MATHEMATICS SL TZ2

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2016 examination session the IB has produced time zone variants of Mathematics SL papers.

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 34	35 - 46	47 - 58	59 - 69	70 - 80	81 - 100

There were three questions on the papers where there was ambiguity around the domain of a given function. Paper 1, question 6 gave an incorrect domain which allowed a second possible solution. In paper 2, questions 3 and 9, the wording was not ideal, as the domains were included in the introduction to the question, rather than in a later subpart where they were needed. In all cases examiners were given instructions to ensure candidates were not disadvantaged by this, and the questions were amended for publication.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 17	18 - 20

The range and suitability of the work submitted

There were a wide variety of explorations with a wide range of quality this session and most clearly related to areas that interested the students. As in the past, certain themes appeared repeatedly — sports, games (board games, video games, gambling), the golden ratio, codebreaking, movies, music. Some were proper explorations and others were questionable in the sense that one could tell from the original topic choice that little mathematics would be involved. Many explorations involved regression although the understanding differed greatly. A lot of these simply used technology to generate the regression equations and graphs but did not go further to demonstrate understanding or to suggest why a particular regression equation was chosen. Others attempted to use chi-squared testing, and were generally not successful, as neither the student nor the teachers seemed to understand the process fully. A number of explorations involved throwing objects or kicking balls and modeling and analyzing the paths of travel. These were usually simplistic models but they often satisfied the requirements and could still score well. The worst cases were explorations that became simple surveys of information and/or a summary of facts about a topic. Students often had a hard time showing much personal engagement or demonstration of understanding. These included logic problems, game theory and topics such as the Golden ratio, the Fibonacci numbers, the origin of e and card counting in blackjack.

A number of explorations from individual schools were very similar to each other. This could be the same topic, the same format of work but with different numbers or following the same process throughout the exploration as though the students had been provided with a template. It was evident in many cases that the teacher may have overly guided the students rather than allowing them to explore their interests.

In addition some students tackled challenging mathematics more suited to higher level. Whether they have actually understood what they have written about was not always clear.

Candidate performance against each criterion

Criterion A

Most work was organized to some degree, with a relevant introduction, a rationale, an aim and some kind of conclusion, but coherence issues were occasionally a problem. Some students would make assumptions about the reader and leave many steps unexplained. Aims were often vague, for example ‘I aim to find out more about…’ and this made the exploration confusing at times since the conclusion could not be linked back to the aim. Most candidates recognize the need for citation and reference lists but there is still a problem with images and data not being cited where they occur in the paper (this is not an issue that is penalized in this criterion but it does speak to organization and is a requirement of all explorations). Weaker students have difficulty writing papers that qualify as complete; stronger ones have more trouble making their explorations concise. There is a balance between those two descriptors which is clearly challenging for students at this level. For instance, it is unnecessary to repeat lengthy calculations. It is inappropriate, in a mathematical exploration, to provide “how to” guides on using specific pieces of software.

Criterion B

This criterion was generally well understood by teachers and students. Most students were able to select appropriate mathematical representations and used terminology in an appropriate manner. A good standard of computer technology was demonstrated here in producing graphs and equations. Some common issues were poor or missing labels on graphs, terms left undefined, and use of calculator notation in the text. A few students put a good deal of effort into phrasing their work in such a way as to avoid having to include mathematical notation. This is not conducive to scoring well in mathematical presentation.

Criterion C

Many students made a point of mentioning a personal interest in their topic. Sometimes this was all the personal engagement present. The fact that a student is interested in the general topic of the paper does not mean that he or she will necessarily score highly in this criterion. Evidence of further engagement must be present in the paper itself. A common “investigation/textbook problem” is unlikely to achieve the higher levels on criterion C unless the students extends this further by asking themselves ‘What if...’ Simply choosing a topic that is above the level of the course does not, by itself, qualify as outstanding personal engagement, although learning new mathematics is one aspect that can contribute to this criterion. In some modelling and statistical explorations obvious opportunities for displaying greater independence, creativity and or personal/interest by designing and collecting their own primary data, rather than using standard secondary data sets, were overlooked.

Criterion D

Some students made an excellent effort to reflect regularly on results as they appeared and what each new set of calculations or derivation means in the context of the aims. The best papers use that reflection to guide the next steps taken in the analysis. Many left reflections to the end and even included a sub-heading for this. Reflection should be more than just restating the conclusions found. These conclusions were often limited or superficial. Rather than just summarizing results students can also consider limitations and possible extensions of their investigations, relative strengths and weaknesses of approaches taken, and alternative perspectives on the topic. Critical reflections included discussions of specific mathematical results in the context of the topic.

