# MATHEMATICS SL TZ2

(IB Africa, Europe & Middle East & IB Asia-Pacific)

## Overall grade boundaries

| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range**: | 0-18 | 19-36 | 37-52 | 53-62 | 63-73 | 74-83 | 84-100 |

## Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2012 examination session the IB has produced time zone variants of the Mathematics SL papers.

## Internal assessment

### Component grade boundaries

| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range**: | 0-7 | 8-13 | 14-19 | 20-23 | 24-28 | 29-33 | 34-40 |

## The range and suitability of the work submitted

The vast majority of schools submitted portfolios taken from the currents sets of tasks prescribed by the IB. Only a few submitted old tasks, and a penalty was applied as per the current policy. In some cases older tasks from IB sources were used. As these were beyond their shelf-life a penalty was also applied. In a few cases schools or teachers had submitted older tasks that had been slightly modified. It is important to note that tasks that resemble older TSM versions too closely are subject to a penalty. The very few cases where teachers had either designed their own tasks or used third-party tasks demonstrated how important it is for these tasks to be previewed in light of how well they address the assessment criteria. Often candidates suffered because the task design did not allow for achievement of the highest levels of some criteria.

## Candidate performance against each criterion

**Criterion A:**

After years of subject reports identifying issues related to poor use of notation one would think that it would be simple enough for most candidates to achieve A2 without much trouble. However, there persists a laxity towards the correct use of appropriate notation and terminology that has resulted in A1 for most candidates. Special concerns revolve around the persistent use of calculator notation and the lack of an appropriate approximately equals symbol for rounded values. In modelling tasks some distinction must be made between functions representing distinct models. Candidates use '*y*' for almost every function without any consideration of the potential ambiguity.

**Criterion B:**

The most successful candidates are those who present their work with clarity and organization, recognizing the effort as more of a mathematical essay than a set of homework exercises. Good graphs are presented as one part of an explanation, including proper labelling and commentary that explains or supports the analysis or results. Diagrams that are poorly drawn hinder effective communication of the ideas that they are meant to support.

**Criterion C:**

Type I:

While some candidates present elegant analyses there are just as many who offer results out of the blue with little or no supporting explanations. The presentation of results without appropriate and sufficient analysis cannot score well in criterion C. Further, once a general statement is conjectured its validity must be tested with new values and checked against the original mathematical pattern.

Type II:

The best work presented clear definitions of variables and some investigation of parameters or constraints. An analytical approach should come next, with the candidate using their mathematical knowledge to propose and develop possible models for consideration. Only then may regression techniques be used to support or refine the best model found. Too many candidates rely on the calculator or computer to generate regression models for consideration, then analyse the best regression model analytically. This defeats the purpose of the criterion. Further, despite their knowledge that certain real-life situations tend to behave according to certain functions, many candidates first seek to match a linear model to the data. Given that a linear model alone is not at the level of the programme the candidate cannot score well unless they have subsequently explored a non-linear model with sufficient analysis. Candidates are expected to extend their model to further data, which has been supplied in the IB tasks. Comments should be offered as to how well the original model fits the new data, and this would satisfy level C5. Modification of the model is addressed in criterion D.

**Criterion D:**

Type I:

Candidates were generally successful in achieving some kind of general statement to at least satisfy level D2. Teachers should note that summation notation does not necessarily represent a general statement. Rather, using $\Sigma$ may only provide a shorthand expression for a part of the analysis that might lead to the appropriate general statement. The scope or limitations may appear obvious but the candidate is responsible for exploring many possible values to check that the proposed limits or scope are truly correct. While a sequence may suggest that $n$ is obviously an integer, is it clear that $n$ starts at 1, or at 0, or can be negative after all? The best work critically considered the pattern of behaviour and sought to analyse the behaviour in a way that explained the result. This achievement of level D5 was rare.

