MATHEMATICS SL TZ2

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 36	37 - 49	50 - 61	62 - 73	74 - 85	86 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2014 examination session the IB has produced time zone variants of Mathematics SL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 17	18 - 20

The range and suitability of the work submitted

A wide range of appropriate topics with mixed quality was submitted. Candidates who had chosen appropriate topics could attain the upper levels of each criterion. A few others, however, produced work that was not commensurate with the level of this course.

Examples of popular themes that were received were card games and gambling, demographics, spread of disease, athletics/sports, and video games. In addition there were many candidates who attempted explorations on 'common investigations' or 'textbook' problems, like the Golden Ratio, Fibonacci numbers, the Birthday paradox, Monty Hall Problem, Pascal's Triangle. There were also numerous modelling activities of real life situations that were very similar in style with the old portfolio modelling tasks. Often in these cases, candidates produced work that was a summary of common facts and or general history of the topic. This would normally demonstrate a lack of personal engagement. Nevertheless, explorations which were based on textbook problems sometimes did also lead to a good exploration, when the students decided to extend the investigation beyond the original problems and/or add something of their own in the exploration. However, this was not in abundance. Most of the explorations based on these common textbook problems or examples revolved around superficial understanding of the concepts, repetition of methods found on the internet and did not lend themselves to anything new, and hence, could not reach the highest levels.

The use of technology to develop regression functions in an attempt to model data was very common. In some cases this was done effectively with suitable mathematical support. However there were cases where the regression model was simply created and applied via technology with very little understanding shown. It is recommended that in future, students will justify their choice of regression model and reflect critically on their choice.

There were a few instances where candidates simply submitted explorations entirely based on old portfolio tasks, which were specifically designed for the old assessment criteria. As a result, such explorations would not necessarily provide the candidates with the opportunity to achieve the highest levels.

The students generally adhered to the recommendation that the exploration be between 6 and 12 pages long. However there were many that were too long. These were often found to be self-penalizing.

Candidate performance against each criterion

Criterion A:

This criterion was addressed well by most of the students, with work being coherent and organized to different extents. In general, they made an attempt to provide a relevant introduction, a rationale, an explicit aim and some sort of conclusion. They also tried to explain relevant concepts and took conciseness into consideration. However, some teachers did miss the subtle differentiation between level 3 and 4, which is about the conciseness and completeness of student work. For instance, very lengthy tables of data may be relegated to an appendix, with a summary in the text where the information is used. Similarly, pages after pages of repetitive calculations would affect the conciseness and flow of the paper; one or two sample calculations would suffice and the rest could be summarized in a table.

Students should be more careful with the stated aim matching what they write and present in their work, and the conclusion they reach. If they find themselves unable to write a paper originally intended because of space constraints or any other reasons, then it would be wise for them to adjust the intended aim accordingly.

In addition, work that depended on a lot of secondary sources tends to have less coherence and is more difficult to follow.

It is crucial that quoted information be correctly cited at the point in the exploration where it appears, both for the flow of the piece and for academic honesty.

Criterion B:

This criterion was appropriately dealt with in most of the explorations. There were plenty of varieties of mathematical presentation displayed but it is essential that students are reminded that these are to be appropriate. Most students were able to use appropriate terminology and notations in their work, including the use of appropriate ICT tools. However, there were students who still used inappropriate computer or calculator notation (this is not an issue if generated by the software), did not define key terms and employed inappropriate presentations, like poorly labelled diagrams or badly scaled graphs. Graphs copied from the Internet were inserted but without any real purpose. Graphs need a purpose and not just included to "use multiple forms of mathematical representation". Mathematical formulas and theorems just taken from the Internet were often included but did not always really add to the students' work.

Criterion C:

This criterion proved to be the most difficult for teachers to assess or interpret and appeared to be the least understood by both students and teachers. It seems that too many teachers stated that they ‘saw’ engagement but this was not supported by the work submitted. They simply assumed that interest in the topic chosen by the student meant high personal engagement. High marks should not be awarded to students who just stated how much they enjoyed the topic or who demonstrated enthusiasm in class, unless this is seen in the exploration itself. Simply stating, "...it interests me..." is not personal engagement.

Students who explored a common investigation/textbook problem without any personal input or extension would not usually achieve the higher achievement levels in this criterion. Nevertheless, a number of teachers did seem to understand this criterion and were able to transmit that information to their students effectively. Some explorations do lend themselves more readily to high levels for personal engagement - for example those where students do their own research and data collection. With the more descriptive or historical topics it is not particularly easy to score highly here.

