May 2015 subject reports

MATHEMATICS SL TZ1

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2015 examination session the IB has produced time zone variants of Mathematics SL papers.

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 30	31 - 43	44 - 55	56 - 68	69 - 81	82 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 17	18 - 20

The range and suitability of the work submitted

As compared to the last session, there was a wider range of topics chosen but also a growing number of similar explorations, for example there appeared to be a growth in the number of real-life modelling tasks. There were some very interesting topics addressed, some of them clearly original to students and sparked by things in their own lives. However, once again there were still many candidates who attempted explorations on common topics such as the Fibonacci sequence, the golden ratio, the Monty Hall problem and the SIR model for epidemics, casino games, cryptography, graphing musical tones, and projectile motion. Many of these stood out as being formulaic and often poorly done; perhaps those topics just appeal to weaker students. The downside of some of these explorations, modelling topics in particular, is that technology does a lot of the "exploring" and candidates were not always able to really demonstrate their understanding. In addition, it was rare for students to find excellent things to expand upon. A number of historical explorations were also submitted, possibly from candidates whose strengths are in areas other than mathematics. Most of these simply echoed information taken from external sources, with little real understanding of mathematics demonstrated. Occasionally, students from some schools appear to all produce explorations of a particular genre, maybe advised by the teacher that certain explorations lend themselves to particular criteria better than others. Although this is not against any IB guidelines, it is intended that, by doing the exploration, students benefit from the mathematical activities undertaken and find them both stimulating and rewarding. On the other hand, schools should not expect or require their students to tackle new mathematics or mathematics beyond the SL curriculum since those that chose a topic above the level of the course often struggled to demonstrate understanding of the mathematics presented in their exploration. Finally, there were still some samples that were based on old portfolio tasks which would prevent the students from achieving the highest levels in certain criteria.

Candidate performance against each criterion

Criterion A

Most students were able to present organized work with a reasonable introduction, provide some sort of rationale, state an aim, make an attempt to explain steps and end with a conclusion. With some coherence, they were able to achieve level 2 here. More explanations, rather than fewer, to clarify the links between one section and the next, as well as showing all steps in their working will improve the coherence of the work. Students should always bear in mind that the target audience for their exploration is their peers. As such, few candidates managed to reach level 4 since their work often suffered from incomplete explanations or an exploration that fully satisfied the stated aim. Both “concise” and “complete” are elusive descriptors. For instance, page after page of repetitive calculations/data/graphs would hurt the conciseness and flow of the paper. In general, explorations that go beyond 18 pages will struggle to be concise.

While more students are including citations, far too many only had a bibliography and did not cite sources of ideas and especially images in the text where those things occurred. This is something that teachers should be aware of, and should require students to correct between the initial and final drafts of the paper.

Criterion B

Most students were able to select appropriate mathematical presentations and employed mostly appropriate notations and symbols for their work, leading to a level 2. Most tables had appropriate headings and most curves were labelled, but endless tables of poorly labelled data are not particularly helpful and do not communicate well. In addition, notation continues to be problematic. The equal sign was often used when the approximation was more appropriate. Use of calculator notation such as * and ^ remains an issue even though it is less than in previous sessions. Different variables were inconsistently used for the same situations. At times variables would change in case (upper or lower) or even in the symbol used, in the middle of calculations or explanations. A lack of clear definition of variables used was also evident. It should be stressed that multiple forms of representations should be attempted whenever applicable. The exposure to, and use of, technology in candidates' explorations varies considerably from one school to the next.

Criterion C

While the understanding of this criterion appears to have improved somewhat from last year and teachers seemed to be more aware that personal engagement requires more than simply stating how much they enjoyed the topic, there are still too many teachers who award levels 3 or 4 without much evidence in the paper itself of the personal engagement. The greatest issue here is that candidates and teachers alike seem to believe that personal interest can be equated with personal engagement. Candidates continue to find making an exploration their own and/or thinking independently one of the harder parts of this internal assessment. Teachers are advised to encourage students to say when something in the exploration was their own original idea to help make the personal engagement more apparent.

Criterion D

The majority of students made some effort on reflecting even though the reflections often consisted solely of repeating results or describing them in terms of the situation. Although some of these reflections were meaningful within the context of their tasks, it would be better if the students could focus on the methods developed, the mathematical process applied, or the implication of the models utilized since truly critical consideration of the implications or limitations was rare. Both students and teachers need to be aware that it is necessary for the student to reflect on the mathematics and what they have learned about it, not just on the real-world phenomenon that they found interesting enough to study. The weaker explorations usually had very little reflection in the body of their work and left the vast majority of their reflection for the conclusion.