Type II:

Candidates mostly arrived at some results that fit the data well or poorly, thus achieving the lower levels of criterion D. The higher levels of criterion D require interpretation in context, exploring and discussing how the model addresses the reality of the scenario. Too often the interpretation centred on the mathematics of function (slope, asymptote, intercept, etc) rather than the meanings behind those mathematics (rate of growth, long-term behaviour and limitations, initial values, etc). Accuracy is also a consideration here; how good must the model be before it reasonably represents the situation? Ultimately the work must consider how well the original model fits other cases, and how that original model can be adapted to make a better fit. Candidates should not be creating a brand new model for level D5.

**Criterion E:**

While various types of software programs have provided more opportunities for candidates to make resourceful use of technology such technology has not always been used to good effect. In addition teachers have provided little information regarding the availability and expectations of technology. Many marks of E3 were unsubstantiated by the work presented or by sufficient evidence provided through teachers' comments. In these cases it is very difficult for moderators to confirm the higher marks. Candidates should take note that "enhances the development of the task" means more than printed output by itself. Quality graphs will explore extreme values or zoom in on critical intervals. They will compare various functions with the intention of showing the comparative quality of fit or behaviour over the long-term. Spreadsheet tables will extend calculations to demonstrate clearly how patterns of results can be extended to further cases. Regression models will be presented in support of analytical models. Suitable commentary explaining the value of each graph or table will accentuate the output presented.

**Criterion F:**

Appropriately, most marks were F1, recognizing that the work satisfied requirements of the task to a reasonable degree. Teachers should be cautious of referencing work to the norm of the class. Rather there should be some absolute standards of excellence identified in the teacher's markscheme that identify expected outcomes worthy of recognition with a mark of

F2. Conversely, F0 should only be used where the work is clearly inadequate relative to expectations. Things such as lateness or sloppiness should not, by themselves, contribute to F0.

## Recommendations for the teaching of future candidates

Students should be taught appropriate mathematical notation and encouraged to use it consistently in their work. Teachers can model this with good use of notation on their assignments and tests. Students should also be required to provide full written answers to short problems so that they can learn to write more in the style expected of the portfolio tasks. Questions that focus on the development of a general statement and how to test its validity are encouraged. Such questions can promote discussion of scope and limitations, as well as allow for explanations to support the statement. For modelling tasks students should be reminded that certain functions fit certain types of behaviour in data plots, and certain scenarios in real life. It is not useful considering model functions that are inappropriate. Once models are developed a thorough discussion on possible interpretations and modifications would be useful. Resourceful use of technology must be explored in the classroom and not left to the students' own devices. The production of pages and pages of printed output does not usually enhance the work. Teachers may also wish to teach students how to use mathematical templates for word processing. Above all teachers must explain each of the assessment criteria to students.

## Further comments

Teachers are reminded that solutions to tasks are essential to the moderators so that they can better understand the teacher's assessment. Comments written directly on the work can also clarify why marks were awarded or where penalties were applied. Summary comments on the form 5/PFCS will also help. Teachers should read the subject reports and feedback forms from past years to get a better idea of what to watch for in the presentation of portfolio tasks. The best professional development in this regard is for teachers to become moderators themselves.

# Paper one

### Component grade boundaries

| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range**: | 0-17 | 18-34 | 35-51 | 52-60 | 61-70 | 71-79 | 80-90 |

## The areas of the programme and examination that appeared difficult for the candidates

- using the binomial theorem with a general exponent
- analyzing circular functions
- conditional probability and probability of compound events
- transformations of functions

- understanding the graph of a function
- equations of lines parallel to the axes

## The areas of the programme and examination in which candidates appeared well prepared

It was pleasing to see that most candidates were able to approach each question in a logical way, and most candidates earned at least partial marks on most of the questions they attempted. Candidates showed good preparation and knowledge in the following areas:

- inverse and composite functions
- solving quadratic equations
- basic probability and tree diagrams
- using the quotient rule to find derivatives
- integration of basic polynomials

## The strengths and weaknesses of the candidates in the treatment of individual questions

**Question 1**

Overall, this question was done well by candidates. In part (a), a surprising number of candidates found the median position (the cumulative frequency) on the *y*-axis, but did not find the median mark on the *x*-axis. Similar misunderstanding was shown by some candidates in part (b), when attempting to find the interquartile range.

