May 2013 subject reports

MATHEMATICS SL TZ2

(IB Africa, Europe & Middle East & IB Asia-Pacific)

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 18	19 - 36	37 - 50	51 - 61	62 - 73	74 - 84	85 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2013 examination session the IB has produced time zone variants of Mathematics SL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 13	14 - 19	20 - 23	24 - 28	29 - 33	34 - 40

The range and suitability of the work submitted

The great majority of samples contained tasks taken from the IB-designed set. By far the most common tasks were Lacsap's Fractions and Gold Medal Heights. A few tasks were teacher-designed with mixed success. There were schools which sent old tasks and a 10-point penalty was applied to these portfolios. In a couple of cases schools had made minor modifications to old tasks and presented them as new. These were adjudicated by the Principal Moderators and where it was deemed that the changes were not significant enough to differ the task from the older IB version, a 10-point penalty was applied. The reason for the shelf-life is that solutions quickly find their way to the internet and if the task is not sufficiently revised then the solutions available give an advantage to candidates using these slightly revised tasks.

Candidate performance against each criterion

Criterion A

Much of the work was done appropriately and consistently against this criterion. However, despite many subject reports and additional notes available on the OCC, some work included computer notation, or used variables inconsistently, or ignored the use of an approximately equals sign for estimations. In Type II tasks, some candidates failed to distinguish “parameters” from “variables” or “constraints”. Candidates must also realize that distinct functions require distinct names in modelling tasks.

Criterion B

Some communication was excellent, with clearly labelled diagrams and graphs, and coherent explanations of analyses and results. Often, though, results appeared without sufficient explanation and axes were not labelled or scales not provided on graphs. Many candidates failed to produce a scatterplot of raw data or created continuous graphs when the domain was discrete. The use of a “Question & Answer” format is still an issue. The portfolio tasks are not homework assignments and should be treated as a mathematical essay, not a series of questions and answers. Some candidates presented detailed or unnecessary explanations of how they used technology, or added theoretical background that did not really improve the quality of work.

Criterion C

Type I:

This criterion requires the generation and organization of data before any analysis is attempted. In certain tasks (especially the Circles task) candidates started with an analytical analysis and then created data from their general statement, or offered none at all. This runs counter to the notion of producing a conjecture from an observed pattern of behaviour and the result was that marks under criterion C were limited to C1 or C2. Validity testing was often done using values that were a part of the analysis that developed the general statement instead of new and further values that were tested against the actual patterns of mathematical behaviour. For example, in the Lacsap’s Fractions task, candidates would simply produce new fractions from their general statement but did not assess the validity of results against patterns that were available within the triangle of fractions.

Type II:

This criterion requires that candidates provide an analytical analysis that leads to a suitable model function using their knowledge of mathematics. The mathematics is expected to be at the level of the syllabus so that candidates should recognize that certain situations require certain approaches. In many cases candidates simply used regression techniques to establish models, or to establish possible models that they subsequently pursued analytically. In some cases efforts were limited to linear regressions with no other consideration of possible model functions. These responses were limited to C2 as the requisite analysis at the level of the program was not achieved.

The manners in which candidates addressed the goodness of fit varied in quality from none at all to careful consideration of the fit at various intervals. Some included a quantitative analysis of fit although this is not required for mathematics SL. Lastly, whereas the level C5 requires that candidates compare their analytical model to a new set of data, many simply created a new model from scratch.

Criterion D

Type I:

Candidates who managed to achieve the general statement did not always properly consider the scope and/or limitations of the statement. Informal explanations were rarely provided and, if so, were poorly expressed. Some candidates simply reiterated the steps they took in their analysis. A few candidates did a good job of explaining where the statement came from, sometimes with good formal methods.