Criterion E

The quality of mathematics and understanding varied widely. Most explorations included mathematics at the appropriate level but it was unusual for candidates to score the top marks, mostly due to a lack of demonstrated understanding of the topic. For instance, a number of candidates used complicated mathematics taken from another source which they had evidently not understood and did not properly explain, use or apply. Such work was basically reduced to substituting values into given formula and had little scope to show mathematical knowledge or understanding. Using only technology to find regression equations without showing any knowledge of how this is accomplished, or trying any type of analytical approach, remains an issue. In such cases, technology often did the work and then the candidate would comment on the superficial results. These are commonly modelling or statistical based explorations, involving linear regressions or Chi-Squared tests, in which the results were generated from technology with no calculations undertaken. Few would go the extra mile and even attempt to explain the how or why behind these results. Too many students restrict themselves to level 2 because they only use mathematics from the prior learning section of the syllabus, even when in other criteria they might score quite well. Teachers could be doing more here to advise candidates on what is required in order to demonstrate understanding. In general, the quantity of mathematics used is not the deciding factor as to the attainment level achieved, but rather the degree of understanding demonstrated by the candidate.

Despite these comments there were some candidates and schools that submitted excellent work which demonstrated very good understanding. They attempted areas of mathematics which were not covered or taught in class by learning the content and techniques on their own.

Recommendations for the teaching of future candidates

- Teachers' understanding of the assessment criteria tended to be correlated with the candidate's performance, and hence, they must familiarize themselves with the expectations of these criteria. This suggests that training opportunities and exposure to a variety of explorations are significant in helping teachers communicate a clearer understanding of what is expected of their students. This can be accomplished through IB teacher training or attention to subject reports and support documents on the Online Curriculum Centre (OCC). Too many times it appears that the mark was generated from some personal understanding of what the criterion represented, rather than a true interpretation of it.
- Teacher guidance is the key in helping candidates choose a focused topic that includes relevant mathematics and provides them with sufficient opportunities to achieve the highest levels across all criteria. It is recommended that candidates choose something they are interested in and are able to actually do than to choose a topic they do not really understand. Students should also be guided on how to select an exploration task that is personally meaningful, allows genuine exploration and provides opportunity to employ mathematics that is commensurate with the level of the course. Standard textbook problems and popular topics that are freely available in the public domain should be avoided. Such a list can be deduced from this and last year's subject reports. These topics generally do not offer much scope for genuine personal engagement and critical reflections.
- The candidates need to be trained to understand the criterion better. This would also mean that teachers need to communicate this to the students through the examples. Teachers should note that a sizable set of new graded examples are available on the OCC and students might be provided access to some of those to help them get an idea of the sort of papers that score well. This highlights the importance of spending reasonable time in introducing the exploration.
- Teachers need to further emphasize the importance of clear referencing and students

should be taught how to provide appropriate in-text citation in their work. A bibliography alone does not help the reader to know when and how the piece of resource had been used in the work. In general, the basic elements of good writing should be taught, including especially the proper way to cite external sources.

- Proper notation should be emphasized, and simple things like how to properly label a graph or present calculations and algebraic arguments should be taught and demonstrated. Many students would benefit from specific instruction in the use of some sort of free software for generating mathematical expressions.
- Personal interest is not, in itself, personal engagement. Candidates should be taught what it means to "explore", and how different situations can be treated in this way. Examples done in class can almost always be reconsidered in terms of "what if...” This would help candidates see how they might ask themselves the same question as they explore their topics.
- Reflection on results should include some consideration of their appropriateness to the situation, their implications (e.g. a model function grows to infinity - is this reasonable?) and their limitations (a probability is near 0 but can it truly be 0?).
- The sequencing of topics studied should be reviewed to allow candidates the greatest flexibility of choice as they prepare their explorations in the middle of the course. Topics such as statistics (including regression) and probability could be treated earlier, as well as function transformations. Trigonometric functions and triangle relationships make for another good topic. Vectors and calculus are generally not accessible topics for the exploration, especially if these have only been treated at an introductory level at the time that the exploration is assigned.

