May 2013 subject reports

MATHEMATICS SL TZ1

(IB Latin America & IB North America)

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 34	35 - 47	48 - 57	58 - 69	70 - 80	81 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2013 examination session the IB has produced time zone variants of Mathematics SL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 13	14 - 19	20 - 23	24 - 28	29 - 33	34 - 40

The range and suitability of the work submitted

The great majority of samples contained tasks taken from the IB-designed set. By far the most common tasks were Lacsap's Fractions and Gold Medal Heights. A few tasks were teacher-designed with mixed success. There were schools which sent old tasks and a 10-point penalty was applied to these portfolios. In a couple of cases schools had made minor modifications to old tasks and presented them as new. These were adjudicated by the Principal Moderators and where it was deemed that the changes were not significant enough to differ the task from the older IB version, a 10-point penalty was applied. The reason for the shelf-life is that solutions quickly find their way to the internet and if the task is not sufficiently revised then the solutions available give an advantage to candidates using these slightly revised tasks.

Candidate performance against each criterion

Criterion A:

Much of the work was done appropriately and consistently against this criterion. However, despite many subject reports and additional notes available on the OCC, some work included computer notation, or used variables inconsistently, or ignored the use of an approximately equals sign for estimations. In Type II tasks, some candidates failed to distinguish “parameters” from “variables” or “constraints”. Candidates must also realize that distinct functions require distinct names in modelling tasks.

Criterion B:

Some communication was excellent, with clearly labelled diagrams and graphs, and coherent explanations of analyses and results. Often, though, results appeared without sufficient explanation and axes were not labelled or scales not provided on graphs. Many candidates failed to produce a scatterplot of raw data or created continuous graphs when the domain was discrete. The use of a “Question & Answer” format is still an issue. The portfolio tasks are not homework assignments and should be treated as a mathematical essay, not a series of questions and answers. Some candidates presented detailed or unnecessary explanations of how they used technology, or added theoretical background that did not really improve the quality of work.

Criterion C:

Type I:

This criterion requires the generation and organization of data before any analysis is attempted. In certain tasks (especially the Circles task) candidates started with an analytical analysis and then created data from their general statement, or offered none at all. This runs counter to the notion of producing a conjecture from an observed pattern of behaviour and the result was that marks under criterion C were limited to C1 or C2. Validity testing was often done using values that were a part of the analysis that developed the general statement instead of new and further values that were tested against the actual patterns of mathematical behaviour. For example, in the Lacsap’s Fractions task, candidates would simply produce new fractions from their general statement but did not assess the validity of results against patterns that were available within the triangle of fractions.

Type II:

This criterion requires that candidates provide an analytical analysis that leads to a suitable model function using their knowledge of mathematics. The mathematics is expected to be at the level of the syllabus so that candidates should recognize that certain situations require certain approaches. In many cases candidates simply used regression techniques to establish models, or to establish possible models that they subsequently pursued analytically. In some cases efforts were limited to linear regressions with no other consideration of possible model functions. These responses were limited to C2 as the requisite analysis at the level of the program was not achieved.

The manners in which candidates addressed the goodness of fit varied in quality from none at all to careful consideration of the fit at various intervals. Some included a quantitative analysis of fit although this is not required for mathematics SL. Lastly, whereas the level C5 requires that candidates compare their analytical model to a new set of data, many simply created a new model from scratch.

Criterion D:

Type I:

Candidates who managed to achieve the general statement did not always properly consider the scope and/or limitations of the statement. Informal explanations were rarely provided and, if so, were poorly expressed. Some candidates simply reiterated the steps they took in their analysis. A few candidates did a good job of explaining where the statement came from, sometimes with good formal methods.

Type II:

Many candidates did not sufficiently interpret their models in the contexts of the tasks. Given that marks of D3 or higher depend upon such interpretation, some very good mathematical analyses did not score well because the interpretation was either poor or ignored completely. A small number of candidates were careful to extend aspects of the model into the real-world scenario, often doing extra research to better understand the situation. The issue of accuracy is often ignored. One aspect of modelling should be consideration of how well the model fits the situation and how accurate it really needs to be to provide a good fit. A model with parameters that have only a few significant digits of accuracy may be nearly as good as a model function with parameters of 10- significant digit accuracy.