**Question 2**

This question was answered correctly by nearly all candidates. In part (b), there were a few who seemed unfamiliar with the notation for composition of functions, and attempted to multiply the functions rather than finding the composite, and there were a few who found the correct composite function but failed to substitute in $x=1$ to find the value.

**Question 3**

In part (a), many candidates were able to successfully write down the value of *a* as instructed by inspecting the graph and seeing the amplitude of the function is 3. Many also used a formulaic approach to reach the correct answer. When finding the value of *b*, there were many candidates who thought *b* was the period of the function, rather than $\frac{2\pi}{\text{period}}$.

In part (b), the directions asked candidates to write down the gradient of the curve at the local minimum point P. However, many candidates spent a good deal of time finding the derivative of the function and finding the value of the derivative for the given value of *x*, rather than simply stating that the gradient of a curve at a minimum point is zero.

For part (c), finding the equation of the normal to the curve, many candidates tried to work with algebraic equations involving negative reciprocal gradients, rather than recognizing that the equation of the vertical line was $x = 2$. There were also candidates who had trouble expressing the correct equation of a line parallel to the *y*-axis.

**Question 4**

The majority of candidates were successful in earning full marks on this question. In part (b), a small number of candidates did not use the correct formula for E(*X*), even though this formula is given in the formula booklet. There were also a few candidates who incorrectly assumed that $p = 0$, forgetting that the sum of the probabilities must equal 1. There were a few candidates who left this question blank, which raises concerns about whether they had been exposed to probability distributions during the course.

**Question 5**

In part (a) of this question, a large number of candidates correctly sketched the graph of $f(-x)$, as asked. A fairly common error, however, was to graph $-f(x)$. In part (b), many candidates seemed to recognize that the value of *a* was related to a vertical stretch, though some omitted the negative required for the vertical reflection. Similarly, some candidates gave a positive value for *b*.

**Question 6**

Most candidates approached this question correctly by using the discriminant, and many were successful in finding both of the required values of *k*. There did seem to be some confusion about the expression "two **equal** real solutions", as some candidates approached the question as though the equation had two distinct real roots, using $b^2 - 4ac > 0$, rather than $b^2 - 4ac = 0$.

There were also a good number who recognized that the quadratic must be a perfect square, although many who used this method found only one of the two possible values of *k*. In addition, there were many unsuccessful candidates who tried to use the entire quadratic formula as though they were solving for *x*, without ever seeming to realize the significance of the discriminant.

**Question 7**

This question proved quite challenging for the majority of candidates, although there were a small number who were able to find the correct value of *n* using algebraic and investigative methods. While most candidates recognized the need to apply the binomial theorem, the majority seemed to have no idea how to do so when the exponent was a variable, *n*, rather than a known integer. Most candidates who attempted this question did expand the quadratic correctly, but many went no further, or simply set the *x*-term of the quadratic equal to 84*x*, ignoring the expansion of the first binomial altogether.

**Question 8**

In part (a), nearly all the candidates recognized that *h* and *k* were the coordinates of the vertex of the parabola, and most were able to successfully show that $a = 3$. Unfortunately, a few candidates did not understand the "show that" command, and simply verified that $a = 3$ would work, rather than showing how to find $a = 3$.

In part (b), most candidates were able to find $f(x)$ in the required form. For a few candidates, algebraic errors kept them from finding the correct function, even though they started with correct values for *a*, *h* and *k*.

In part (c), nearly all candidates knew that they needed to integrate to find the area, but errors in integration, and algebraic and arithmetic errors prevented many from finding the correct area.

**Question 9**

Part (a) of this question was answered correctly by the large majority of candidates. There were some who did not follow the instruction to copy and complete the tree diagram on their separate paper, and simply filled in the blanks on the exam paper.

In part (b), many candidates struggled with finding the compound probability, and did not use the provided information in the appropriate manner. Quite a few candidates seemed to be confused about when they should add the probabilities or when they should multiply.

In part (c), many recognized that the question dealt with conditional probability, and many tried to use the formula from the information booklet, but failed to realize that they had already found the required values for the numerator and denominator in their working for part (b).