Type II:

Many candidates did not sufficiently interpret their models in the contexts of the tasks. Given that marks of D3 or higher depend upon such interpretation, some very good mathematical analyses did not score well because the interpretation was either poor or ignored completely. A small number of candidates were careful to extend aspects of the model into the real-world scenario, often doing extra research to better understand the situation. The issue of accuracy is often ignored. One aspect of modelling should be consideration of how well the model fits the situation and how accurate it really needs to be to provide a good fit. A model with parameters that have only a few significant digits of accuracy may be nearly as good as a model function with parameters of 10- significant digit accuracy.

Criterion E

Some tasks are better suited to the use of technology, yet all tasks require this. The use in Type II tasks was generally good while the use in Type I tasks was generally poor. There was little effort to use graphs to explore possible relations, nor to use technology to test the scope and limitations. In Type II tasks there was good use of graphing technology although some candidates appeared to believe that any graph would suffice. Some graphs were not effective in demonstrating the issue at hand due to a poor window choice. The opportunity for full and resourceful use of graphs, such as using multiple functions on the same grid or expanded windows to show long-term behaviour, was missed by a good number of candidates.

Criterion F

For the most part candidates were appropriately awarded F1. In a few cases teachers had too lenient or too strict standards for this criterion. It would appear that some teachers are imposing their own classroom standards here. It was often unclear on what basis a particular candidate may have been awarded a mark of F2.

Recommendations and guidance for the teaching of future candidates

If teachers and candidates carefully reviewed the assessment criteria and considered their work in this light then candidates would be much more successful. Given that the portfolio is a criterion-referenced piece of work it is not sufficient for the candidate to produce work that is holistically assessed as “good work”. Each accomplishment within the work worth assessing must reflect some aspect of the assessment criteria.

Candidates should better learn to provide proper notation and communication. This can be accomplished in the classroom through short exercises that require such things in the solution of homework problems. It would help if the teacher set a standard for what is appropriate and maintained this standard throughout the course.

The process of developing a general statement and of producing a model function can be taught. Many syllabus topics lend themselves nicely to this, such as arithmetic and geometric sequences and exponential or sinusoidal functions. The connection between these topics and the portfolio tasks should be consciously made in class.

Many application problems in classrooms are often dealt with as simple exercises to get “the answer”. Exploring these applications by changing parameters or applying the results to similar situations will help candidates see the connection between the maths and the real world.

In general, given that most candidates presented word-processed work, one would expect that a basic spreadsheet and some internet resources are available. The teacher is responsible for building their own sense of confidence with these and other technological tools that are available so that they may make clear the expectations regarding their use.

Further comments

As we move to a new model of internal assessment next year it will be ever more important that teachers and candidates come to know and understand the criteria. It is essential that teachers explain the criteria levels to candidates so that they know what is expected of them. As teachers prepare their samples for moderation they should also stop and consider that the moderation will go much better if the proper background knowledge and expectations regarding solutions and the use of technology are provided. Comments should be made freely and directly on the candidates’ work so that the moderator can better understand why the given mark was awarded.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 49	50 - 59	60 - 68	69 - 78	79 - 90

General comments

In general, the questions in this year's paper did not require a great deal of arithmetic or algebraic manipulation, but they did require good understanding of the concepts. Candidates who understood the concepts were able to answer questions with a minimum amount of work. Unfortunately, candidates who did not understand the concepts often found themselves stuck with convoluted and unnecessary working which often did not lead them to an answer.

The areas of the programme and examination which appeared difficult for the candidates

- Recognizing the symmetry of a quadratic function
- Understanding the difference between area and the definite integral
- Integrating exponential expressions
- Transformations of circular functions
- Rules of logarithms
- Recognizing the value of a trigonometric ratio for an angle in the first quadrant

- Necessary conditions for points of inflexion
- Reasoning skills, and answering questions with non-traditional contexts

The areas of the programme and examination in which candidates appeared well prepared

It was pleasing to note that the large majority of candidates were able to make a good attempt on each question, and very few questions were left entirely blank. Time did not seem to be a factor, as it appeared that candidates were not rushing through the later questions. In addition, candidates did not seem to have any trouble adapting to the new answer booklets. In general, candidates showed good preparation and knowledge in the following areas:

- Simple probability and percentages
- Differentiation and integration of simple polynomials
- Matrix multiplication
- Composite functions
- Relationship between velocity and displacement
- Interpreting information presented in tables and graphs
- Answering questions in straightforward context, using formulaic approaches

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Inverse and Composite functions

The overwhelming majority of candidates answered both parts of this question correctly. There were a few who seemed unfamiliar with the inverse notation and answered part (a) with the derivative or the reciprocal of the function. A few candidates made arithmetic errors in part (b) which kept them from finding the correct answer.