Criterion E:

Some tasks are better suited to the use of technology, yet all tasks require this. The use in Type II tasks was generally good while the use in Type I tasks was generally poor. There was little effort to use graphs to explore possible relations, nor to use technology to test the scope and limitations. In Type II tasks there was good use of graphing technology although some candidates appeared to believe that any graph would suffice. Some graphs were not effective in demonstrating the issue at hand due to a poor window choice. The opportunity for full and resourceful use of graphs, such as using multiple functions on the same grid or expanded windows to show long-term behaviour, was missed by a good number of candidates.

Criterion F:

For the most part candidates were appropriately awarded F1. In a few cases teachers had too lenient or too strict standards for this criterion. It would appear that some teachers are imposing their own classroom standards here. It was often unclear on what basis a particular candidate may have been awarded a mark of F2.

Recommendations and guidance for the teaching of future candidates

If teachers and candidates carefully reviewed the assessment criteria and considered their work in this light then candidates would be much more successful. Given that the portfolio is a criterion-referenced piece of work it is not sufficient for the candidate to produce work that is holistically assessed as “good work”. Each accomplishment within the work worth assessing must reflect some aspect of the assessment criteria.

Candidates should better learn to provide proper notation and communication. This can be accomplished in the classroom through short exercises that require such things in the solution of homework problems. It would help if the teacher set a standard for what is appropriate and maintained this standard throughout the course.

The process of developing a general statement and of producing a model function can be taught. Many syllabus topics lend themselves nicely to this, such as arithmetic and geometric sequences and exponential or sinusoidal functions. The connection between these topics and the portfolio tasks should be consciously made in class.

Many application problems in classrooms are often dealt with as simple exercises to get “the answer”. Exploring these applications by changing parameters or applying the results to similar situations will help candidates see the connection between the maths and the real world.

In general, given that most candidates presented word-processed work, one would expect that a basic spreadsheet and some internet resources are available. The teacher is responsible for building their own sense of confidence with these and other technological tools that are available so that they may make clear the expectations regarding their use.

Further comments

As we move to a new model of internal assessment next year it will be ever more important that teachers and candidates come to know and understand the criteria. It is essential that teachers explain the criteria levels to candidates so that they know what is expected of them. As teachers prepare their samples for moderation they should also stop and consider that the moderation will go much better if the proper background knowledge and expectations regarding solutions and the use of technology are provided. Comments should be made freely and directly on the candidates’ work so that the moderator can better understand why the given mark was awarded.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 30	31 - 41	42 - 51	52 - 62	63 - 72	73 - 90

General comments

In general, the questions in this year's paper did not require a great deal of arithmetic or algebraic manipulation, but they did require good understanding of the concepts. Candidates who understood the concepts were able to answer questions with a minimum amount of work. Unfortunately, candidates who did not understand the concepts often found themselves stuck with convoluted and unnecessary working which often did not lead them to an answer.

The areas of the programme and examination which appeared difficult for the candidates

- Matrix algebra, including multiplication of matrices
- Integration with a boundary condition
- Expressions with a logarithm in the exponent
- Expected value in a probability experiment
- Conditional probability
- Differentiation involving the chain rule
- Second derivative test for point of inflexion

- Curve sketching from information on derivatives

The areas of the programme and examination in which candidates appeared well prepared

- Basic vector algebra, including perpendicularity and scalar product
- Quadratic functions and graphs
- Differentiation involving the product rule
- Inverse of a matrix
- Inverse functions
- Subtraction of logarithms
- Simple probability without replacement

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: vector algebra, magnitude

Most candidates comfortably applied algebraic techniques to find new vectors. However, a significant number of candidates answered part (b) as the absolute numerical value of the vector components, which suggests a misunderstanding of the modulus notation. Those who understood the notation easily made the calculation.

Question 2: quadratic graphs

Most candidates recognized the values of the *x*-intercepts from the factorized form of the function. Candidates also showed little difficulty finding the vertex of the graph, and employed a variety of techniques: averaging *x*-intercepts, using $x = \frac{-b}{2a}$, completing the square.