Throughout this question, it was discouraging to see the large number of candidates making arithmetic errors. There were a surprising number of candidates who multiplied fractions incorrectly, or found an incorrect value for simple multiplication such as $2 \times 4 = 6$ or $6 \times 7 = 43$.

**Question 10**

While most candidates answered part (a) correctly, there were some who did not show quite enough work for a "show that" question. A very small number of candidates did not follow the instruction to use the quotient rule.

In part (b), most candidates knew that they needed to solve the equation $f'(x) = 0$, and many were successful in answering this question correctly. However, some candidates failed to find both values of *x*, or made other algebraic errors in their solutions. One common error was to find only one solution for $x^2 = 1$; another was to work with the denominator equal to zero, rather than the numerator.

In part (c), a significant number of candidates seemed to think that the line $y = k$ was a vertical line, and attempted to find the vertical asymptotes. Others tried looking for a horizontal asymptote. Fortunately, there were still a good number of intuitive candidates who recognized the link with the graph and with part (b), and realized that the horizontal line must pass through the space between the given local minimum and the local maximum they had found in part (b).

## Recommendations and guidance for the teaching of future candidates for both papers

Candidates should be given the chance to become familiar with the style of the examination by looking at past exams for practice, and working under timed conditions. It appears that some candidates spent too much time on earlier questions and were rushed when they got to the later parts of the exam. In addition, candidates should be encouraged to stop and think about what a question is asking and to look for clues within the given information. Too many times, we see candidates looking for a formulaic approach when an intuitive approach is better and faster. This was often the case in questions 3(b) and (c), in question 9(c), and in question 10(c).

Candidates also need to be aware of the different command terms and the requirements of these. For example, in question 3(b), the command was "write down", and there was only one mark available. Yet many candidates obviously spent a great deal of time on this question. Another example is in 8(aii), where the command term was "show that", yet some candidates did not show how to get $a = 3$; rather they worked backwards from the given answer.

As always, candidates should be encouraged to show all their working in a neat, organized manner. Whether the final answer is correct or incorrect, it is much easier for examiners to award marks for a correct method when the working is easy to follow and not randomly scattered all over the page. If a mistake is made, it is best to simply draw an "X" or a line through any unwanted working.

# Paper two

### Component grade boundaries

| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range**: | 0-16 | 17-32 | 33-44 | 45-53 | 54-62 | 63-71 | 72-90 |

## The areas of the programme and examination that appeared difficult for the candidates

Almost all students attempted to answer all the questions of the exam, although in some centres there appeared to be some areas of the syllabus which proved difficult for the students:

- Binomial and conditional probability.
- “Show that” questions.
- Matrix algebra.
- Chain rule.

- Vector equation.
- Sketching a function appropriately and accurately
- Kinematics as an application of calculus.
- Recognizing the need to use the graphic display calculator (GDC) to solve equations.

## The areas of the programme and examination in which candidates appeared well prepared

The following topics were well understood by a significant number of candidates:

- Angle between two vectors.
- Trigonometry - solution of triangles.
- Finding the inverse of a matrix with the use of their GDC.
- Normal distribution.
- Vector diagrams and manipulation of vectors

## The strengths and weaknesses of the candidates in the treatment of individual questions

**Question 1: Triangle Trigonometry**

This question was attempted in a satisfactory manner. The sine rule was applied satisfactory in part (b) but some obtained an incorrect answer due to having their calculators in radian mode. Some incorrect substitutions were seen, either by choosing an incorrect side or substituting 70 instead of $\sin 70^\circ$. Approaches using a combination of the cosine rule and/or right-angled triangle trigonometry were seen, especially in part (c) to calculate the area of the triangle.

A few candidates set about finding the height, then used the formula for the area of a right-angled triangle.

**Question 2: Differentiation**

Many students failed in applying the chain rule to find the correct derivative, and some inappropriately used the product rule. However, many of those obtained full follow through marks in part (b) for the sketch of the function they found in part (a).

Most candidates sketched an approximately correct shape in the given domain, though there were some that did not realize they had to set their GDC to radians, producing a meaningless sketch.