Further comments

- Technology must be seen as a tool for the exploration; not a driver of the exploration. For instance, in modelling tasks, students should be encouraged to show at least why a certain regression model is chosen and as much as possible provide an understanding of the algebraic development of the chosen model.
- Often mathematics was used that was not from the SL course. Applying the mathematics students have discovered or used should be emphasized. Teachers were often awarding high marks for work that was either copied from the internet with no real understanding shown by the candidate. Mathematics used from Higher Level does not guarantee the highest grade when no real understanding has been shown. Similarly, complicated mathematics used from some other source which is not understood should be discouraged. A good number of these would be explorations based on topics in Physics; there was always significant mathematics involved (which was readily found in a standard textbook) and thus very hard to be sure that the student really understood the derivations. In general, only the very best candidates are likely to be successful with a goal that requires learning a whole new area of mathematics.
- Teachers appear not to always rigorously check calculations. They overlook both obscure and obvious errors made by students or do not annotate these errors if they have been seen. It is obviously helpful in moderation if mathematical errors are highlighted.
- Some schools only submitted photocopies of the student work. This loses the colour that the original printouts apparently had and therefore has the potential to affect the

communication mark. It also tends to make things more difficult to read.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 28	29 - 39	40 - 51	52 - 62	63 - 74	75 - 90

General comments

Candidates are to be congratulated overall for their demonstration and application of understanding. The following comments ought to be reviewed carefully when preparing future candidates for the examination.

The areas of the programme and examination which appeared difficult for the candidates

- Expected value from a discrete probability table
- Perimeter of a major sector
- Inverse functions
- Finding the cosine of an obtuse angle
- Identifying a complicated discriminant
- Clear justification of function features using calculus
- Recognising infinite geometric series – inquiry based thinking
- Application of vectors to geometric figures

The areas of the programme and examination in which candidates appeared well prepared

- Arc length
- Solving an equation using indices laws
- Trigonometric identities
- Calculus – using gradient function.
- Basic vectors - finding $\overrightarrow{AB}$, magnitude, scalar product
- Integration to find areas under curves
- Probability – summing to 1, simple tree diagram
- Graphing $f(-x)$

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: discrete random variables

Most candidates were able to find *p*, however expectation emerged as surprisingly more difficult. Quite often E(*X*)/4 was found or candidates wrote the formula with no further work.

Question 2: circular trigonometry

Most candidates were able to find the minor arc length. Similarly most candidates successfully found the major arc length in part b) but did not go on to add the two radii. Quite a few candidates worked with decimal approximations, rather than in terms of π.

Question 3: solving an equation using indices laws

Indices laws were well understood with many candidates solving the equation correctly. Some candidates used logs, which took longer, and errors crept in.

Question 4: functions

Typically candidates were more successful in finding the composite function than the inverse. Some students tried to find the function, rather than read values from the given graph. The sketch of $f(-x)$ was often well done, with the most common error being a reflection in the x-axis.

Question 5: trigonometric identities

Many candidates were able to find the cosine ratio of $\frac{\sqrt{7}}{4}$ but did not take into account the information about the obtuse angle and seldom selected the negative answer. Finding $\cos 2x$ proved easier; the most common error seen was $\cos 2x = 2\cos x$.

Question 6: discriminant

Many candidates were able to identify the discriminant correctly and continued with good algebraic manipulation. A commonly seen mistake was identifying the constant as $\frac{5}{4}p$ instead of $\frac{5}{4}p-5$. Mostly a correct approach to part b) was seen $(\Delta = 0)$, with the common error being only one answer given for p, even though the question said values (plural).

Question 7: definite integrals – trigonometry

Most candidates recognised that a definite integral was required and many were able to set up a correct equation. Incorrect integration leading to $-\sin x$ was quite common and poor notation was frequently seen. Some candidates appeared to guess their value from the graph, showing little supporting work.

Question 8: vectors

a) Finding $\overrightarrow{AB}$ and its magnitude were mostly well done.
b) Mostly correct answers with common errors being using both position vectors or writing it as " L = “instead of "***r*** =".
c) Many candidates assumed that $\overrightarrow{AB} = \overrightarrow{BC}$, although this was not indicated on the diagram nor given in the question.
d) Mostly this was well answered. A surprising number of candidates wrote the scalar product as a vector (0, -2, 2). In part b) many missed the clue given by the phrase “hence, write down” and carried out a calculation for cosine theta using the scalar product again.
e) This part was poorly done. Few candidates realised how to directly calculate the area based on their previous work and could not see the “height” of the obtuse triangle as $\left|\overline{OC}\right|$. Those who tried to use $A = \frac{1}{2}ab\sin C$ had trouble generating the angle. Those who subtracted areas ($\Delta OAC - \Delta OBC$) were usually successful.