Question 2: Matrices

While most candidates were successful on both parts of this question, there were some who seemed unfamiliar with matrix multiplication. Many candidates realized that they did not need to do the entire matrix multiplication to find the elements they needed. Unfortunately, there were a few who selected the wrong element to find *q*, ending up with the equation $3+0q=3$.

Question 3: Working with Logarithms

This question proved to be surprisingly challenging for many candidates. A common misunderstanding was to set *p* equal to 6 and *q* equal to 7. A large number of candidates had trouble applying the rules of logarithms, and made multiple errors in each part of the question. Common types of errors included incorrect working such as $\log_3 p^2 = 36$ in part (a), $\log_3\left(\frac{p}{q}\right)=\frac{\log_3 6}{\log_3 7}$ or $\log_3\left(\frac{p}{q}\right)=\log_3 6-\log_3 7$ in part (b), and $\log_3(9p)=54$ in part (c).

Question 4: Inverse Functions and Graphs

In part (a) of this question, most candidates were able to find the value of $f(2)$ correctly, while some had trouble finding $f^{-1}(-1)$. Many candidates tried to find an equation for the function, or to make tables of values to help them find their answers. The intent of this question was to read the answers from the given graph. Candidates should be reminded that when the command term is "write down", there is no need for them to do large amounts of working.

In part (b) of this question, candidates were generally successful in reversing the *x* and *y* coordinates of key points or reflecting in the $y = x$ line to correctly sketch the graph of the inverse function. Common errors included not sketching the graph for the appropriate domain, or sketching the graph of $f(-x)$ or the graph of $-f(x)$.

Question 5: Graph of Circular Function

Many candidates were able to answer all three parts of this question with no difficulty. Some candidates ran into problems when they attempted to substitute into the equation of the function with the parameters *p*, *q* and *r*. The successful candidates were able to find the answers using the given points and their understanding of the different transformations.

Part (b) seemed to be the most difficult, with some candidates not understanding the relationship between *q* and the period of the function. There were also some candidates who showed working such as $\frac{2\pi}{b}$ without explaining what *b* represented.

Question 6: Kinematics

A good number of candidates earned full marks on this question, and many others were able to earn at least half of the available marks. Most candidates knew to integrate, but there were quite a few who tried to find the derivative instead. Many candidates integrated the term $6e^{2t}$ incorrectly, but most were able to earn some further method marks for substituting into their integrated function. The majority of candidates who substituted (0, 10) into their integrated function knew that $e^0 = 1$.

Question 7: Area and Integrals

There was a minor error on the diagram, where the point on the *y*-axis was labelled 2 (to indicate the length of the radius), rather than -2. Examiners were instructed to notify the IB assessment centre of any candidates adversely affected. Candidate scripts did not indicate any adverse effect.

While most candidates were able to correctly find the area of the quarter circle in part (a), very few considered that the value of the definite integral is negative for the part of the function below the *x*-axis. In part (b), most went on to earn full marks by subtracting the area of the quarter circle from 3π.

Candidates who did not understand the connection between area and the value of the integral often tried to find a function to integrate. These candidates were not successful using this method.

Question 8: Frequency and probability

Overall, candidates were very successful in parts (a), (b) and (c) of this question. Most of the errors in these parts had to do with candidates not understanding terms such as "at least" or "less than".

Part (d) was quite challenging for candidates, who may not have read the question carefully and studied the values in the diagram. Many seemed confused by the idea that not all the girls who were given a second chance were selected. In part (d)(ii), many did not find the percentage of the whole group, but rather the percentage of the girls who were given a second chance.

