May 2014 subject reports

MATHEMATICS SL TZ1

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 34	35 - 48	49 - 60	61 - 71	72 - 83	84 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2014 examination session the IB has produced time zone variants of Mathematics SL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 17	18 - 20

The range and suitability of the work submitted

A wide range of appropriate topics with mixed quality was submitted. Candidates who had chosen appropriate topics could attain the upper levels of each criterion. A few others, however, produced work that was not commensurate with the level of this course.

Examples of popular themes that were received were card games and gambling, demographics, spread of disease, athletics/sports, and video games. In addition there were many candidates who attempted explorations on 'common investigations' or 'textbook' problems, like the Golden Ratio, Fibonacci numbers, the Birthday paradox, Monty Hall Problem, Pascal's Triangle. There were also numerous modelling activities of real life situations that were very similar in style with the old portfolio modelling tasks. Often in these cases, candidates produced work that was a summary of common facts and or general history of the topic. This would normally demonstrate a lack of personal engagement. Nevertheless, explorations which were based on textbook problems sometimes did also lead to a good exploration, when the students decided to extend the investigation beyond the original problems and/or add something of their own in the exploration. However, this was not in abundance. Most of the explorations based on these common textbook problems or examples revolved around superficial understanding of the concepts, repetition of methods found on the internet and did not lend themselves to anything new, and hence, could not reach the highest levels.

The use of technology to develop regression functions in an attempt to model data was very common. In some cases this was done effectively with suitable mathematical support. However there were cases where the regression model was simply created and applied via technology with very little understanding shown. It is recommended that in future, students will justify their choice of regression model and reflect critically on their choice.

There were a few instances where candidates simply submitted explorations entirely based on old portfolio tasks, which were specifically designed for the old assessment criteria. As a result, such explorations would not necessarily provide the candidates with the opportunity to achieve the highest levels.

The students generally adhered to the recommendation that the exploration be between 6 and 12 pages long. However there were many that were too long. These were often found to be self-penalizing.

Candidate performance against each criterion

Criterion A:

This criterion was addressed well by most of the students, with work being coherent and organized to different extents. In general, they made an attempt to provide a relevant introduction, a rationale, an explicit aim and some sort of conclusion. They also tried to explain relevant concepts and took conciseness into consideration. However, some teachers did miss the subtle differentiation between level 3 and 4, which is about the conciseness and completeness of student work. For instance, very lengthy tables of data may be relegated to an appendix, with a summary in the text where the information is used. Similarly, pages after pages of repetitive calculations would affect the conciseness and flow of the paper; one or two sample calculations would suffice and the rest could be summarized in a table.

Students should be more careful with the stated aim matching what they write and present in their work, and the conclusion they reach. If they find themselves unable to write a paper originally intended because of space constraints or any other reasons, then it would be wise for them to adjust the intended aim accordingly.

In addition, work that depended on a lot of secondary sources tends to have less coherence and is more difficult to follow.

It is crucial that quoted information be correctly cited at the point in the exploration where it appears, both for the flow of the piece and for academic honesty.

Criterion B:

This criterion was appropriately dealt with in most of the explorations. There were plenty of varieties of mathematical presentation displayed but it is essential that students are reminded that these are to be appropriate. Most students were able to use appropriate terminology and notations in their work, including the use of appropriate ICT tools. However, there were students who still used inappropriate computer or calculator notation (this is not an issue if generated by the software), did not define key terms and employed inappropriate presentations, like poorly labelled diagrams or badly scaled graphs. Graphs copied from the Internet were inserted but without any real purpose. Graphs need a purpose and not just included to "use multiple forms of mathematical representation". Mathematical formulas and theorems just taken from the Internet were often included but did not always really add to the students' work.

Criterion C:

This criterion proved to be the most difficult for teachers to assess or interpret and appeared to be the least understood by both students and teachers. It seems that too many teachers stated that they 'saw' engagement but this was not supported by the work submitted. They simply assumed that interest in the topic chosen by the student meant high personal engagement. High marks should not be awarded to students who just stated how much they enjoyed the topic or who demonstrated enthusiasm in class, unless this is seen in the exploration itself. Simply stating, "...it interests me..." is not personal engagement.