Question 9: calculus

a) Well answered and candidates coped well with *k* in the expression.
b) Mostly answered well with the common error being to substitute into f' instead of f''
c) A straightforward question that was typically answered correctly.
d) Some candidates recalculated the gradient, not realising this had already been found in part c). Many understood they were finding a linear equation but were hampered by arithmetic errors.
e) Using change of sign of the first derivative was the most common approach used with a sign chart or written explanation. However, few candidates then supported their approach by calculating suitable values for f’(x). This was necessary because the question already identified a local maximum, hence candidates needed to explain why this was so. Some candidates did not mention the ‘first derivative’ just that ‘it’ was increasing/decreasing. Few candidates used the more efficient second derivative test.

Question 10: probability and geometric series

Some teachers’ comments suggested that the word ‘loses’ in the diagram was misleading, But candidate scripts did not indicate any adverse effect.

a) Very well answered.
b) i) Probabilities *p* and *q* were typically found correctly. ii) Fewer candidates identified the common ratio and number of rolls correctly.

Few candidates recognized that this was an infinite geometric sum although some did see that a geometric progression was involved.

Recommendations and guidance for the teaching of future candidates

Candidates should be encouraged to use correct notation and lay out their solutions in a logical, mathematical way. This should be emphasised throughout the teaching of the two-year course. Standard errors such as vector equations in the form "L=" are still frequently seen. Often question parts use answers from previous working but this is often forgotten with work unnecessarily repeated – candidates should read the whole question and identify where parts link together. Be mindful of efficient solutions, commensurate with the marks allocated. Non-calculator papers do not require awkward, longwinded arithmetic. If a solution develops into this, it may be best to look for an alternative approach. Candidates should read questions carefully, looking for words like 'hence', which is an obvious clue to use their previous work. Similarly, 'values' indicates there is more than one answer. These clues in questions can avoid loss of marks or time spent unnecessarily. The command terms (e.g. hence, show that, find, explain, and write down) should be specifically addressed in class so candidates know what is expected from each. Candidates need to be exposed more to inquiry style questions where they can develop a pattern and apply it or form a conjecture. Candidates need to understand that it is not acceptable to substitute given results in a show that question, nor make assumptions. It is better not to cross out working that has not been replaced.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 28	29 - 39	40 - 49	50 - 59	60 - 69	70 - 90

The areas of the programme and examination which appeared difficult for the candidates

Candidates in this session had difficulties in the following areas of the programme:

- Using a graphic display calculator (GDC) to find a regression line and a correlation coefficient
- Curve sketching within a given domain
- Identifying the need for and applying the chain rule for differentiation
- Relationship between the graphs of functions and their derivatives
- Definite integrals as they pertain to regions below the x-axis
- Recognizing and finding probabilities for a binomial distribution

The areas of the programme and examination in which candidates appeared well prepared

For students who were well prepared, there was ample opportunity to demonstrate a high level of knowledge and understanding on this paper. The following areas of the programme were handled well by most students.

- Arithmetic sequences and series
- Using the equation of a regression line to interpolate
- Finding specific features of a graph using a GDC
- Triangle trigonometry including sine and cosine rules

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: regression and correlation

Candidates continue to have difficulty using their GDCs to find and correctly identify the coefficients of a linear regression. Both the *r* and r^2 values were often given as candidates were hedging their bets and were not entirely clear which one to give. Candidates frequently were unable to find the correct values for *a* and *b* suggesting a lack of familiarity working with GDCs. It was also surprising to see so many candidates leave these values to only one significant figure sacrificing all the marks for this part. Subsequent use of their line to find *y* for a given *x* was not difficult for most, but answers were not often given to the required accuracy of one decimal place.

Question 2: binomial expansion

This is a common question and yet it was not unusual to see candidates writing out the expansion in full or using Pascal's triangle to find the correct binomial coefficient. Of those candidates who managed to identify the correct term, many omitted the parentheses around $2x$ which led to an incorrect answer. Most candidates were able to distinguish between "the term in x^3" and the coefficient. There are still a significant number of candidates who add the parts of a term rather than multiply them and this approach gained no marks.

Question 3: arithmetic sequences and series

In general, candidates showed confidence in this area of the syllabus. Appropriate formulae were chosen for parts (b) and (c) and many candidates were able to achieve full marks. However, many candidates found the common difference to be +1.5 in part (a) by subtracting $u_{10} - u_{11}$ or believing that the common difference should always be positive.

Question 4: rational functions

Part (a) was generally well done with candidates using both algebraic and graphical approaches to obtain solutions. There are still some who do not identify their asymptotes using equations. Candidates rarely appreciated the relevance of the horizontal asymptote in (b), and often attempted a long, and often unsuccessful, algebraic approach to find the limit.

