematics of a suitable level. Some explorations were based on physics which did not allow much understanding to be demonstrated as they were often based on formulae that were just quoted and had values substituted in. As ever, common textbook problems or examples that are easy to find online but were not generally extended or personalized in any way by the candidate were evident.

A few explorations only used topics taken from previous knowledge and equally very few used mathematics at a level higher than the course although there were still some that did.

Some schools obviously coached their candidates to follow a particular format, sometimes producing near identical modelling style explorations. Schools are strongly discouraged from this approach. In some schools where modelling was encouraged strongly, candidates would choose a model without considering the nature of the data; either they started with one polynomial function in mind and never considered anything else, or they tried many regression models and chose one based on an R^2 value thus not scoring well in personal engagement, reflection, or use of mathematics.

Candidate performance against each criterion

Criterion A

Candidates generally do well in this criterion. The majority reached at least level 2 although level 4 has proved hard to achieve, often due to a lack of conciseness. Communication was well understood as many candidates started with a suitable introduction and a plan with an aim that was answered in a conclusion with clear mathematical flow in between. In order for this to be true it is imperative that the aim is clearly stated.

The higher attainment levels are distinguished by the quality of coherence. Coherence issues were occasionally a problem where some steps in mathematical calculations were left unclear to the reader. Repetitive calculations that affected the conciseness of the paper were also evident.

Criterion B

Candidates generally select appropriate mathematical presentations leading to at least level 2 in this criterion. A good standard of technology was demonstrated in producing graphs and equations. However, it is important that all symbols are clearly defined. Some other common issues are poor or missing labels on graphs and the lack of use of approximation sign. Inappropriate notations, like “*” for multiplication and “E” for power of ten are still used in many explorations. The same variable is written inconsistently both as capital and small letters.

Having said this, in general the majority of candidates are doing an adequate job typing mathematical expressions with correct notation.

Criterion C

This is the area in which there was most inconsistency due to the varied expectations of teachers. There are still many teachers who award levels 3 or 4 without much evidence in the paper itself of the personal engagement. Just being interested in the topic does not warrant a 3 or 4 although clearly it is a contributing factor. Common textbook topics still do not show the expected personal engagement and should be discouraged unless an interesting extension or perspective is added to it. In addition, candidates often chose topics that would self-limit the amount of personal engagement possible. For example, there were a number of statistics tasks correlating two sets of data (e.g. GDP and another variable). It is hard to demonstrate much personal engagement in topics like these unless candidates collect their own primary data.

More candidates seem to be making explicit connections with other DP courses (business management, environmental systems and societies, economics) which demonstrated some personal engagement.

Criterion D

Most candidates could not critically reflect on their work. There was a large number of fairly descriptive explorations that did not actually focus on what the mathematics itself was revealing or the problems behind the data collection itself.

There was also some success in providing reflection throughout the exploration although the conclusion itself in many cases tended to be fairly superficial. Simply stating results without considering validity, strengths, weaknesses, alternative mathematical approaches and limitations was still common across samples. In short, many did not consider the implications of their results.

Reflection over the appropriate degree of accuracy, given the context of the work, is often neglected by candidates

Criterion E

It was notable that in many explorations the mathematics explored was either part of the syllabus or in some cases beyond. Level 6 remains hard to attain, mostly due to a lack of demonstrating thorough understanding. There was still an issue with regression analysis being conducted using technology only without demonstrating understanding or justifying the chosen

model; candidates did not explain why certain functions were chosen and they could not interpret the results adequately. Some candidates limited themselves to level 2 because they used only very simple mathematics.

Recommendations for the teaching of future candidates

Internal Assessments need to be discussed alongside the curriculum, and not be treated in isolation. It was good to observe that in many explorations, candidates explored their interest in different subjects and this should be encouraged rather than setting a template for what candidates should do.

When teaching topics whether functions, calculus or statistics etc. candidates should be exposed to possible explorations. Equally early interaction with the criteria is important. Thus mini-explorations and assignments before the exploration can show the candidates what is needed and how to earn the higher marks. This can then be combined with old explorations so that they get a better understanding of these criteria.

Candidates should be guided on how to select an exploration that provides opportunity to employ mathematics that is commensurate with the level of the course. It is helpful to be realistic about what topics, outside of the curriculum, different candidates might be able to cope with. Knowing the abilities of one's candidates is useful in guiding them to a suitable exploration that both interests them, and will allow them to access the higher attainment levels on criterion E.

Candidates should be encouraged to use equation editors whenever possible to ensure correct mathematical notation. Ensure candidates check for notation mistakes.

Teachers should be more explicit in explaining the use of accuracy or approximation in their mathematical teaching so that this is not overlooked in the exploration. Similarly, the correct use of the approximate sign when given rounded values, consistent use of mathematical notation, labelling graphs and the defining of variables should be demonstrated by the teacher.