Students who explored a common investigation/textbook problem without any personal input or extension would not usually achieve the higher achievement levels in this criterion. Nevertheless, a number of teachers did seem to understand this criterion and were able to transmit that information to their students effectively. Some explorations do lend themselves more readily to high levels for personal engagement - for example those where students do their own research and data collection. With the more descriptive or historical topics it is not particularly easy to score highly here.

It is important to note that this criterion cannot be used to penalize late submission of work.

Criterion D:

This criterion was clearly understood well by many teachers and students and there was a wide range of achievement here. Many students simply described the results in their explorations. They also sometimes reflected on why they found the exploration interesting or enjoyed learning about it. Less often did they reflect on the analytical process of exploring. Many reflections were superficial. There were also cases where students would undertake explorations similar to the old portfolio tasks using the same questioning technique to reflect on the process. This did not lend itself to meaningful reflection.

Many students were under the impression that reflection could only come through in the conclusion and hence missed out on the opportunity of demonstrating substantial evidence of critical reflection throughout the exploration.

Higher levels were generally awarded to those students who considered further exploration, discussed implications of results, compared strengths and weaknesses of mathematical approaches and contemplated different perspectives.

Criterion E:

There was a wide range of achievement in this criterion from the lowest to the highest. Most students employed relevant mathematics that was commensurate with the level of the course, since the majority of topics were chosen to allow students to demonstrate at least 'some' mathematics. Students could be seen using many areas of mathematics from sequences to differential equations. In general, most teachers were able to determine whether or not the work was commensurate with the level of the course.

Regression analysis was used extensively but not with thorough understanding demonstrated. There were cases where the regression model was simply created and applied via technology with very little justification of their choice of regression model.

Teachers occasionally awarded high marks for work with numerous calculations even if no clear understanding was shown. In addition just showing the correct answer is not the same as showing understanding; it must be demonstrated. Students who did an excellent job of explaining their reasoning throughout their papers would gain the higher marks. A considerable number of students chose topics where the mathematics involved was clearly beyond their understanding. This was often because it had been taken from other sources. Although students did reference these sources, it was clear that many did not understand what they were doing mathematically.

The mathematics used need only be what is required to support the development of the exploration. This could be a few small topics or even a single topic from the syllabus. It needs to be made clearer to the students that it is better to do a few things well rather than trying to do more mathematics badly. If the mathematics used is relevant to the topic being explored, commensurate with the course, and understood by the student, then it can achieve a high level.

Recommendations and guidance for the teaching of future candidates

- Teachers need to become familiar with the assessment criteria. There are numerous examples located in the Teacher Support Material (TSM) on the Online Curriculum Centre (OCC), and they should familiarize themselves with the goals of the exploration, and how achievement levels for the new criteria are awarded by referring to the annotated student work in the TSM.
- One of the major issues was the lack of annotation and/or comments specific to individual student work provided by the teachers. In many cases, where comments were provided, these tended to be paraphrased from the descriptors. Teachers are reminded that it states in the TSM that one of their responsibilities is to assess the work accurately, annotating it appropriately to indicate where achievement levels have been awarded. This includes marking the mathematics and identifying any errors. These comments greatly help to confirm the level awarded by the teacher. Without supporting comments, changes are more likely.
- Teachers should provide relevant background information about their courses where appropriate.
- Teachers need to follow the procedures in the guide which allows students to submit a first draft. This way, teachers can assess the suitability of the topic, check the general organization and coherence, orally test the students' knowledge of the mathematics and most importantly, ensure that the work is that of the student and not just a regurgitation of Wolfram, Wikipedia and other math sites.
- If students are going to type their work, they need to use an appropriate equation editor to avoid errors in notation. Students should not insert screenshots of equations and formulas from Wikipedia or Wolfram. This habit is a good indicator that the work is not their own. They would also benefit greatly from explicit instruction in the use of those tools.
- Teachers should discourage students from choosing common or textbook topics without a clear plan on how to make them into a proper exploration (with enough relevant mathematics in it) since these tended to be research based with very little personalization or creativity demonstrated. Yet teachers should not predefine certain topics for their students or limit them to a certain area of mathematics.
- Teachers need to emphasize the importance of clear referencing and proper citation in student work. Many students provided a bibliography or work cited page at the end of their documents without identifying how these resources have been used in the body of their work. Students should be guided to cite all resources used in the body of their work including all data and images from secondary sources.
- Teachers must annotate and write comments on student work. Those schools where teachers did this were more likely to have scores confirmed, as would be expected; the reasoning behind the scores awarded was readily apparent.
- Teachers could issue practice explorations to hone/practice certain specific skills.
- Students should be reminded of the guidelines that the exploration should be between 6 and 12 pages long.
- Schools should be strongly discouraged from mandating a particular type of exploration. Rather students should be free to explore an area of their choice.