It is important to note that this criterion cannot be used to penalize late submission of work.

Criterion D:

This criterion was clearly understood well by many teachers and students and there was a wide range of achievement here. Many students simply described the results in their explorations. They also sometimes reflected on why they found the exploration interesting or enjoyed learning about it. Less often did they reflect on the analytical process of exploring. Many reflections were superficial. There were also cases where students would undertake explorations similar to the old portfolio tasks using the same questioning technique to reflect on the process. This did not lend itself to meaningful reflection.

Many students were under the impression that reflection could only come through in the conclusion and hence missed out on the opportunity of demonstrating substantial evidence of critical reflection throughout the exploration.

Higher levels were generally awarded to those students who considered further exploration, discussed implications of results, compared strengths and weaknesses of mathematical approaches and contemplated different perspectives.

Criterion E:

There was a wide range of achievement in this criterion from the lowest to the highest. Most students employed relevant mathematics that was commensurate with the level of the course, since the majority of topics were chosen to allow students to demonstrate at least ‘some’ mathematics. Students could be seen using many areas of mathematics from sequences to differential equations. In general, most teachers were able to determine whether or not the work was commensurate with the level of the course.

Regression analysis was used extensively but not with thorough understanding demonstrated. There were cases where the regression model was simply created and applied via technology with very little justification of their choice of regression model.

Teachers occasionally awarded high marks for work with numerous calculations even if no clear understanding was shown. In addition just showing the correct answer is not the same as showing understanding; it must be demonstrated. Students who did an excellent job of explaining their reasoning throughout their papers would gain the higher marks. A considerable number of students chose topics where the mathematics involved was clearly beyond their understanding. This was often because it had been taken from other sources. Although students did reference these sources, it was clear that many did not understand what they were doing mathematically.

The mathematics used need only be what is required to support the development of the exploration. This could be a few small topics or even a single topic from the syllabus. It needs to be made clearer to the students that it is better to do a few things well rather than trying to do more mathematics badly. If the mathematics used is relevant to the topic being explored, commensurate with the course, and understood by the student, then it can achieve a high level.

Recommendations and guidance for the teaching of future candidates

- Teachers need to become familiar with the assessment criteria. There are numerous examples located in the Teacher Support Material (TSM) on the Online Curriculum Centre (OCC), and they should familiarize themselves with the goals of the exploration, and how achievement levels for the new criteria are awarded by referring to the annotated student work in the TSM.
- One of the major issues was the lack of annotation and/or comments specific to individual student work provided by the teachers. In many cases, where comments were provided, these tended to be paraphrased from the descriptors. Teachers are reminded that it states in the TSM that one of their responsibilities is to assess the work accurately, annotating it appropriately to indicate where achievement levels have been awarded. This includes marking the mathematics and identifying any errors. These comments greatly help to confirm the level awarded by the teacher. Without supporting comments, changes are more likely.
- Teachers should provide relevant background information about their courses where appropriate.
- Teachers need to follow the procedures in the guide which allows students to submit a first draft. This way, teachers can assess the suitability of the topic, check the general organization and coherence, orally test the students' knowledge of the mathematics and most importantly, ensure that the work is that of the student and not just a regurgitation of Wolfram, Wikipedia and other math sites.
- If students are going to type their work, they need to use an appropriate equation editor to avoid errors in notation. Students should not insert screenshots of equations and formulas from Wikipedia or Wolfram. This habit is a good indicator that the work is not their own. They would also benefit greatly from explicit instruction in the use of those tools.
- Teachers should discourage students from choosing common or textbook topics without a clear plan on how to make them into a proper exploration (with enough relevant mathematics in it) since these tended to be research based with very little personalization or creativity demonstrated. Yet teachers should not predefine certain topics for their students or limit them to a certain area of mathematics.
- Teachers need to emphasize the importance of clear referencing and proper citation in student work. Many students provided a bibliography or work cited page at the end of their documents without identifying how these resources have been used in the body of their work. Students should be guided to cite all resources used in the body of their work including all data and images from secondary sources.
- Teachers must annotate and write comments on student work. Those schools where teachers did this were more likely to have scores confirmed, as would be expected; the reasoning behind the scores awarded was readily apparent.
- Teachers could issue practice explorations to hone/practice certain specific skills.
- Students should be reminded of the guidelines that the exploration should be between 6 and 12 pages long.
- Schools should be strongly discouraged from mandating a particular type of exploration. Rather students should be free to explore an area of their choice.