Criterion E

Some students presented topics that were clearly not commensurate with mathematics SLand were taken from the prior learning topics, or chose topics that would lead to difficult mathematics beyond their level of understanding. Regression models were often treated with technology and showed little understanding as it was not clear why a particular regression model was chosen or was appropriate. Another common shortcoming is that complicated formulas were used and applied but without any evidence to support student understanding of how and why these formulas actually worked. The top level of 6 is still difficult to reach, and teachers have sometimes awarding this level to work in which the student has significant errors which have apparently gone undetected. This is a concern. It is surely difficult to thoroughly check the explorations for mathematical errors when every student has a different topic, but it is nevertheless important that teachers make a good faith effort to do so. One of the key differentiators between attainment levels is the degree of understanding that is demonstrated within the student work. It is not the degree of difficulty but the level of understanding that is assessed in this criterion.

Recommendations for the teaching of future candidates

In general students need more exposure to exploratory mathematics before they are assigned the IA Exploration. As concepts are introduced in class there may be room for short explorations or activities that allow students to learn what it means to explore and also learn the intent of each criterion. This may also encourage students to seek out new and novel topics instead of falling back on well-known topics from a mathematical text or other sources. Before settling on topics, students should be afforded the opportunity to read some of the better-scoring papers in the Teacher Support Material. Teachers can challenge students to explain what mathematics they will do on their own before approving a topic. Teachers need to ensure that students meet internal deadlines and have obtained relevant feedback to their first drafts. Explorations that are poorly planned often do not score well against the assessment criteria. Students could teach the new mathematics they are attempting to explore/learn to their peers to assess how successful their understanding is as well as what type of questions could arise. They could then use this in their explorations.

Some additional recommendations regarding the criteria:

- The notion of coherence needs better explaining to teachers and students. Work should not appear "out of nowhere". Students should not leave any results or methods in the work without comment and interpretation.
- The value of having a clear aim should be highlighted as this can make for a better exploration overall.
- It is recommended that schools explicitly instruct their students in the use of one of the many tools, free or otherwise, that produce correct mathematical notation and graphs.
- Candidates need to be encouraged to handle mathematics within their capacity of understanding. Use of higher mathematical concepts with minimum understanding does not help them score well.

Further comments

- Teachers should be strongly encouraged to complete the background information. It is also essential for them to indicate their markings on the student work. Similarly, it would be helpful if they give elaborate comments against each criterion. Teacher comments on the work also need to be clear and preferably legible.
- Schools in which teachers provide specific comments both on the forms 5/EXCS and on student work seem far less likely to have their marks changed by the moderators.

Teachers who do this appear to understand the criteria better and the annotations make it easier for moderators to understand the teachers' reasoning.

- However, a number of schools continue to submit samples with no or very few annotations on the work. With reference to past feedback forms, such repeat offenders certainly made the moderation much more difficult.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 40	41 - 50	51 - 60	61 - 70	71 - 90

General comments

On the whole, candidates were generally well-prepared for this paper. There were a couple of exceptions to this, which will be highlighted in this report. Most candidates were able to make a good attempt at each of the questions, earning at least some of the available marks, with stronger candidates able to earn very high marks on the paper.

The areas of the programme and examination which appeared difficult for the candidates

- The effect of a constant change to a data set
- Working with vectors and the concept of a unit vector
- Recognizing the relationship between a tree diagram and conditional probability
- Integration using substitution or inspection
- Recognizing trigonometric identities

The areas of the programme and examination in which candidates appeared well prepared

- Quadratic functions and solving quadratic equations
- Simple probability, especially using tree diagrams and Venn diagrams
- Geometric sequences
- Properties of logarithms
- Composite functions

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: quadratic function

Nearly all candidates performed well on this question, earning full marks on all three question parts. In part (b), there were some candidates who factored the quadratic expression correctly, but went on to give negative values for a and b.

Question 2: mean and variance of a data set

While most candidates were able to answer part (a) of this question correctly, they were not as successful in part (b). It seems that the item on the Maths SL syllabus dealing with the "effect of constant changes to the original data" was skipped over in many schools.

Question 3: logarithms

Most candidates were able to earn some or all the marks on this question. Part (a) was answered correctly by nearly all candidates. In part (b), the majority of candidates knew they needed to factor 45, though some did not apply the log rules correctly to earn all the available marks here.

Question 4: geometric sequence and solving quadratic equation

Nearly all candidates attempted to set up an expression, or pair of expressions, for the common ratio of the geometric sequence. When done correctly, these expressions led to a quadratic equation which was solved correctly by many candidates.