Further comments

- There were instances of potential plagiarism, with many students using sources such as

Wolfram or Wikipedia to copy content, formulas or ideas.

- The mathematical content was often lacking even though the exploration itself was well written and scored well.
- The general feeling after seeing the variety of explorations is that this new IA has provided the students with a great opportunity to explore what they wished, and given them the opportunity to appreciate mathematics in their own way.
- The exemplar materials and the frequently asked questions in the TSM have been/will be updated after the first live session. Teachers should make sure that they read these documents carefully, along with the updated guidance on applying the criteria.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 34	35 - 46	47 - 56	57 - 65	66 - 75	76 - 90

General comments

The areas of the programme and examination which appeared difficult for the candidates

- Integration, definite and indefinite
- Probability, combined and conditional
- Application of discriminant
- Direction vectors of lines
- Infinite geometric series in a generalized context

The areas of the programme and examination in which candidates appeared well prepared

- Quadratic functions
- Arithmetic sequences and series
- Logarithms
- Derivatives of polynomials
- Intersection of lines in vector form
- Probability on a tree diagram

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: the quadratic function and its graph

Candidates showed great familiarity with the quadratic function and the vertex. A common error was to give a negative value for h. In solving for a, many found the substitution and subsequent algebra to be straightforward. Occasionally a candidate would expand the quadratic before substituting the given values, which led to a more burdensome expression.

Question 2: arithmetic sequences and series

A clear majority of candidates performed well in this question, finding little difficulty when working with arithmetic formulas.

Question 3: definite integrals, volumes of revolution

Many candidates recognized the link between the definite integral and the volume. However, a surprising number of candidates did not substitute limits into a correctly integrated function. Common errors included substituting limits into the original function, or into its derivative function. Some integrated x^2 and then squared that result before substituting limits. Some candidates gave a numerical value which included $+C$. These are but a few of the creative misconceptions shown by candidates in what was meant to be a straightforward definite integral.

Question 4: logarithms

The values of the three logarithms were well known to many candidates, although a number of candidates gave an answer of 2 for $\log_{16} 4$. Some attempted a change of base, suggesting a formulaic approach to logarithms rather than as a concept of exponents. Many could use their values to create the equation, although many had difficulty working with an exponent of $\frac{3}{2}$ and did not reach a finished integer result.

Question 5: probability of combined events

Candidates who recognized the need for the intersection of Y and F often completed the question successfully. Those who created a Venn diagram in their working tended to be more successful, while those who employed a formula tended toward greater misconceptions. A number of candidates confused the concepts of intersection and union, and so substituted incorrectly into their formula. Some assumed independence when calculating $\mathrm{P}(Y \cap F)$, while others multiplied the complements of each individual probability. Others correctly found the probability of the intersection, but did not recognize that the complement is needed to answer the question.

Question 6: definite integral of a composite trig function.