Question 9: Simple derivatives and quadratic functions

In part (a), most candidates were able to correctly find the derivative of the function. In part (b), many candidates did not understand the significance of the axis of symmetry and the known point (0, 5), and so were unable to find $g(4)$ using symmetry. A few used more complicated manipulations of the function, but many algebraic errors were seen.

In part (c), a large number of candidates were able to simply write down the correct value of *h*, as intended by the command term in this question. A few candidates wrote down the incorrect negative value. Most candidates attempted to substitute the *x* and *y* values of the known point correctly into the function, but again many arithmetic and algebraic errors kept them from finding the correct value for *a*.

Part (d) required the candidates to find the derivative of *g*, and to equate that to their answer from part (a). Although many candidates were able to simplify their equation to $\cos x = 0$, many did not know how to solve for *x* at this point. Candidates who had made errors in parts (a) and/or (c) were still able to earn follow-through marks in part (d).

Question 10: Calculus

Nearly all candidates who attempted to answer parts (a) and (c) did so correctly, as these questions simply required them to understand the notation being used and to read the values from the given table.

In part (b), the majority of candidates earned one mark for stating that $h''(x) = 0$ at point P. As this is not enough to determine a point of inflexion, very few candidates earned full marks on this question.

Part (d) proved to be quite challenging for even the strongest candidates, as almost none of them used the product rule to find $h'(3)$. The most common error was to say $h'(3) = f'(3) \times g'(3)$. Despite this error, many candidates were able to earn further method marks for their work in finding the equation of the normal. There were also a small number of candidates who were able to find the equation for $h''(x)$, and from that $h'(x)$. These candidates were often successful in earning full marks, although this method was quite time-consuming.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 31	32 - 43	44 - 53	54 - 64	65 - 74	75 - 90

The areas of the programme and examination which appeared difficult for the candidates

- Solving equations with the graphic display calculator (GDC)
- Algebraic manipulations
- Binomial expansion

- Conditional probability
- Expected value
- Sketching a graph over a specific domain
- Area between curves

The areas of the programme and examination in which candidates appeared well prepared

Candidates demonstrated a good level of knowledge and understanding with most topics. Strengths included:

- Matrices with the GDC
- Normal distribution
- Triangle trigonometry
- Vectors
- Binomial probability

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Matrix inverse and equation

This question was easily accessible to most candidates. The majority of candidates worked both parts with their calculators, although a few candidates attempted to use a system of equations in part (b). A surprising number of candidates gave $\boldsymbol{X} = (-4, 4, 5)$ in part (b), missing the negative for the third element. Occasionally candidates obtained the correct answer in part (b) from the incorrect working of $\boldsymbol{X} = \boldsymbol{B}\boldsymbol{A}^{-1}$ which cost them a mark.

Question 2: Normal distribution

The normal distribution was handled better than in previous years with many candidates successful in both parts and very few blank responses. Some candidates used tables and *z*-scores while others used the GDC directly; the GDC approach earned full marks more often than the *z*-score approach. A common error in part (b) was to set the expression for *z*-score equal to the probability. Many candidates had difficulty giving answers correct to three significant figures; this was particularly an issue if no working was shown.

Question 3: Triangle trigonometry

The vast majority of candidates were very successful with this question. A small minority drew an altitude from C and used right triangle trigonometry. Errors included working in radian mode, assuming that the angle at C was 90°, and incorrectly applying the order of operations when evaluating the cosine rule.

Question 4: Intersection of lines in 3-D

Most students were able to set up one or more equations, but few chose to use their GDCs to solve the resulting system. Algebraic errors prevented many of these candidates from obtaining the final three marks. Some candidates stopped after finding the value of *s* and/or *t*.