It is very important to stress to students that although they are asked to produce a sketch, it is still necessary to show its key features such as domain and range, stationary points and intercepts.

**Question 3: Geometric Sequences**

In part (a), although most candidates substituted correctly into the formula for the sum of a geometric series and set it equal to 324.8, some used the formula for the sum to infinity and a few the formula for the sum of an arithmetic series. The overwhelming error made was in attempting to solve the equation algebraically and getting nowhere, or getting a wrong answer. The great majority did not recognize the need to use the GDC to find the value of $r$.

In part (b) many did not obtain any marks since they weren't able to find an answer to part (a). Those who were able to get a value for $r$ in part (a) generally went on to gain full marks in (b). However, this was one of the most common places for rounding errors to be made.

**Question 4: Normal Distribution**

There were many completely successful attempts at this question, with good use of formulae and calculator features.

However, in part (b) some candidates did not recognize the need to find the standardized value and set their equation equal to the probability given in the question, thus earning only one mark.

**Question 5: Applications of Differentiation and Integration (velocity, acceleration, displacement)**

This question was well answered by many candidates, although there were some who did not recognize the relationship between velocity, acceleration and displacement. Many of them substituted into the original expression given for the velocity, losing most of the marks. Very few appear to have used their GDC for the integration.

**Question 6: Matrix Algebra**

Most candidates were able to find the correct inverse for part (a) with their GDCs, but many seemed unaware of the importance of the order of operations with matrices leading to errors in part (b). Many candidates operated with matrices as if they were real numbers, using that since $\boldsymbol{A} \cdot \boldsymbol{A}^{-1} = \boldsymbol{I}$, then $\boldsymbol{C} = \boldsymbol{B}$.

Some candidates got engaged in a maze of algebra trying to find the inverse of the matrix, without the use of the GDC, most were unsuccessful.

**Question 7: Binomial and Conditional Probability**

Although candidates seemed more confident in attempting binomial probabilities than in previous years, some of them failed to recognize the binomial nature of the question in part (a). Many knew that the complement was required, but often used $1-\mathrm{P}(X=1)$ or $1-\mathrm{P}(X \le 1)$ instead of $1-\mathrm{P}(X=0)$.

Part (b) was poorly answered. Whilst some candidates recognized that it was a conditional probability, very few were able to correctly apply the formula, identify the outcomes and follow on to achieve the correct result.

Only a few could find the intersection of the events correctly. Several thought the numerator was a product (i.e. $\mathrm{P}(\text{at most } 2) \times \mathrm{P}(\text{at least } 1)$), and then cancelled common factors with the denominator. Others realized that $x = 1$ and $x = 2$ were required but multiplied their probabilities.

This was the most commonly missed out question from Section A.

**Question 8: Vectors**

Although a large proportion of candidates managed to answer this question, their biggest challenge was the use of a proper notation to represent the vectors and vector equations of lines.

In part (a), finding $\vec{\mathrm{OB}}$ and $\vec{\mathrm{OF}}$ was generally well done, although many lost the mark for (iii) due to poor working or not clearly showing the result.

Part (b) was very poorly done. Not all the students recognized which correct position vectors they had to use to write the equations of the lines. It was seen that they frequently failed to present the equations in the required format, which prevented these candidates from achieving full marks. The notations generally seen were $AG = \boldsymbol{a} + \boldsymbol{b}t$, $\mathbf{r} = 4 + t(4,3,2)$ or $\boldsymbol{L} = \boldsymbol{a} + \boldsymbol{b}t$.

Most achieved the correct result in part (c) with many others gaining most of the marks as follow through from choosing incorrect vectors. Some students did not state which vectors had been used, another cause for losing marks. A few showed poor notation, including ***i***, ***j*** and ***k*** in the working.

**Question 9: Functions. Finding and Solving a 3x3 System of Equations**

Part (a) was generally well done, with a few candidates failing to show a detailed substitution. Some substituted 2 in place of *x*, but didn't make it clear that they had substituted in *y* as well.