Further comments

A list of URLs is not an adequate bibliography, and yet some candidates persist in only including an unordered list of those. Also, URLs are required, according to p. 14 of the document "Effective citing and referencing" and not all candidates are including them on internet sources. Sources of images and information must be cited at the point in the paper where they appear. While a bibliography is also important, its presence does not remove the requirement for in-text citations.

The Teacher Support Material states two of the responsibilities of the teacher are

- To verify the accuracy of all calculations,
- To assess the work accurately, annotating it appropriately to indicate where achievement levels have been awarded.

It is essential that the teacher indicate where calculations have been found to be both incorrect and correct in the candidate work. It is also essential that the work be marked up to indicate where the teacher has seen features that led to the criteria levels they ultimately awarded.

Teachers should be encouraged to write comments within the exploration as it allows clarity in marking. It is essential that annotations are included on the candidate work that show why and where a level has been awarded. Teachers are advised to check all documents prior to upload to ensure all pages are present and oriented correctly, and any comment boxes added electronically are expanded and not blocking any text. Examiners will only see a static image of the work and cannot expand or move comment boxes.

Some schools have done an excellent job removing candidate details from the work. However, a few schools are still using the old 5/EXCS form, and many more have candidate names, school names, candidate numbers, and so on in the text. Teachers should take note that it is expected that candidate work be appropriately anonymized and this should be emphasized for the next examination session

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–19	20–38	39–50	51–58	59–67	68–75	76–90

The areas of the programme and examination which appeared difficult for the candidates

- Integration using substitution or inspection
- Vector geometry
- Differentiation using the product rule
- Interpreting the area under and between curves; area and definite integrals
- Solving trigonometric equations with solutions in a given domain
- Interpreting the vector equation of a line
- Using correct mathematical notation

The areas of the programme and examination in which candidates appeared well prepared

- Using formulas for arithmetic sequences and series
- Reasoning involving simple patterns
- Rules of logarithms
- Vector addition, scalar product, and angle between vectors
- Normal curve, using area and symmetry to find probabilities
- Simple operations with vectors
- Properties of logarithms

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Arithmetic sequence

This question was answered correctly by the large majority of candidates. There were occasional arithmetic errors, and a few candidates treated the sequence as geometric, rather than arithmetic, as was stated in the question.

Question 2: Vectors

Most candidates performed well on this question. A common incorrect answer in part (a) was $k = 2$, though candidates who obtained this result usually earned full follow-through marks in part (b). There were some candidates who did not recognize the scalar product of perpendicular vectors must be equal to zero, and some who attempted to use the cosine formula without realizing the cosine of 90° is zero. It is unfortunate to observe that there were some candidates who seemed not to have been exposed to vectors at all during the course.

Question 3: Normal curve

Although this question was generally very well done, it was surprising to note that some candidates left it blank, and many candidates seemed unfamiliar with the symmetry of the normal curve. There were a few candidates who had trouble interpreting the inequality symbols, giving an answer of 0.76 rather than 0.24 in part (a), for example.

Question 4: Recognizing and generalizing a pattern

The majority of candidates were successful in answering this question. In part (a), there were a number of candidates who confused p and q , perhaps not reading the question carefully. In part (c), some candidates tried to write a formula in terms of p and q , rather than writing it in terms of n .

Question 5: Integration using substitution or inspection

This was the most challenging question in Section A for the large majority of candidates, with many earning only one or two method marks in this question. Although most candidates knew to integrate $f'(x)$, very few did so correctly. The most common errors involved attempts to use natural log functions, rather than applying the power rule for integrals. However, even with the plethora of integration errors, most candidates did attempt to substitute the initial conditions in an attempt to solve for the unknown constant.

Question 6: Interpreting values of a function, product rule for differentiation

Nearly all candidates answered part (a) of this question correctly, but the large majority were not successful in part (b), with only a few attempting to use the product rule. The most common error in part (b) was to simply multiply $f'(8) \times g'(8)$, leading to an incorrect answer of -15 .

Question 7: Properties of logarithms, solving trigonometric equation

A number of candidates were able to make a good start on this question, correctly applying rules of logarithms and recognizing the double angle formula to simplify the given equation. However, solving the resulting trigonometric equation proved to be more difficult for candidates.

Even those candidates who recognized $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ were rarely able to find both of the correct answers within the given domain.

Question 8: Cumulative frequency curve

Candidates were generally successful answering all parts of the question, with the majority earning most, if not all, of the available marks. In part (a)(i), there were some candidates who did not interpret the median correctly, giving incorrect answers of 80 or 35. In part (c), while most candidates recognized that 25 hours must be worked, many did not give the correct number of 18 employees.