Question 5: trigonometry

In part (a) of this question, the large majority of candidates substituted correctly into the area formula for the triangle, though algebraic errors kept some of them from simplifying the equation to $\sin \mathrm{A\hat{B}C} = \frac{1}{2}$. Unfortunately, a number of candidates who got to this point often did not know the correct angles that correspond with this sine value.

In part (b), many candidates realized that $\mathrm{C\hat{B}D}$ was the supplement of $\mathrm{A\hat{B}C}$. However, at this point many candidates substituted $30°$, or their follow-through angle in degrees, into the formula for the area of a sector found in the formula booklet, not understanding that this formula only works for angles in radians.

Question 6: composite functions and trigonometric identities

In part (a), nearly all candidates found the correct composite function in terms of $\cos x$, though many did not get any further than this first step in their solution to the question. While some candidates seemed to recognize the need to use trigonometric identities, most were unsuccessful in finding the correct expression in the required form. In part (b), very few candidates were able to provide the correct range of the function.

Question 7: vectors

Most of the candidates recognized that the scalar product of the vectors must be zero. However, some did not find the correct scalar product because they did not multiply the correct corresponding vector components of $\boldsymbol{u}$ and $\boldsymbol{v}$. In addition, the majority of candidates did not attempt to use the fact that the unit vector $\boldsymbol{v}$ has a magnitude of 1. For the small number of candidates who were successful in solving for m and/or n, some did not clearly present the correct pairs of answers.

Question 8: probability

On the whole, candidates were very successful on this question, with the majority of candidates earning most of the available marks. The most common error was seen in part (c)(ii), where many candidates did not earn the mark. It is also interesting to note that many of the candidates who answered this part correctly did so by using the formula for conditional probability, rather than recognizing that the required probability is given to them in the second branch of the tree diagram.

Question 9: calculus

Many candidates answered part (a) of this question correctly, though some seemed to be working backwards from the given expression for area, which is not the intention of a "show that" question. In part (b), while many candidates found the correct derivative, some did so using cumbersome methods such as the quotient rule, rather than using the simpler power rule.

It was disappointing to see the number of candidates who did not recognize that the derivative they had just found in part (b) would have to be equal to zero in order for the surface area to be a minimum. For the candidates who did set their derivative equal to zero, most were able to find the correct height.

In part (d) of this question, there were some arithmetic errors which kept candidates from finding the correct area. The most common error here, by far, was not considering that the number of tins purchased must be an integer.

Question 10: calculus

As is typically the case with question 10, this proved to be quite a challenging question for many candidates. In part (a), while many candidates seemed to recognize that there was some relationship between the given derivative and the gradient of the tangent line, most did not substitute zero for the x-value, and were unable to find the correct gradient of the line.

In part (b), nearly every candidate understood that the area was equal to the integral of f from 0 to a, very few were able to integrate correctly using either substitution or inspection. Many candidates did not even attempt to integrate, stopping after writing the integral expression.

In part (c), most candidates started with a correct expression for the area of the triangle such as $\frac{ab}{2}$. However, very few were able to substitute their expression for b from part (a)(ii), and therefore did not find a value for k.

Recommendations and guidance for the teaching of future candidates

Teachers and students need to be familiar with the current Mathematics SL guide, particularly with respect to the syllabus content and the notation list. It is evident that there are areas of the syllabus which are not being covered in some schools.

Candidates and teachers need to be familiar with the marking principles and command terms used in these papers. For example, candidates are expected to show some type of working on a question with the command term "find". If a candidate writes only their answer, with no working shown, they are likely to lose marks, even if their answer is correct. On questions with the command term "show that", candidates are expected to show how to obtain the given result. They should not simply substitute a given value into a formula and show that it "works". Of course this answer will "work", as it has been given as the correct answer.

As always, candidates should always show their working in a neat and orderly manner. It should be clear what question or question part the working is for, and working should be shown at the place where it is used. In addition, candidates should be told to simply cross out any incorrect working they do not wish the examiners to consider. When a candidate uses an invalid method, or has errors in their working which are not crossed out, that work will be considered as part of their answer even if they switch to a different method later in their working.