Further comments

- There were instances of potential plagiarism, with many students using sources such as

Wolfram or Wikipedia to copy content, formulas or ideas.

- The mathematical content was often lacking even though the exploration itself was well written and scored well.
- The general feeling after seeing the variety of explorations is that this new IA has provided the students with a great opportunity to explore what they wished, and given them the opportunity to appreciate mathematics in their own way.
- The exemplar materials and the frequently asked questions in the TSM have been/will be updated after the first live session. Teachers should make sure that they read these documents carefully, along with the updated guidance on applying the criteria.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 31	32 - 45	46 - 56	57 - 67	68 – 78	79 - 90

The areas of the programme and examination which appeared difficult for the candidates

- integration, particularly using substitution methods
- understanding the relationship between 2-D vector and Cartesian representations
- recognizing the relationships between functions and their inverses
- relating vector models to real-world situations
- looking for and generalizing patterns
- graphical representation of a derivative function
- trigonometric ratios of special angles
- reasoning skills, and answering questions with non-traditional contexts

The areas of the programme and examination in which candidates appeared well prepared

- using the quotient rule for derivatives
- working with quadratic functions in vertex form and in standard form
- finding the discriminant of a quadratic
- using basic properties to evaluate logarithms
- using double-angle cosine formulas
- scalar product and showing two vectors are perpendicular
- answering questions in straightforward context, using formulaic approaches

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: trigonometry

The majority of candidates answered both parts of this question correctly. In part (a), most candidates used Pythagoras' Theorem, however some candidates were familiar with the 5:12:13 triangle ratio, which was also acceptable. In part (b), most candidates were successful in using one of the double-angle formulas to find $\cos 2A$. Unfortunately, there were quite a few candidates who made arithmetic errors, and some who made the mistake of simply doubling their answer from part (a). There was an error on the French paper, where the "*A*" disappeared from $\cos 2A$. This was dealt with in the marking, to ensure no candidate was disadvantaged.

Question 2: logarithms

While most candidates were successful on all three parts of this question, there were some who were unable to evaluate their expressions after using the appropriate properties of logarithms. There were a few candidates who did not seem to be aware that the necessary formulas for logarithm properties are now in the formula booklet, as of May 2014.

Question 3: inverse functions

This question proved to be surprisingly challenging for many candidates. Although a good number of candidates were successful in part (c), sketching the graph of the inverse function, they struggled with parts (a) and (b) of the question. Many candidates did not seem to be aware of the link between the range of a function and the domain of its inverse. In part (c), there were a few who sketched the graph of $-f(x)$ or $f(-x)$, rather than the inverse function.

Question 4: two-dimensional vectors

The majority of candidates struggled with part (a) of this question, which seems to indicate that they are unaware of the relationship between Cartesian and vector components in two dimensions. In part (b), most candidates were able to earn some marks by substituting either the gradient or the coordinates into the equation of a line, although some went on to make algebraic errors which prevented them from getting full marks. Part (c), writing the vector equation of a line, seemed to be the easiest part of this question for candidates, although many candidates still make the mistake of setting their equation equal to L, which is not a vector, but rather the name of the line.

Question 5: integration with a given condition

While the majority of candidates seemed to know what was required in this question, few were able to carry through and earn full marks. Many candidates did not know how to integrate the given cosine function, other than to write some expression involving sine. Among those who did integrate correctly, many did not know the value of $\sin\frac{\pi}{6}$ needed to solve for c.

Question 6: derivatives

Many candidates earned full marks on part (a) of this question, which required them to sketch the graph of the derivative function given the graph of $y = f(x)$, and many others were able to earn partial credit for showing the x-intercepts of the derivative graph. Despite this, there were a number of candidates who left part (a) blank, or sketched completely incorrect functions. In part (b), candidates who considered the value of the function, its first derivative and its second derivative at the necessary points were successful. However, many candidates just seemed to be guessing at the order, and gave incorrect answers.

Question 7: sequences

This question proved to be quite a challenge for candidates, who did not know how to find the individual terms from the given sums. Candidates who simply attempted to apply one of the arithmetic or geometric sum formulas were not successful. In both parts of this question, the most common error was to assume the sequence was purely arithmetic, which of course did not lead to a correct generalization in part (b). Candidates often seemed to be trying to create their own patterns, making assumptions rather than looking for the pattern which could be found with the given information.