Question 3: differentiation with product rule, gradient at a point

Many candidates correctly applied the product rule for the derivative, although a common error was to answer $f'(x) = 2x\cos x$. Candidates generally understood that the gradient of the curve uses the derivative, although in some cases the substitution was made in the original function. Some candidates did not know the values of sine and cosine at $\frac{\pi}{2}$.

Question 4: matrix algebra

While most candidates first subtracted B from C, many incorrectly left-multiplied by $\boldsymbol{A}^{-1}$ to find the expression for X. Furthermore, a significant number of candidates attempted to divide by A, which suggests a misunderstanding of basic matrix algebra. For those who left-multiplied, full follow-through marks were available in part (b), while few further marks were earned by those who attempted to divide matrices. Although a good number of candidates were able to multiply matrices correctly, a similar number could not carry out the calculation without error. Many simply did not know the correct row-column process.

Question 5: inverse functions

Candidates often found an inverse function in which to substitute the value of 2. Some astute candidates set the function equal to 2 and solved for *x*. Occasionally a candidate misunderstood the notation as asking for a derivative, or used $\frac{1}{f(x)}$. For part (b), many candidates recognized that if $g(30)=3$ then $g^{-1}(3)=30$, and typically completed the question successfully. Occasionally, however, a candidate incorrectly answered $\sqrt{25}=\pm 5$.

Some candidates created their own function that satisfied $g(30)=3$, such as $g(x)=\frac{x}{10}$. While this was not the method envisioned in the question, it is a valid approach that could earn full marks.

Question 6: integration with a boundary condition

While some candidates correctly integrated the function, many missed the division by 2 and answered $12\ln(2x-5)$. Other common incorrect responses included $\frac{12x}{x^2-5x}$ and $-12(2x-5)^{-2}$. Finding the constant of integration also proved elusive for many. Some either did not remember the +*C* or did not try to find its value, while others misunderstood the boundary condition and attempted to calculate the definite integral from 0 to 4.

Question 7: properties of exponents and logarithms

Many candidates readily earned marks in part (a). Some interpreted $\log_2 40-\log_2 5$ to mean $\frac{\log_2 40}{\log_2 5}$, an error which led to no further marks. Others left the answer as $\log_2 8$ where an integer answer is expected. Part (b) proved challenging for most candidates, with few recognizing that changing 8 to base 2 is a helpful move. Some made it as far as $2^{3\log_2 5}$ yet could not make that final leap to an integer.

Question 8: vector equations of line

While many candidates can find a vector given two points, few could write down a fully correct vector equation of a line. Most candidates wrote their equation as "$L_1 =$", which misrepresents that the resulting equation must still be a vector.

Those who recognized that vector perpendicularity means the scalar product is zero found little difficulty answering part (b). Occasionally a candidate would use the given $p=-6$ to show the scalar product is zero. However, working backward from the given answer earns no marks in a question that requires candidates to **show** that this value is achieved.

While many candidates knew to set the lines equal to find an intersection point, a surprising number could not carry the process to correct completion. Some could not solve a simultaneous pair of equations, and for those who did, some did not know what to do with the parameter value. Another common error was to set the vector equations equal using the same parameter, from which the candidates did not recognize a system to solve. Furthermore, it is interesting to note that while only one parameter value is needed to answer the question, most candidates find or attempt to find both, presumably out of habit in the algorithm.

Question 9: conditional probability, expected value

Many candidates correctly found the probability of selecting no green marbles in two draws, although some candidates treated the second draw as if replacing the first. When finding the probability for exactly one green marble, candidates often failed to recognize two pathways for selecting one of each color.

Few candidates understood the concept of expected value in this context, often leaving this blank or treating as if a binomial experiment. Successful candidates often made a distribution table before making the final calculation.

Most candidates answered part (c) correctly. However, many overcomplicated (cii) by using the conditional probability formula. Those with a clear understanding of the concept easily followed the "write down" instruction.