Question 5: Geometric series

Many candidates were able to successfully obtain two equations in two variables, but far fewer were able to correctly solve for the value of *r*. Some candidates had misread errors for either 440 or 62.755, with some candidates taking the French and Spanish exams mistaking the decimal comma for a thousands comma. Many candidates who attempted to solve algebraically did not cancel the $1-r$ from both sides and ended up with a 4th degree equation that they could not solve. Some of these candidates obtained the extraneous answer of $r=1$ as well. Some candidates used a minimum of algebra to eliminate the first term and then quickly solved the resulting equation on their GDC.

Question 6: Binomial theorem

Many candidates struggled with this question. Some had difficulty with the binomial expansion, while others did not understand that the constant term had no x, while still others were unable to simplify a ratio of exponentials with a common base. Some candidates found $r=3$ using algebraic methods while others found it by writing out the first several terms. In some cases, candidates just set the entire expansion equal to 1280.

Question 7: Circle trigonometry

As to be expected, candidates found this problem challenging. In part (a), many were able to use right angle trigonometry to find the length of OC. Those who used a systematic approach in part (b) were more successful than those whose work was scattered about the page. While a pleasing number of candidates successfully found the area of sector AOB, far fewer were able to find the area of triangle BOC. Candidates who took an analytic approach to solving the resulting equation were generally less successful than those who used their GDC. Candidates who converted the angle to degrees generally were not very successful.

Question 8: Angle between vectors

The majority of candidates successfully found the vectors between the given points in part (a). In part (b), while most candidates correctly found the value of *a*, many unnecessarily worked with the magnitudes of the vectors, sometimes leading to algebra errors. Some candidates showed a minimum of working in part (c)(i); in a "show that" question, candidates need to ensure that their working clearly leads to the answer given. A common error was simplifying the magnitude of vector AC to $\sqrt{20a^2}$ instead of $\sqrt{20+a^2}$. In part (c)(ii), a disappointing number of candidates embarked on a usually fruitless quest for an algebraic solution rather than simply solving the resulting equation with their GDC. Many of these candidates showed quite weak algebra manipulation skills, with errors involving the square root occurring in a myriad of ways.

Question 9: Binomial probability

Parts (a)(i) and (ii) were generally well done, with candidates either using a tree diagram or a binomial approach. Part (a)(iii) proved difficult, with many either having trouble finding $\mathrm{P}(X=2)$ or using $\mathrm{E}(X)=np$. A great majority were confident solving part (b) with the GDC, although some did write the binomial term. Those candidates who did not use the binomial function on the GDC had more difficulty in part (c), although a pleasing number were still able to identify that they were seeking $\mathrm{P}(X\le 5)$. While most candidate knew to use conditional probability in part (d), fewer were able to do so successfully, and even fewer still correctly rounded their answer to two decimal places. The most common error was to multiply probabilities to find the intersection needed for the conditional probability formula. Overall, candidates seemed better prepared for probability.

Question 10: Area between curves

There was a flaw with the domain noted in this question. While not an error in itself, it meant that part (b) no longer assessed what was intended. The markscheme included a variety of solutions based on candidate work seen, and examiners were instructed to notify the IB assessment centre of any candidates adversely affected, and these were looked at during the grade award meeting.

While some candidates sketched accurate graphs on the given domain, the majority did not. Besides the common domain error, some exponential curves were graphed with several concavity changes. In part (a)(ii), most candidates found the intersection correctly. Those who used their GDC to evaluate the integral numerically were usually successful, unlike those who attempted to solve with antiderivatives. A common error was to find the area of the region enclosed by f and g (although it involved a point of intersection outside of the given domain), rather than the area of the region enclosed by f and g and the y-axis. While some candidates were able to show some good reasoning in part (b), fewer were able to find the value of m which maximized the area of the region. In addition to the answer obtained from the restricted domain, full marks were awarded for the answer obtained by using the point of tangency.

Recommendations and guidance for the teaching of future candidates – paper 1 & 2

Candidates should be encouraged to show all their working in a neat, organized manner. Mathematical working does not only mean algebraic steps, but also the reasoning in obtaining a solution. Correct mathematical notation is not specifically assessed in the examination papers, but a strong foundation in notation is an essential feature of mathematical learning. Sloppy notation by candidates can be forgiven if the mathematical elements of a question are reasonably expressed. However, there are times when sloppy notation interferes with the mathematical expression of a concept.