Question 8: quadratic functions

Overall, candidates were very successful in this question. In part (a), the word “value” was meant to convey 0, but some candidates wrote the expression for the discriminant, $36-12p$. The majority of candidates were able to successfully show that $p=3$, although a few incorrectly worked backwards, using the given value of p in their working. Most candidates were able to answer parts (b), (c) and (d) correctly, and used a variety of appropriate methods to find the x-coordinate of the vertex and the solution to $f(x)=0$. Many missed the link between these three parts of the question, however, and spent time doing unnecessary additional work in parts (c) and (d). A common error in part (d) was to give $h=-1$, rather than $h=1$. There were mixed results in part (e), where many candidates knew what to do, but did not find the correct answer due to algebraic errors.

Question 9: application of three-dimensional vectors

In part (a), very few candidates knew how to find the velocity of the airplane by using the direction vector, despite this being mentioned in the syllabus. In part (b), only a very small number of candidates earned full marks, though most earned partial marks. While most candidates realized they needed to substitute $t=2$ into the equation, the overwhelming majority gave their final answers as $\begin{pmatrix}-3\\10\\8\end{pmatrix}$, which is the position vector of the airplane after 2 seconds, or as $\sqrt{173}$, the magnitude of this position vector. In part (c), most candidates were successful in using the scalar product to show that the paths of the two airplanes are perpendicular. In part (d), most candidates knew how to begin, though some algebraic errors kept them from earning full marks. There were a few candidates who successfully solved for s and t, but did not fully answer the question by indicating that Jack's airplane took off 3 seconds after Ryan's.

Question 10: calculus

Part (a) of this question required candidates to use the quotient rule to show how to obtain the given derivative, and nearly all of them did so correctly. Only a few did not show all the algebraic steps necessary to get the final given answer, which is required in a "show that" question. On the other hand, candidates were not nearly as successful in part (b), which required them to integrate the given function using substitution or inspection. It was apparent that many candidates were not familiar with this new topic of the syllabus. A few of these candidates seemed to have some idea that the integrated function involved ln, while others wrote nonsensical answers such as simply integrating the numerator and denominator of the given function. Most candidates who attempted part (c) had a good idea of how to start, and many who answered part (b) correctly also earned full marks in part (c). Those who answered part (b) incorrectly usually earned at least some follow-through marks in part (c), however their convoluted answers from part (b) often prevented them from being able to finish part (c).

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 19	20 - 39	40 - 48	49 - 57	58 - 65	66 - 74	75 - 90

The areas of the programme and examination which appeared difficult for the candidates

- Effective use of the graphic display calculator (GDC), including working to the required degree of accuracy
- Considering answers in context of a question (given domains, interpreting values, appropriateness of decimal vs integer answers)
- Transformations of trigonometric functions
- Binomial theorem involving a product
- Integration applications
- Moving from one probability distribution to another

The areas of the programme and examination in which candidates appeared well prepared

- Triangle trigonometry: sine and cosine rule
- Arc length and sector area
- Solving exponential equations

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: arcs and sectors

The appropriate formulae were identified and used correctly by most students.

Question 2: volume of revolution

Most candidates found the x-intercepts correctly. Many were able to write a correct integral expression for the volume but did not use their calculators to easily find the value. Instead, they attempted to use “analytic” approaches, such expanding the square, often incorrectly.

Question 3: regression model

It was surprising to see that a significant number of candidates tried to find the values of a and b by choosing two points from the table and solving simultaneous equations, as opposed to using the regression feature on their calculator. A large number could not interpret the gradient and failed to consider the specific context of the question (eg. 'per km'). Premature rounding of answers in part (a) often led to the loss of a mark in part (b).

Question 4: Venn diagram and probability

Some candidates assumed the events were mutually exclusive thus adding the probabilities of A and *B* directly so that in part (b), 0.9 was a frequently seen answer. A great diversity of answers was seen for the shading of $A \cap B'$. Despite incorrect shadings, many students could still find the probability in part (c)(ii) satisfactorily, as if parts (i) and (ii) were unrelated.

Question 5: triangle trigonometry

Most candidates successfully recognized which formulae to choose and applied them well. A common mistake in part (a) was to give only the acute angle answer or finding 131.8 $\left(90^{\circ}+41.8^{\circ}\right)$ as their obtuse angle.

Question 6: circular functions

There was mixed success in determining the values of p, q, and r. In particular, it was difficult for candidates to recognize that the period was 24. Those candidates who had one or more of these values incorrect typically did not write all their solutions within the given domain. Some candidates did not recognize the difference between solving $f(x)=7$ and evaluating $f(7)$.