Only a handful of candidates correctly applied conditional probability to find $\mathrm{P}(A|R)$ in part (d). While some wrote down the formula, or drew a tree diagram, few correctly calculated $\mathrm{P}(\text{red}) = \frac{5}{8}$. A common error was to combine the marbles in the two jars to give $\mathrm{P}(\text{red}) = \frac{9}{16}$.

Question 10: functions and derivatives, curve sketching

Many candidates left their answer to part (a) as $\ln 1$. While this shows an understanding for substituting a value into a function, it leaves an unfinished answer that should be expressed as an integer.

Candidates who attempted to consider where *f* is increasing generally understood the derivative is needed. However, a number of candidates did not apply the chain rule, which commonly led to answers such as "increasing for all *x*". Many set their derivative equal to zero, while neglecting to indicate in their working that $f'(x) > 0$ for an increasing function. Some created a diagram of signs, which provides appropriate evidence as long as it is clear that the signs represent *f'*.

Finding $f''(1)$ proved no challenge, however, using this value to **show that** no point of inflexion exists proved elusive for many. Some candidates recognized the signs must not change in the second derivative. Few candidates presented evidence in the form of a calculation, which follows from the "hence" command of the question. In this case, a sign diagram without numerical evidence was not sufficient.

Few candidates created a correct graph from the information given or found in the question. This included the point (0, 0), the fact that the function is always increasing for $x > 0$, the concavity at $x = 1$, and the change in concavity at the given point of inflexion. Many incorrect attempts showed a graph concave down to the right of $x = 0$, changing to concave up.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 33	34 - 43	44 - 52	53 - 62	63 - 71	72 - 90

The areas of the programme and examination which appeared difficult for the candidates

Candidates in this session had difficulties in the following areas of the programme:

- Using a graphic display calculator (GDC) to find mean and variance
- Domain, range, curve sketching, showing key features of graphs
- Distance travelled
- Applying transformations to functions and points
- Normal distribution
- Interpreting the graphs of functions and their derivatives; conditions for maximum rates of change

The areas of the programme and examination in which candidates appeared well prepared

For students who were well prepared, there was ample opportunity to demonstrate a high level of knowledge and understanding on this paper. The following areas of the programme were handled well by most students.

- Arithmetic sequences and series
- Binomial expansion
- Matrices and their inverses
- Basic use of sine and cosine rules
- Differentiation techniques

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1 Arithmetic Sequences and Series

The majority of candidates had little difficulty with this question. If errors were made, they were normally made out of carelessness. A very few candidates mistakenly used the formulas for geometric sequences and series.

Question 2 Frequency Distributions

Candidates had little problem determining a missing frequency from a cumulative frequency table, however in part (b), few used the GDC to their advantage to correctly find the mean and variance. There were numerous unsuccessful attempts at using the formulae for mean and variance, most resulting in algebraic errors along the way. Candidates recognized the concept of variance but were often unable to determine what value should be squared.

Question 3 Binomial Expansion

Candidates frequently made reasonable attempts at both parts of the question. Those who correctly stated the values in (a) were generally successful in part (b). Many candidates offered the whole term rather than the coefficient in part (b) and lost the final mark. Some candidates appeared to have misread the order of the variables, stating that $p=7$ (instead of $r=7$), $q=5$ (instead of $p=5$) and $r=5$ or 7 (instead of $q=5$ or 7). A large number of candidates did not make the connection between parts (a) and (b).

Question 4 Matrices and their Inverses

This question was well done by the majority of candidates. A number of candidates transposed the coefficient matrix and then obtained correct follow through marks in the remainder of the question. Some candidates did not obey the command term "hence" in part (b) and proceeded to solve the system using long, algebraic methods rather than simply using the GDC to find $\boldsymbol{A}^{-1}\boldsymbol{b}$. In part (b), the order of the matrices was often reversed resulting in a one mark deduction when the correct answer was found.

Question 5 Kinematics

There was a minor error on this question, where the units for velocity were given as ms^{-2} rather than ms^{-1}. Examiners were instructed to notify the IB assessment centre of any candidates adversely affected, and these were considered at the grade award meeting.