Few candidates were able to integrate the function correctly, often giving $2\sin 2x$ or $-\frac{1}{2}\sin 2x$ as a result. Some convoluted their process by substituting a double-angle identity for $\cos 2x$, and often substituted limits into this identity. While a number of candidates recognized that $\sin 2\pi = 0$, few recognized that $\sin 2a = 1$ gives $2a = \frac{\pi}{2}$. Even fewer candidates clearly indicated that $\frac{5\pi}{4}$ is the only value in the given domain.

Question 7: derivative of polynomial function, discriminant of quadratic

Most candidates could derive the polynomial function correctly, and while some recognized the discriminant nature of part (b), only a handful reasoned that a non-negative quadratic requires $\Delta \le 0$.

Question 8: vector equations of lines

Noteworthy is the number of candidates who gave a correct vector equation of a line, yet when asked for the direction vector, calculated a magnitude or gave a position vector instead, even after being prompted to find $\overrightarrow{AB}$. Some answered with phrases such as “to the right”, which further suggests the concept is not well understood.

Many candidates wrote the equation of the line using $L_1 =$, which suggests a significant misunderstanding of the geometric nature of vector equations.

Many candidates found the point of intersection of two lines, although a common error was to use the same parameter for each line.

In finding the angle between lines, a number of candidates mistakenly used position vectors in the formula. At times the candidate would write an incorrect direction vector in part (i), and then proceed to use correct vectors in part (ii), which again reveals a fundamental misunderstanding of what is meant by "direction" in a vector equation.

Question 9: conditional probability

Candidates know how to complete a tree diagram of probabilities, and most showed the interpretive ability to multiply along one set of branches to find a probability.

Candidates had some difficulty when finding the probability of at least one win, as this required looking at three of the four pathways along the tree, with many neglecting to include the probability that Bill wins both games. A number of candidates cleverly employed the complement principle with little error.

Far fewer candidates could interpret the conditional probability correctly, often missing that the numerator of the formula is equivalent to Bill winning both games. Common errors were to calculate the conditional probability by using $\dfrac{\frac{2}{3}\times\frac{14}{15}}{\frac{14}{15}}$, or giving $\mathrm{P}(\text{Bill wins both}) = \frac{2}{3}$ as a final result.

Question 10: geometric sequences and series

While many candidates completed the table correctly, many incorrectly gave $x_2 = 6$, which suggests students were looking for an easy pattern rather than thinking about the geometric nature of the situation. Continuing the pattern to find A_6 proved easy for many.

While some candidates recognized the need for an infinite geometric series in part (c), most attempted to work numerically, assuming 32 for the first term. Few set their equation equal to k, and fewer still found the correct first term in terms of k. Some candidates employed a guess and check strategy with mixed results.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 32	33 - 44	45 - 54	55 - 63	64 - 73	74 - 90

The areas of the programme and examination which appeared difficult for the candidates

Candidates in this session had difficulties in the following areas of the programme:

- Using a graphic display calculator (GDC) to find a regression line and a correlation coefficient
- Geometric interpretation of vectors and scalar product
- Interpreting derivative as instantaneous rate of change
- Displacement vs. distance travelled
- Applying the quotient rule to find the gradient at a point
- Estimating the mean and standard deviation of grouped data from a GDC
- Curve sketching
- Algebraic simplification

The areas of the programme and examination in which candidates appeared well prepared

For students who were well prepared, there was ample opportunity to demonstrate a high level of knowledge and understanding on this paper. The following areas of the programme were handled well by most students.

- Triangle trigonometry including sine and cosine rules
- Binomial expansion
- Scalar product and perpendicular vectors
- Cumulative frequency
- Normal and binomial distributions

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: triangle trigonometry

Candidates generally did well on this question apart from a few candidates who performed the required calculations in radian mode. A few candidates assumed incorrectly that the triangle was right – angled.

Question 2: binomial theorem

The majority of candidates were able to state the number of terms in the expansion, and many made good progress in part (b) working with the formula for finding a term. While most clearly recognized the binomial expansion, a significant number of candidates painstakingly wrote out the first 10 rows of Pascal's triangle and/or attempted to expand the expression $(x+3)^{10}$ manually. There are still a significant number of candidates who add the parts of a term rather than multiply them.