If a mistake is made, it is best to simply draw an "X" or a line through any unwanted working. Incorrect working that is not crossed out will be considered as part of the answer, and therefore marked according to the markscheme, even if different working is seen later.

Candidates should be exposed to past IB exams and markschemes, and should use these for practice. Working past exams under timed conditions exposes candidates to different types of questions and helps them learn to pace themselves so they can more easily complete the exam in the time allotted.

There were comments in the teacher feedback forms suggesting that students were not familiar with some of the notation and terminology used in the examination. The notation and terminology used in the question papers are published in Mathematics SL subject guide. Using questions from past papers will help students become familiar with the style and form of questions, as well as with the notation and terminology.

Looking at the markschemes can help students and teachers understand what is required by the different command terms, such as "write down", "find", "sketch", or "explain". Graph paper is generally not required for a sketch. Graphs should be carefully sketched, paying attention to, and labelling, key features, either general ones, such as intercepts, or specific ones identified in a question. In a “show that” question, it should be obvious from the penultimate step that the next line gives the required result. This may require candidates to show additional simplification steps.

Candidates should be encouraged to read each question carefully and consider what information is given and what a question is asking them to do before they begin their work. Candidates can also look for clues within the given information. They should check when transcribing information in order to avoid errors.

Candidates are not making the connections between different parts of the same question. Most questions are designed around a “theme” and students are normally expected to use the results of one part in subsequent parts, but they often fail to see the significance of their results. Candidates should be taught to reflect on the meaning of results by asking questions like, “why is this part being asked?” or “how is it relevant to the next part?”

Students need to be given experience working with ‘non-conventional’ questions on different areas of the syllabus which will help them to transfer their learning from one area of mathematics to another. Encourage students to view mathematics as a whole, rather than a set of discrete topics. When studying the expansion of binomials, candidates should practice finding specific terms. An understanding of conditional probability should extend beyond substituting into the formula. Probability and statistics continues to be the weakest area of knowledge demonstrated by candidates. Schools should be encouraged to ensure that candidates have been fully prepared in all aspects of this course.

Paper 1

While many students will often reach for a formula when encountering questions in conditional probability, few have the conceptual understanding to go beyond the information booklet. In situations where a tree diagram is not provided, it may be helpful to instruct students to create their own diagram, in an effort to more visually determine that the denominator in the formula is calculated by two pathways.

Paper 2

Paper 2 is a GDC required paper, not simply a GDC allowed paper. Candidates should be encouraged to consider whether use of the GDC is appropriate when answering any question on Paper 2. Although basic GDC skills are improving, there are still candidates who are opting for an analytical approach rather than a more efficient GDC approach particularly with the less obvious applications of solving equations, finding intersections or evaluating definite integrals. This often leads to simple algebraic errors and consumes valuable time. It should be emphasized that once an equation is established, no algebraic working is needed to support an answer. Teachers should place greater emphasis on integrating the use of technology as a tool for learning and for better understanding key concepts as well as for solving problems by communicating solutions clearly.

Many candidates continue to struggle with what work to show when using technology. Working should be used to show any set up required before using the GDC. Mathematical notation should be used, not calculator notation. Writing “used GDC” is not enough evidence of a valid approach. Examples of this may be seen in the student solutions for the May 2010 papers, included in the subject reports on the OCC.

Candidates should ensure that their GDCs are in the correct mode (e.g. radian/degrees). They should be taught not simply to transcribe graphs from their GDC without considering their intrinsic knowledge of key features and behaviours of functions. They should be encouraged to use the appropriate GDC tools to find and label key features of graphs.

Numerical values (including answers given correct to three significant figures) should be stored in the memory, and the more accurate “long” value used if needed in subsequent parts. Inaccurate values or premature rounding of values can lead to wrong final answers.