Question 5: exponential modelling

The majority of candidates were able to sketch the shape of the graph accurately, but graph sketching is an area of the syllabus in which candidates continue to lose marks. In this particular question, candidates often did not consider the given domain or failed to accurately show the behaviour of the graph close to the horizontal asymptote as $x \to \infty$. In (b), most candidates were able to identify the initial approach by finding $G(45)$, but missed the fact that function defined the cost per guest and not the total cost.

Question 6: tangents and normals

Few candidates were completely successful with this question. Students using an analytical approach were aware of the relationship between the gradients of the tangent and normal, but were often unable to find a correct derivative initially. The solution was made significantly easier when the GDC was used effectively and the few candidates who used this approach, were generally successful. Attempting to find the equation of the tangent and/or the normal were common, ineffective approaches.

Question 7: transformations

The coordinates of the minimum point was correctly given by most candidates, although some opted for an analytical approach which was often futile and time consuming. In part (b), few students appreciated that the solution set consisted of two **intervals** often giving only one correct interval or equalities. The most common, incorrect approach was an attempt to use the discriminant.

Question 8: triangle trigonometry

Many candidates were successful with this question and their knowledge and use of the sine and/or cosine rules. There were a considerable number of candidates who did not appreciate the ambiguous case of the sine rule and often found the two angles in part (b) to be the same. Students would be well advised to consider the reasonableness of their answers as they pertain to the given diagram and in particular, whether the expected angle is acute or obtuse. There were a few successful approaches using right triangle trigonometry, but this was a less efficient method and not really expected.

Question 9: probability Distributions

This question saw many candidates competently using their GDCs to obtain required values, although a surprising number in part (b) chose to use an inefficient ‘guess and check’ method to try and obtain the standard deviation. Those using a correct approach often used a rounded z-score to find σ leading to an inaccurate final answer. In part (c), some candidates did not recognize or understand how to apply the given condition. In part (d), the binomial distribution, although often recognized, was not applied successfully.

Question 10: graphs of derivatives and definite integrals

In part (a), many candidates did not get full marks in justifying that p = 6 was where the maximum occurs. The derivative changing from positive to negative was not sufficient since there are cases where the derivative changes signs at a value where there is no turning point. Part (c) was very poorly done as most candidates did not recognize the use of the chain rule to find the derivative of $\ln\left(f\left(x\right)\right)$, a fairly basic application for Mathematics SL. In part (d), candidates appeared to have difficulty with the command term “verify”, and even if they were successful, did not make the connection to part (e) where they attempted a variety of interesting ways to find $g\left(5\right)$ - the most common approach was to set up two incorrect integrals involving areas A and B. Many students did not realize that integrating a function over an interval where the function is negative gives the opposite of the area between the function and the *x*-axis.

Recommendations and guidance for the teaching of future candidates

Read the subject reports each session as they continue to repeat recommendations regarding skills that are absolutely essential for Mathematics SL but are still not well understood or applied.

Cover all aspects of the syllabus as the examining team endeavours to ensure complete syllabus coverage across both papers

Candidates still do not have a clear understanding of how to round answers correctly to three significant figures. They should also be advised to work with a minimum of four significant figures in the case of non-exact values, and only round to three significant figures at the end of a question part.

Candidates are still not using their GDCs to their full advantage on paper 2. There are certain skills that they are expected to use – finding regression lines, equations of tangents, numerical derivatives and integrals, finding probabilities in a given distribution, etc., that are not handled well by many candidates. Candidates must also be aware of how to set their GDCs to display relevant information such as the correlation coefficient.

Teachers should continue to provide sufficient opportunities for students to practice sketching functions on the GDC and to find key features (asymptotes, maximums and minimums) within a specified domain.

Students should be given opportunities to develop their graphical calculator skills in tandem with algebraic approaches to different parts of the syllabus to improve their overall understanding e.g. algebraic techniques of differentiation and the interpretation of the graph of the derivative function.

Many students received partial credit for showing their method and their reasoning, but this still needs work – especially when communicating how they're using their GDCs. In addition, "found using GDC" accompanying an answer is not enough of an explanation and many students are still using calculator-specific language - binomcdf, normalcdf,… that may result in a loss of marks if an examiner is not familiar with the syntax of a particular model of GDC.

There were various comments on the G2s on all components about terminology and notation, but these were consistent with the guide. Teachers sometimes seem unaware of the notation list, neither do they realise that there are differing terms for the same thing (especially in French and Spanish), but IB policy is to use the guide.

Teachers should realise that the examination paper is designed such that the harder questions are generally 6 and 7 in section A and 9,10 in section B. These questions will usually assess conceptual understanding as well as procedural competency.