Finally, candidates should be exposed to past IB examinations, and they should practice working through these papers under examination conditions. This will allow them to become familiar with the format of the papers, help them learn to budget their time appropriately, and give them an opportunity to practice showing their work in an organized manner for both sections of the paper.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 18	19 - 37	38 - 46	47 - 55	56 - 63	64 - 72	73 - 90

The areas of the programme and examination which appeared difficult for the candidates

Most of the students attempted to answer all the questions of the exam, although in some centres there appeared to be some areas of the syllabus which proved difficult for the students:

- Recognize the need to find the period of a trigonometric function within a context.
- The concept of independent events.
- Problems involving kinematics.
- Use of chain rule to find the derivative of a function involving an exponential.
- Considering the domain when graphing a function
- Using a GDC to solve an equation that is not easily solved algebraically
- Percentages and their relation to geometric sequences.
- Recognizing when a problem has more than one solution

The areas of the programme and examination in which candidates appeared well prepared

The following topics were well understood by a significant number of candidates:

- Arithmetic sequences.
- Stating the number of terms of a binomial expansion.
- Use of the graphic display calculator (GDC) to find intercepts and for simple curve sketching.
- The trigonometry in non-right angled triangles.
- Linear regression with the use of their GDC.
- Vector geometry: magnitude, vector equation of a line and position vectors
- Using the GDC to find the probability of a normal distribution

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: arithmetic sequences

Most candidates found this question straightforward and accessible. They could find the correct difference and substituted correctly into term and sum formula respectively.

Question 2: sine and cosine rules

Most candidates solved part (a) correctly, recognizing the need for the law of sines.

In part (b), some recognized they had to use cosine rule but substituted incorrectly. There were a few who used Pythagoras theorem or overly long approaches using the sine rule for 2(b).

Some used the calculator in degree mode instead of radian mode, not recognizing that the angles were given in radians.

Question 3: graph sketching

In part a), most candidates were successful at finding the intercepts with the x and y axis, though many failed to write the horizontal asymptote as an equation. Some candidates gave the answer for the horizontal asymptote as $y \neq -2$.

For part b), a considerable number of candidates could sketch the exponential function providing an approximately correct shape, although many of them did not use the correct domain, making it go beyond x = 4. Others plotted an incorrect value of y at x = 4, resulting in the loss of a mark.

Considering that all the question requires from students is to copy the graph off the GDC, it is important to stress which are the features that cannot be missed.

Question 4: trigonometric functions

Candidates did quite well at part a). Most substituted correctly but considered $\cos 0 = 0$, obtaining an incorrect answer of 17.

Most candidates understood that they needed to solve $h(t) = 20$, but could not do it. A considerable number of students tried to solve the equation algebraically and the most common errors were to obtain $\cos k = \dfrac{-0.2}{1.2}$ or $k = \dfrac{-3}{15\cos 1.2}$.

Part (c) proved difficult as many students had difficulties recognizing they needed to find the period of the function and many who could, did not round the final answer to one decimal place.

Question 5: binomial expansions

Although slightly challenging, this question aimed at assessing candidates' fluency at using the binomial theorem to find the coefficient of a term.

In part a), most candidates realized that the expansion had 11 terms, although a few answered 10.

In part b), many candidates attempted to answer and knew what they needed to find. However, the execution of the plan was not always successful. A fair amount of students had difficulties with the powers of the factors of the required term and could only earn the first method mark for a valid approach. Some candidates gave the term instead of the coefficient as the answer. A few of them attempted to expand the binomial algebraically and very few added instead of multiplied, losing all marks.

Question 6: normal distribution and independent events

The first part of this question was a direct application of the normal distribution and most candidates who attempted the question obtained the correct value. In some cases, candidates gave the answer to 2 or 1 sf, losing a mark and taking the risk of obtaining an incorrect answer in the following question.

Part b) proved challenging for various reasons. Many did not recognize that 0.01 was the probability of an intersection. Others did not know how to find that probability using the fact that the events were independent. Some candidates thought that the independence formula was $P(A)+P(B)=0.01$ instead of $P(A)\times P(B)=0.01$.

Of those that were able to find the correct value of $P(R<x)$, only some continued to find the value of x .

Premature rounding in the answer to (a) sometimes caused the final mark in (b) to be lost unnecessarily.

Question 7: kinematics

Most candidates realized that they needed to calculate the integral of the velocity, and did it correctly. However, only a few realized that there were two possible positions for the particle, as it could move in two directions. In general, the only equation candidates wrote was $3p^2-6p=2$, that gave solutions outside the given domain. Candidates failed to differentiate between displacement and distance travelled.

Question 8: linear regression and geometric sequence

Although the question talked about the regression equation, a few students tried to find the values of *a* and *b* by forming two equations with the coordinates of two points from the table. A considerable number of candidates did not write the value of the correlation coefficient or gave an incorrect one. It can be that a GDC feature (Diagnostics) from some calculators was turned off.