Question 9: Vectors

The topic of vectors continues to be a challenging one for many candidates, and this question was no exception. Most candidates answered part (a) correctly, either by recognizing the coordinates of A from the given vector equation of the line, or by substituting $t = 0$ into the equation of the line. In part (b), many candidates found \vec{OB} rather than \vec{AB} , though most were able to correctly find the magnitude of their vector. In part (c), most candidates appropriately chose to use the cosine formula for vectors, though some did not recognize the need to use the vectors \vec{AB} and \vec{AC} , and therefore did not obtain the correct value of $\cos \hat{BAC}$. A few candidates were successful in using the cosine rule for triangles in this part of the question. Most of the candidates who attempted part (d) seemed to recognize that P_1 and P_2 were the points B and C, though fewer were able to correctly find the coordinates of these points or the vector \vec{BC} . Some of the candidates who found the correct vector \vec{BC} stopped there, rather than finding the magnitude of this vector.

Question 10: Patterns in derivatives

While nearly all candidates found the correct derivative in part (a)(i) of this question, fewer recognized the need to substitute k into their derivative to answer part (a)(ii). While some candidates were successful in part (b), many were not able to show the desired result. Common errors were to simply assume B was halfway between the origin and C, or to work backward from the given x -coordinate. In part (c), a good number of candidates tried to substitute the lengths into the formula for the area of a triangle, but many used $-\frac{k}{2}$ for BC, obtaining a negative answer for the area. Part (d) was extremely challenging for candidates, with very few able to find a correct expression for R . Only a few were able to recognize the simplest solution, which was to integrate the function from $-k$ to 0, then subtract the area of the triangle they had found in part (c).

Recommendations and guidance for the teaching of future candidates

It is important for teachers to encourage their candidates to write their solutions in a neat, organized manner, using proper mathematical notation. We too often see candidates who are confused by their own working, either because of improper notation such as missing brackets, or because the work is not written in a way that is easy to follow. Candidates should also indicate which part of a question they are answering, rather than writing work, seemingly at random, all over the page. It is also very helpful for candidates to be given an opportunity to practise working with past IB examinations, in order to be familiar with the style and structure of the questions. Teachers and candidates should be aware of ideas like “method marks” and “follow-through marks”; it is troubling to see candidates quit working on a whole question after the first part, as there are still marks they may be able to gain in later question parts. Candidates should also practise answering non-routine questions, where they are required to do more than substitute values into given formulas.

As always, it is necessary for both teachers and candidates to be familiar with the whole of the Mathematics SL guide, including the notation list, and it is important for candidates to be exposed to the entire syllabus. For example, in this paper, as has been the case in past papers, it remains clear that many candidates have had little or no exposure to topics such as vectors or integration using substitution.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–17	18–35	36–45	46–54	55–63	64–72	73–90

The areas of the programme and examination which appeared difficult for the candidates

- Finding navigational bearings
- Manipulation of the inequality sign when solving an inequality that involves negative numbers or logarithms
- Sketching a graph over the correct domain, with correct features such as endpoints and relative extrema
- Recognizing the period of a trigonometric function
- Binomial distribution
- Interpreting movement and position of a particle using distance or displacement in kinematic problems
- The concept of rate of change
- Recognizing and finding compound or conditional probabilities
- Using the GDC to find a volume of revolution

The areas of the programme and examination in which candidates appeared well prepared

- Arc length and sector area
- The use of a GDC to find intercepts and sketch functions
- The trigonometry in non-right angled triangles
- Using sine and cosine rules
- Linear regression with the use of their GDC
- Finding the common ratio and general term of a geometric sequence given the first two terms
- Expected value

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Arc Length and Sector Area

As expected, this question was accessible for most candidates, though a few tried to use degrees. Other candidates could not answer part (b), which was not solved directly from a formula.

Question 2: Linear regression

Most candidates could answer this question correctly, recognizing that it involved linear regression.

However, there were candidates who tried to find the values of a and b by forming a set of simultaneous equations using points from the given table. It was observed that some candidates used their TI calculators incorrectly, using “L2” as the frequency list.

Part (b) was generally well done, with many candidates earning follow through marks. Rounding the final answer in part (b) was not correctly done by some candidates.

Question 3: Transformations, domain and range

In part (a), not all the candidates seemed familiar with the concept of range and some lost marks for not including the end points. Others struggled to use a correct notation when writing the domain and range.

Part (b) was generally well done, although some candidates translated the graph to the left or down.

Part (c) proved easy, earning follow through marks for those who had solved part (b) incorrectly.

Question 4: Trigonometric function

Most candidates had some success with this question, although there were some blank responses.

In part (b) some candidates could not recognize the period of the function. Many found this part difficult due to the need to use inverse cosine before dividing by 7.