A great majority could find the two equations in part (b). However there were a significant number of candidates who failed to identify that the gradient of the tangent is zero at a minimum point, thus getting the incorrect equation $3a + 2b = 4$. A considerable number of candidates only had 2 equations, so that they either had a hard time trying to come up with a third equation (incorrectly combining some of the information given in the question) to solve part (c) or they completely failed to solve it.

Despite obtaining three correct equations many used long elimination methods that caused algebraic errors. Pages of calculations leading nowhere were seen.

Those who used matrix methods were almost completely successful.

**Question 10: Functions – First and Second Derivatives**

This exercise seemed to be challenging for the great majority of the candidates, in particular parts (b), (c) and (d).

Part (a) was generally attempted using the cosine rule, but many failed to substitute correctly into the right hand side or skipped important steps. A high percentage could not arrive at the given expression due to a lack of knowledge of trigonometric identities or making algebraic errors, and tried to force their way to the given answer.

The most common errors included taking the square root too soon, and sign errors when distributing the negative after substituting $\cos 2\theta$ by $1-2\sin^2\theta$.

In part (b), most candidates understood what was required but could not find the correct length of the arc PRQ mainly due to substituting the angle by $\theta$ instead of $2\theta$.

Regarding part (c), many valid approaches were seen for the graph of *f*, making a good use of their GDC. A common error was finding a second or third solution outside the domain. A considerable amount of sketches were missing a scale.

There were candidates who achieved the correct equation but failed to realize they could use their GDC to solve it.

Part (d) was attempted by very few, and of those who achieved the correct answer not many were able to show the method they used.

## Recommendations and guidance for the teaching of future candidates for both papers

- Candidates need to be encouraged to follow instructions, especially those on giving answers exactly or correct to three significant figures. Marks may be lost if answers are not given to three significant figures. There are many who still interpret three significant figures as three decimal places. Candidates should further be encouraged to show their working, as answers left to 1 or 2 significant figures with no working may achieve no marks. They should also be encouraged to avoid premature rounding, as this may lead to incorrect answers.

- There seems to be an increasing tendency for some candidates to omit to label the sub-parts of questions. This makes marking very difficult, as the examiner does not know precisely what part of the question the candidate is trying to answer. Teachers should encourage students to label each part of their answer exactly as the sub-heading is given in the question and only use graph paper for graphs and not for writing answers to questions.

- GDC skills continue to need emphasis. Its use was beneficial in this paper as there were some questions (or parts of questions) that required a graphical or GDC approach rather than analytical.

  It is evident that many candidates, although able to obtain a reasonable graph of a function over a given domain on their GDC, are unable to use it to obtain accurate values for intersections or roots or see the relationship between this and the solution of an equation. Furthermore, they still attempt an algebraic strategy without considering how complicated –or impossible- it might be, instead of adopting a calculator solution as soon as they realize that the equation is other than a very simple one.

- Teachers must also stress to students the importance of checking the mode of their calculators to determine if they are using radians or degrees when working with angles and trigonometric functions, and that it is probable that students may have to switch from one to the other during the exam.

- More work needs to be done with students to enable them to learn how to show their method and their reasoning –especially when communicating how they're using their GDCs. In addition, "found using GDC" accompanying an answer is not enough of an explanation and many students are still using calculator-specific language (i.e. Binompd).

- More practice is needed in "show that" questions, where clear logical steps are required. Part of the work done in class should address this, allowing students to analyze if a solution clearly shows what is being asked or not.

## Further comments

This paper seemed fair and straightforward, and enabled candidates to perform well.

It was evident that students felt confident until question 6 and there were some questions whose correct solution depended on the good knowledge of the topic.

It was nice to see that many students reached the end, and attempted to answer all the questions, and fewer "blank" spaces were seen.

Teachers should emphasize that candidates should look for links where one part of a question is following on from another. This is particularly true when given information can be used to earn marks on a later question part, even if the given information cannot be shown to be true by the candidate.
Candidates should be aware of the command terms used in questions; i.e. "write down" means that the answer can be found without showing working while "find" indicates that there is working to be shown.