Question 3: linear regression

It was surprising the number of candidates who could not use their calculator to perform a simple linear regression. Many candidates incorrectly, attempted to use two points from the table to find the equation of a straight line to identify the values of a and b. Candidates were generally able to use their equation to interpolate correctly. In part (b), the correlation was successfully described but many could not work out how to find the r value, as they seemed unaware of how to turn the calculator diagnostics back on after a mandatory reset.

Question 4: vectors and scalar product

This question was well done by the majority of candidates. That said, there was a great deal of sloppy work in part (a) with candidates not joining endpoints of vectors, not drawing straight lines or failing to include the direction arrow. Sloppy work here often cost candidates two marks. In part (b), if candidates recognized that for perpendicular vectors the scalar product is zero, then they had little difficulty finding the required value. There were however, many that failed to recognize this important relationship.

Question 5: rate of change

Part (a) was generally well done with most candidates correctly finding $P(5)$. Some did not read the question carefully and set $t = 4$ and many candidates lost the final mark when they failed to write their answer as an integer number of deer, rounded down. The relationship between the rate of change and the derivative was not well understood and very few were able to interpret instantaneous rate of change correctly in the context of the problem. Yet again, many candidates incorrectly worked in degree mode.

Question 6: kinematics

While the majority of candidates understood the connection between velocity and displacement, few were able to work through the question systematically and use all the information provided to obtain the overall displacement. Most students realized that integration of V_R was needed, however their solutions often broke down when the constant of integration was omitted. In a valid approach using a definite integral, disappointingly few candidates used their GDCs to evaluate their integral and quite a few candidates failed to take account of the fact that both people were 60 m from Buenos Aires when $t = 0$. Although the units were inappropriate for the context, feedback from examiners indicated that students did not seem to be adversely affected by this.

Question 7: equation of a normal

This was an accessible problem that created difficulties for candidates. Few realized that the quotient rule was required to find the gradient of the tangent. Many who were unsuccessful stated that the gradient was $\frac{5}{2}$, with only a few giving work to support this, i.e. $\frac{g'(2)}{h'(2)}$. Candidates often incorrectly assumed that $g(x)$ and $h(x)$ were linear, and they attempted to find these. Other careless and common errors included incorrect substitution into the quotient rule; incorrect evaluation of the tangent gradient; not evaluating the normal gradient from their tangent gradient; incorrect evaluation of the normal gradient; and incorrect substitution into the straight line equation to get the final answer.

Question 8: cumulative frequency, distributions

Those students who had adequately covered this area of the syllabus found this question straightforward. Parts (a) and (b) were generally well done. Some candidates found difficulty interpreting the cumulative frequency curve and expressing the values in terms of a frequency table. In part (c), many students used the upper limit given in the frequency table that did not affect the standard deviation but did affect the value of the mean. Again, surprisingly, calculators were often not used effectively to find these values with many candidates resorting to manual approaches. In parts (d) and (e), candidates generally recognized the nature of the distributions in question and often obtained full marks or full follow through if errors were made in (c). However, candidates who did not show their working in these parts obtained no follow through marks.

Question 9: circular functions

Candidates could obtain a great deal of marks in this question with little thinking and analysis and good use of their GDC. However, those who did not explain their reasoning lost a great deal of marks for incorrect answers. Part (a) was normally well done with some well – graphs that showed the key features – scale, shape, zeros, and max/min. That said, it was disappointing that many candidates continue to submit sloppy work where care has not been taken in drawing the curve. As marks were allocated for approximately correct domain and range, candidates who did not give any indication of scale lost two of the three marks. Part (b) could be read directly from the graph, with finding the x–coordinates of the maximum and minimum, the only working required. Candidates stated their answer to part (b) showing a great deal of variety in representing inequalities, even inventing their own notations! Part (c) had mixed results with some candidates being able to “write down” the answers from detailed graphs in part (a) while others struggled to recognize that *a* represented the amplitude and c, the phase (horizontal) shift.