The correct use of technology was an issue for some in part (c). Some candidates were in degree mode rather than radian. Others did not correctly group the symbols, for example entering $2.2\cos\left(\frac{\pi}{7}\right)10+7.5$ rather than $2.2\cos\left(\frac{\pi}{7}\times 10\right)+7.5$.

Question 5: Geometric sequence

Most candidates were able to find the common ratio and the expression for the general term of the sequence. Many candidates who were able to answer correctly often did not take advantage of technology to solve their equation or inequality, but rather solved analytically using logarithms. This method proved quite challenging and time consuming, as the manipulation of the inequalities required careful attention.

Question 6: Composite function and graph sketching

Candidates were generally successful with part (a) of this question. In part (b) most of the candidates lost one mark as the point at $x = 2.25$ was inaccurate.

Part (c) seemed challenging. Only a small number of candidates interpreted the question correctly. The great majority tried to use the discriminant to find the values of k .

Question 7: Linear motion

Candidates struggled with this question. Only the most capable could obtain full marks. Although most recognized they had to integrate, the attempts of the vast majority only earned the two first method marks. A significant number of candidates failed to recognize that two integrals were required, and most added the results of the two integrals. Many candidates struggled to recognize the difference between displacement and distance. Able candidates earned the first 4 marks without difficulty but did not appreciate the subtlety of the last part of the question. Only the occasional candidate managed to explain the answer successfully.

Question 8: Analysis of a function and volume of revolution

Candidates generally did not encounter any problems in parts (a) and (b) to find the value of p and the coordinates of point A. Some tried to find the rate of change analytically, getting a decimal number and losing that mark. Some candidates did not seem to know what a rate of change was.

In part (c) most candidates could find the x -coordinate of B correctly using the second derivative but many incorrectly obtained a y -coordinate of 6 for point B.

In part (d) the majority knew that they had to integrate but many either did not square the function or had incorrect limits or made mistakes in using their GDC.

Question 9: Trigonometry

Surprisingly, not many candidates understood what was meant by bearing, giving an answer of 79° for part (a). In part (b) they generally earned full marks, understanding that the sine rule was needed. Most were also able to identify the need to use the cosine rule in part (c), though some candidates had their GDC in radian mode, losing the final marks in all the subparts.

In part (d) the majority understood that BE was perpendicular to AC but they assumed that the triangle ACE was isosceles, thus dividing AC into 2 equal parts.

Question 10: Probability distribution and Binomial probability

Part (a) was well handled by most candidates, who were able to find the values of p , q and r correctly. However, it was surprising to see that many candidates who could find the constants for the function g , could not correctly write the function using the constants. In particular, candidates did not use correct brackets of the horizontal shift.

In part (b)(i) not all the candidates could explain that the probability of drawing 3 white marbles was the same as that of drawing 0 blue. Part (b)(iii) seemed difficult for most candidates, and only a few showed a correct equation to find $w = 6$. However, many correct answers were seen with no working at all.

Part (c) was usually correctly answered by the majority, realizing that a Binomial probability was needed. Part (d) proved challenging for most candidates: not many valid approaches were offered as solutions to this subpart; often as they did not recognize one prize had to be obtained in the first 7 attempts. Many found the probability of winning 2 prizes in 8 attempts.

Recommendations and guidance for the teaching of future candidates

- It is very important to emphasize the need of presenting their work neatly and to clearly label each question part that they answer.
- It should be emphasized to candidates that they should avoid premature rounding of answers in intermediate steps and instead work with values of at least 4 significant figures (or preferably more) in their working. Working with less figures may result in inaccurate final answers.
- Teachers should stress on the importance of checking the mode of their calculators to determine if they are using radians or degrees when working with angles and trigonometric functions. Candidates are required to know that they may need to switch from one to the other during the exam.
- Graphs should be based on the given domain, be neat and contain all the essential attributes.
- Candidates should be sure to take advantage of the technology available during the paper 2 examination. Most candidates demonstrated that they could successfully use a calculator to find the points of intersection they were directed to find in question 4(a).

In questions 6(b) and 7(b), when candidates could use the same techniques to solve the equations on a calculator, many attempted to find the solution algebraically, rather than with technology. Some of those candidates were successful and others were not. All candidates could have used their time more efficiently, if they would realize that they could directly solve on the calculator.

- Teachers should ensure that their candidates can decide when their calculators are needed.
- More emphasis should be given to linear motion problems.
- Teachers should provide examples and practise questions that will help candidates analyse the difference between distance and displacement.
- Encourage candidates to explain their reasoning and to relate their answer to the context.
- Encourage candidates to sketch graphs to aid in their explanations and show understanding.
- Practise writing and using inequalities.
- Understanding that “write down” means that little or no work should be required.