Part (b) was generally well done, with many candidates earning follow through marks. There were some difficulties in rounding the answer to the nearest 100 dollars

Part (c) was attempted in two different ways: recognizing the correct rate 0.95 and then finding the price of the car after 6 years. Some of these candidates used a formula similar to the one for terms of a geometric sequence, $P\times(rate)^{t-1}$, but substituted t by 6 and hence, got an incorrect result.

Others listed all six values to obtain the answer. When using this method, the problem was using less accurate intermediate results and hence, not getting the first 5 correct values of the car.

Many candidates either missed out questions 8 (c) and (d) or multiplied either $0.05\times6\times16100$ or $0.95\times6\times16100$ and failed to notice that the answer did not make sense. Other students tried to use the sum formula for a geometric series.

Part (d) was attempted using a graphical approach as well as analytically using logarithms to find the year in which Lina would sell the car, though many failed in giving the correct year. Common answers were “in the ninth year” or “in 2020”. The same happened to those candidates who used a table of values and found the price of the car after 9 years and 10 years. These candidates should be reminded to show both “crossover” values for a table method to be valid.

Question 9: calculus

Part (a) was in general well answered. Many candidates lost the marks for writing 2 or $y \neq 2$ instead of $y = 2$.

In part (b) some candidates got confused and found $f^{-1}(x)$ instead of $f'(x)$. When calculating the derivative, two types of approaches were seen. Most of the ones who rewrote the function as $f(x) = (x-1)^{-1} + 2$, applied the chain rule correctly. Those who tried to apply the quotient rule made various mistakes: incorrect derivative of a constant, incorrect multiplication by zero, wrong subtraction order in the numerator, omitted the negative sign in the answer.

In (c), most candidates were coherent and obtained the same value as the one written in part (a).

In part (d) many candidates did not manage to differentiate the function g correctly. Of those who could, the equation was generally well solved algebraically.

For part (e), not many candidates wrote a correct equation with their derivatives. There was mixed performance for this question, as those who knew they needed to use their GDC managed to obtain an answer, while many got tangled in unsuccessful attempts to solve the equation algebraically. Many candidates tried to solve quite complex equations ‘manually’ instead of trying to graph the expressions on their calculators and finding the value of x at the point of intersection. Of those students who tried to solve graphically only a small percentage actually sketched the two curves that they were considering. This sketch is particularly useful to examiners to see how the student is thinking, or what steps s/he is taking to solve the equations.

Only a few realized that the question asked for the gradient, which was represented by the y-coordinate of the point of intersection, rather than the x-coordinate.

Question 10: vectors, vector equation and relationship between magnitudes

Parts (a) and (b) were attempted by the great majority of the candidates and appropriate approaches were seen, earning at least the method marks.

Part (c) was generally well done, with many candidates writing the equation of the line as $L = a + tb$, losing one mark.

Part (d) was also well answered by a great majority of students. Even those candidates who had part (a) incorrect, could gain all the marks here.

Part (e) was the most challenging of the paper. For many a major problem was to set up the equation $\sqrt{117} = \sqrt{52t^2}$ and, hence, realize that D could have two positions.

Recommendations and guidance for the teaching of future candidates

Ensure that candidates are exposed to past IB papers so that they are aware of the type of questions being asked and of the standard of responses required.

Insist that some working ought to be shown to clarify the method being used.

Students should be taught to avoid premature rounding of intermediate answers, and to carry through more than 4 significant figures in their working.

Students need to read questions carefully and follow any specific instructions. For example, giving a final answer to the accuracy stated in the question Q4(c) and Q8(b) or giving a coefficient and not the entire term Q5(b).

Students need practice curve sketching even when a GDC is allowed. In particular, students need to learn to restrict graphs to the given domain and correctly show key features such as intercepts and asymptotic behavior. This requires candidates to have a deeper and conceptual understanding of what they are doing with their calculators. Not only do they need to know which buttons to push, but mainly what they are pushing them for. There seems to be too much reliance on the GDC programmes and not enough on understanding and application.

Emphasize that the GDC can be used to solve most equations but MUST be used in equations which equate a transcendental function to a polynomial or rational function. Also, candidates should limit the graphing window to the given domain so they are not even tempted to give solutions outside of the domain.

Teachers should also stress to students the importance of checking the mode of their calculators to determine if they are using radians or degrees when working with angles and trigonometric functions, and that it is probable that students will have to switch from one to the other during the exam, if it is required. It is important to remind students to turn diagnostics on after TI84 GDCs have been reset and also to remember that GDCs are in radian mode after resetting.

Emphasize the need to present work clearly and to label each sub-part of each question that is being answered.

Candidates should be familiar with all aspects of the mathematics SL guide, including the different command terms (write down, find, calculate).