Question 10: rational functions

Most candidates were successful with part (a) correctly identifying the horizontal and vertical asymptotes. Of course, there were some that did not write them down as equations and lost both marks. Part (b) was poorly done, as many did not recognize the need for the intersection point of the asymptotes. Rather, they often substituted the point $(1, 3)$ into f to find q, which of course did not work, as this was not a point on the graph of f. Although part (c) required only some rather simple algebra, most candidates could either not carry it through correctly or had no idea how to start it. Part (d), the discriminator, was handled well by a minority of only the strongest candidates.

Recommendations and guidance for the teaching of future candidates – paper 1 and 2

General

Teachers need to ensure that they are familiar with all the changes for May 2014. This includes the syllabus and approaches to teaching and learning, as well as the command terms and the new formula booklet. There was evidence in all papers that not all the changes had been taken on board. All the required information is available on the OCC.

Some teachers commented that questions of an investigative nature are not appropriate for a timed examination, that it is a type of question used in the old IA portfolios. However, as expressed on page 11 of the Mathematics SL Guide, it must be recognized that such questions are part of the new course, and that such an inquiry-based approach to learning is integral to the teaching of the course. While paper authors recognize the timed nature of the examination, students who are trained to make and justify conjectures will find such questions more accessible.

Candidates need to be familiar with the vocabulary and underlying concepts, rather than simply practicing standard routines and processes. Candidates should be encouraged to read each question carefully and consider what information is given and what a question is asking them to do before they begin their work. It is clear that candidates are much more comfortable with questions which allow them to simply reach for the formula booklet and substitute into a formula. For example, many students will often reach for a formula when encountering questions in conditional probability, but most do not have the conceptual understanding to go beyond the formula booklet.

They should be exposed to past IB exams and markschemes, and are encouraged to use these for practice. Looking at the markschemes can help both the students and the teachers understand what is required by the different command terms, such as "write down", "find", "show that", or "hence".

Candidates should be encouraged to show all their working in a neat, organized manner, using proper mathematical notation. Being familiar with the format of the examination paper will help candidates with this. If a mistake is made, it is best to simply draw an "X" or a line through any unwanted working.

Paper 1

In general, the questions did not require a great deal of arithmetic or algebraic manipulation, but they did require good understanding of the concepts. Candidates who understood the concepts were able to answer questions with a minimum amount of work. Unfortunately, candidates who did not understand the concepts often found themselves stuck with convoluted and unnecessary working which often did not lead them to an answer.

Paper 2

Paper 2 is a GDC required paper, not simply a GDC allowed paper. Candidates should be encouraged to consider whether use of the GDC is appropriate when answering any question on Paper 2. Although basic GDC skills are improving, there are still candidates who are opting for an analytical approach rather than a more efficient GDC approach eg definite integrals, evaluating numerical derivatives, solving equations, etc. Alternative methods often lead to simple algebraic errors and consume valuable time. It should be emphasized that once an equation is established, no algebraic working is needed to support an answer. Teachers should place greater emphasis on integrating the use of technology as a tool for learning and for better understanding key concepts as well as for solving problems by communicating solutions clearly. Many candidates continue to struggle with what work to show when using technology. “Found using GDC” is not a valid explanation and using calculator specific language in the working should be discouraged.

Candidates continue to lose marks sketching poor graphs that do not show the required features. This would seem a straightforward problem to overcome by practicing more often when the need arises during class. However, students should be taught not simply to transcribe graphs from their GDC without considering their intrinsic knowledge of key features and behaviours of functions. They should be encouraged to use the appropriate GDC tools to find and label key features of graphs.

Candidates should ensure that their GDCs are in the correct mode (eg radian/degrees). Numerical values needed in subsequent parts should be stored in the GDC memory (the more accurate “long” value). Inaccurate values or premature rounding of values can lead to wrong final answers.