Candidates continue to produce sloppy graphs resulting in loss of marks. Although the shape was often correctly drawn, students were careless when considering the domain and other key features such as the root and the location of the maximum point. The fact that most candidates with poorly drawn graphs correctly found the root in (b)(i), clearly emphasized the disconnect between geometric and algebraic approaches to problems. In (b)(ii), most appreciated that the definite integral would give the distance travelled but few could write a valid expression and normally just integrated from $t=0$ to $t=5$ without considering the part of the graph below the *t* – axis. Again, analytic approaches to evaluating their integral predominated over simpler GDC approaches and some candidates had their calculator set in degree mode rather than radian mode.

Question 6 Transformations

Part (a) was frequently done well but a lack of understanding of the notation $f(x+1)$ often led to an incorrect value for *p*. In part (b), candidates did not recognize the simplicity of the problem. Most seemed to be unable to correctly recognize the two transformations implied in the question and were thus unable to attempt a geometric solution. Flawed algebraic approaches to part (b) were common and many could not interpret the notation $g(3x)$ as multiplying the *x* – value by $\frac{1}{3}$.

Question 7 Normal Distribution

This was an accessible problem that created difficulties for candidates. Although they recognized and often wrote down a formula for IQR, most did not understand the conceptual nature of the first and third quartiles. Those who did could solve the problem effectively using their GDC in relatively few steps. Candidates that were able to start this question often drew the normal curve and gave quartile values at 140 and 160. This generally led to a solution which while wrong, was also clearly inadequate for the indicated 7 marks.

Question 8 Triangle Trigonometry

There was an error on this question, where the measurements were inconsistent. Whichever method a candidate used to answer the question, the inconsistencies did not cause a problem. The markscheme included a variety of solutions based on possible combinations of solutions, and examiners were instructed to notify the IB assessment centre of any candidates adversely affected. Candidate scripts did not indicate any adverse effect.

Despite this unfortunate error, the question posed few difficulties for candidates and most approached the problem as intended. Although there were other ways to approach the problem (using properties of cyclic quadrilaterals) few considered this, likely due to the fact that cyclic quadrilaterals is not part of the syllabus. Candidates were proficient in their use of sine and cosine rules and most could find the area of the required triangle in part (c). Those who made errors in this question either had their GDC in the wrong mode or were rounding values prematurely while some misinformed candidates treated ADC as a right-angled triangle. In part (d), most candidates recognized what to do and often obtained follow through marks from errors made in previous parts.

Question 9 Calculus and Rate of Change

Candidates had little difficulty with parts (a), (b) and (c). Successful analytical approaches were often used in part (b) but again, this was not the most efficient or expected method. In part (c), candidates gained marks by correctly identifying upper and lower bounds but often did not express them properly using an appropriate notation. In part (d), the majority of candidates opted to use the quotient rule and did so with some degree of competency, but failed to recognize the command term "show that" and consequently did not show enough to gain full marks. Approaches involving the chain rule were also successful but with the same point regarding sufficiency of work. Part (e) was poorly done as most were unable to interpret what was required. There were a few responses involving the use of the "trace" feature of the GDC which often led to inaccurate answers and a number of candidates incorrectly reported $x = 19.6$ as their final answer. Some found the maximum value of *f* rather than f'.

Question 10 Trigonometric Functions

Most candidates were successful with part (a) but a surprising number had difficulty producing enough work to show that the period was 25; writing down the exact value of *b* also overwhelmed a number of candidates. In part (c), candidates did not recognize that the seat on the Ferris wheel is a minimum at $t = 0$ thereby making the value of *a* negative. Incorrect values of 61 were often seen with correct follow through obtained when sketching the graph in part (d). Graphs again frequently failed to show key features in approximately correct locations and candidates lost marks for incorrect domains and ranges. Part (e) was very poorly done for those who attempted the question and most did not make the connection between height, time and probability. The idea of linking probability with a real – life scenario proved beyond most candidates. That said, there were a few novel approaches from the strongest of candidates using circles and angles to solve this part of question 10.