Question 7: binomial theorem

As this was an unstructured question, many candidates struggled to get further than writing out some or all of the expansion or writing down Pascal's triangle. Some recognized which term could lead to the correct result but faltered on the algebra and arithmetic. Very few candidates included $k=-2$ as the second answer.

Question 8: exponential models

As a whole this question was well answered and candidates demonstrated good techniques. Answers in context questions should be carefully considered eg when an integer answer is appropriate, or indeed asked for in the question.

Question 9: kinematics

In part (b) most candidates found the correct answer but quite often it was obtained from incorrect working (eg $t^{6}-64=0$) and so lost two marks. Finding the total distance travelled proved challenging with many candidates attempting to integrate but finding displacement instead. Most candidates correctly differentiated the velocity but caution is needed to demonstrate all algebraic processes used to obtain the given answer. Similarly, attempts in part (e) often lacked supporting working/explanations (eg graph) and answers were given without consideration to the given restricted domain.

Question 10: probability

Simple probability was handled well and even when using wrong values from part (a), candidates knew what they had to do. Most candidates recognized that conditional probability was required in part (a)(ii) though to find the intersection of $H>60$ and $H>70$, many candidates assumed the events were independent and multiplied the probabilities. Some marks were obtained by mentioning binomial probability in part (c) or interpreting $X \geq 25$ correctly but full answers were quite rare.

Recommendations and guidance for the teaching of future candidates – paper 1 & 2

General

Teachers need to ensure that they are familiar with all the changes for May 2014. This includes the syllabus and approaches to teaching and learning, as well as the command terms and the new formula booklet. There was evidence in all papers that not all the changes had been taken on board. All the required information is available on the OCC.

Some teachers commented that questions of an investigative nature are not appropriate for a timed examination, that it is a type of question used in the old IA portfolios. However, as expressed on page 11 of the Mathematics SL Guide, it must be recognized that such questions are part of the new course, and that such an inquiry-based approach to learning is integral to the teaching of the course. While paper authors recognize the timed nature of the examination, students who are trained to make and justify conjectures will find such questions more accessible.

Candidates need to be familiar with the vocabulary and underlying concepts, rather than simply practicing standard routines and processes. Candidates should be encouraged to read each question carefully and consider what information is given and what a question is asking them to do before they begin their work. It is clear that candidates are much more comfortable with questions which allow them to simply reach for the formula booklet and substitute into a formula. For example, many students will often reach for a formula when encountering questions in conditional probability, but most do not have the conceptual understanding to go beyond the formula booklet.

They should be exposed to past IB exams and markschemes, and are encouraged to use these for practice. Looking at the markschemes can help both the students and the teachers understand what is required by the different command terms, such as "write down", "find", "show that", or "hence".

Candidates should be encouraged to show all their working in a neat, organized manner, using proper mathematical notation. Being familiar with the format of the examination paper will help candidates with this. If a mistake is made, it is best to simply draw an "X" or a line through any unwanted working.

Paper 1

In general, the questions did not require a great deal of arithmetic or algebraic manipulation, but they did require good understanding of the concepts. Candidates who understood the concepts were able to answer questions with a minimum amount of work. Unfortunately, candidates who did not understand the concepts often found themselves stuck with convoluted and unnecessary working which often did not lead them to an answer.

Paper 2

Paper 2 is a GDC required paper, not simply a GDC allowed paper. Candidates should be encouraged to consider whether use of the GDC is appropriate when answering any question on Paper 2. Although basic GDC skills are improving, there are still candidates who are opting for an analytical approach rather than a more efficient GDC approach eg definite integrals, evaluating numerical derivatives, solving equations, etc. Alternative methods often lead to simple algebraic errors and consume valuable time. It should be emphasized that once an equation is established, no algebraic working is needed to support an answer. Teachers should place greater emphasis on integrating the use of technology as a tool for learning and for better understanding key concepts as well as for solving problems by communicating solutions clearly. Many candidates continue to struggle with what work to show when using technology. “Found using GDC” is not a valid explanation and using calculator specific language in the working should be discouraged.

Candidates continue to lose marks sketching poor graphs that do not show the required features. This would seem a straightforward problem to overcome by practicing more often when the need arises during class. However, students should be taught not simply to transcribe graphs from their GDC without considering their intrinsic knowledge of key features and behaviours of functions. They should be encouraged to use the appropriate GDC tools to find and label key features of graphs.

Candidates should ensure that their GDCs are in the correct mode (eg radian/degrees). Numerical values needed in subsequent parts should be stored in the GDC memory (the more accurate “long” value). Inaccurate values or premature rounding of values can lead to wrong final answers.