Recommendations and guidance for the teaching of future candidates – paper 1 and 2

Candidates should be encouraged to show all their working in a neat, organized manner. Mathematical working does not only mean algebraic steps, but also the reasoning in obtaining a solution. Correct mathematical notation is not specifically assessed in the examination papers, but a strong foundation in notation is an essential feature of mathematical learning. Sloppy notation by candidates can be forgiven if the mathematical elements of a question are reasonably expressed. However, there are times when sloppy notation interferes with the mathematical expression of a concept.

If a mistake is made, it is best to simply draw an "X" or a line through any unwanted working. Incorrect working that is not crossed out will be considered as part of the answer, and therefore marked according to the markscheme, even if different working is seen later.

Candidates should be exposed to past IB exams and markschemes, and should use these for practice. Working past exams under timed conditions exposes candidates to different types of questions and helps them learn to pace themselves so they can more easily complete the exam in the time allotted.

There were comments in the teacher feedback forms suggesting that students were not familiar with some of the notation and terminology used in the examination. The notation and terminology used in the question papers are published in Mathematics SL subject guide. Using questions from past papers will help students become familiar with the style and form of questions, as well as with the notation and terminology.

Looking at the markschemes can help students and teachers understand what is required by the different command terms, such as "write down", "find", "sketch", or "explain". Graph paper is generally not required for a sketch. Graphs should be carefully sketched, paying attention to, and labelling, key features, either general ones, such as intercepts, or specific ones identified in a question. In a “show that” question, it should be obvious from the penultimate step that the next line gives the required result. This may require candidates to show additional simplification steps.

Candidates should be encouraged to read each question carefully and consider what information is given and what a question is asking them to do before they begin their work. Candidates can also look for clues within the given information. They should check when transcribing information in order to avoid errors.

Candidates are not making the connections between different parts of the same question. Most questions are designed around a “theme” and students are normally expected to use the results of one part in subsequent parts, but they often fail to see the significance of their results. Candidates should be taught to reflect on the meaning of results by asking questions like, “why is this part being asked?” or “how is it relevant to the next part?”

Students need to be given experience working with ‘non-conventional’ questions on different areas of the syllabus which will help them to transfer their learning from one area of mathematics to another. Encourage students to view mathematics as a whole, rather than a set of discrete topics.

When studying the expansion of binomials, candidates should practice finding specific terms.

An understanding of conditional probability should extend beyond substituting into the formula.

Probability and statistics continues to be the weakest area of knowledge demonstrated by candidates. Schools should be encouraged to ensure that candidates have been fully prepared in all aspects of this course.

Paper 1

While many students will often reach for a formula when encountering questions in conditional probability, few have the conceptual understanding to go beyond the information booklet. In situations where a tree diagram is not provided, it may be helpful to instruct students to create their own diagram, in an effort to more visually determine that the denominator in the formula is calculated by two pathways.

Paper 2

Paper 2 is a GDC required paper, not simply a GDC allowed paper. Candidates should be encouraged to consider whether use of the GDC is appropriate when answering any question on Paper 2. Although basic GDC skills are improving, there are still candidates who are opting for an analytical approach rather than a more efficient GDC approach particularly with the less obvious applications of solving equations, finding intersections or evaluating definite integrals. This often leads to simple algebraic errors and consumes valuable time. It should be emphasized that once an equation is established, no algebraic working is needed to support an answer. Teachers should place greater emphasis on integrating the use of technology as a tool for learning and for better understanding key concepts as well as for solving problems by communicating solutions clearly.

Many candidates continue to struggle with what work to show when using technology. Working should be used to show any set up required before using the GDC. Mathematical notation should be used, not calculator notation. Writing “used GDC” is not enough evidence of a valid approach. Examples of this may be seen in the student solutions for the May 2010 papers, included in the subject reports on the OCC.

Candidates should ensure that their GDCs are in the correct mode (eg radian/degrees). They should be taught not simply to transcribe graphs from their GDC without considering their intrinsic knowledge of key features and behaviours of functions. They should be encouraged to use the appropriate GDC tools to find and label key features of graphs.

Numerical values (including answers given correct to three significant figures) should be stored in the memory, and the more accurate “long” value used if needed in subsequent parts. Inaccurate values or premature rounding of values can lead to wrong final answers.