MATHEMATICS SL TZ1

(IB Latin America & IB North America)

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-17	18-33	34-45	46-57	58-69	70-81	82-100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2012 examination session the IB has produced time zone variants of the Mathematics SL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-7	8-13	14-19	20-23	24-28	29-33	34-40

The range and suitability of the work submitted

The vast majority of schools submitted portfolios taken from the currents sets of tasks prescribed by the IB. Only a few submitted old tasks, and a penalty was applied as per the current policy. In some cases older tasks from IB sources were used. As these were beyond their shelf-life a penalty was also applied. In a few cases schools or teachers had submitted older tasks that had been slightly modified. It is important to note that tasks that resemble older TSM versions too closely are subject to a penalty. The very few cases where teachers had either designed their own tasks or used third-party tasks demonstrated how important it is for these tasks to be previewed in light of how well they address the assessment criteria. Often candidates suffered because the task design did not allow for achievement of the highest levels of some criteria.

Candidate performance against each criterion

Criterion A:

After years of subject reports identifying issues related to poor use of notation one would think that it would be simple enough for most candidates to achieve A2 without much trouble. However, there persists a laxity towards the correct use of appropriate notation and terminology that has resulted in A1 for most candidates. Special concerns revolve around the persistent use of calculator notation and the lack of an appropriate approximately equals symbol for rounded values. In modelling tasks some distinction must be made between functions representing distinct models. Candidates use '*y*' for almost every function without any consideration of the potential ambiguity.

Criterion B:

The most successful candidates are those who present their work with clarity and organization, recognizing the effort as more of a mathematical essay than a set of homework exercises. Good graphs are presented as one part of an explanation, including proper labelling and commentary that explains or supports the analysis or results. Diagrams that are poorly drawn hinder effective communication of the ideas that they are meant to support.

Criterion C:

Type I:

While some candidates present elegant analyses there are just as many who offer results out of the blue with little or no supporting explanations. The presentation of results without appropriate and sufficient analysis cannot score well in criterion C. Further, once a general statement is conjectured its validity must be tested with new values and checked against the original mathematical pattern.

Type II:

The best work presented clear definitions of variables and some investigation of parameters or constraints. An analytical approach should come next, with the candidate using their mathematical knowledge to propose and develop possible models for consideration. Only then may regression techniques be used to support or refine the best model found. Too many candidates rely on the calculator or computer to generate regression models for consideration, then analyse the best regression model analytically. This defeats the purpose of the criterion. Further, despite their knowledge that certain real-life situations tend to behave according to certain functions, many candidates first seek to match a linear model to the data. Given that a linear model alone is not at the level of the programme the candidate cannot score well unless they have subsequently explored a non-linear model with sufficient analysis. Candidates are expected to extend their model to further data, which has been supplied in the IB tasks. Comments should be offered as to how well the original model fits the new data, and this would satisfy level C5. Modification of the model is addressed in criterion D.

Criterion D:

Type I:

Candidates were generally successful in achieving some kind of general statement to at least satisfy level D2. Teachers should note that summation notation does not necessarily represent a general statement. Rather, using Σ may only provide a shorthand expression for a part of the analysis that might lead to the appropriate general statement. The scope or limitations may appear obvious but the candidate is responsible for exploring many possible values to check that the proposed limits or scope are truly correct. While a sequence may suggest that n is obviously an integer, is it clear that n starts at 1, or at 0, or can be negative after all? The best work critically considered the pattern of behaviour and sought to analyse the behaviour in a way that explained the result. This achievement of level D5 was rare.

Type II:

Candidates mostly arrived at some results that fit the data well or poorly, thus achieving the lower levels of criterion D. The higher levels of criterion D require interpretation in context, exploring and discussing how the model addresses the reality of the scenario. Too often the interpretation centred on the mathematics of function (slope, asymptote, intercept, etc) rather than the meanings behind those mathematics (rate of growth, long-term behaviour and limitations, initial values, etc). Accuracy is also a consideration here; how good must the model be before it reasonably represents the situation? Ultimately the work must consider how well the original model fits other cases, and how that original model can be adapted to make a better fit. Candidates should not be creating a brand new model for level D5.

Criterion E:

While various types of software programs have provided more opportunities for candidates to make resourceful use of technology such technology has not always been used to good effect. In addition teachers have provided little information regarding the availability and expectations of technology. Many marks of E3 were unsubstantiated by the work presented or by sufficient evidence provided through teachers' comments. In these cases it is very difficult for moderators to confirm the higher marks. Candidates should take note that "enhances the development of the task" means more than printed output by itself. Quality graphs will explore extreme values or zoom in on critical intervals. They will compare various functions with the intention of showing the comparative quality of fit or behaviour over the long-term. Spreadsheet tables will extend calculations to demonstrate clearly how patterns of results can be extended to further cases. Regression models will be presented in support of analytical models. Suitable commentary explaining the value of each graph or table will accentuate the output presented.

Criterion F:

Appropriately, most marks were F1, recognizing that the work satisfied requirements of the task to a reasonable degree. Teachers should be cautious of referencing work to the norm of the class. Rather there should be some absolute standards of excellence identified in the teacher's markscheme that identify expected outcomes worthy of recognition with a mark of

F2. Conversely, F0 should only be used where the work is clearly inadequate relative to expectations. Things such as lateness or sloppiness should not, by themselves, contribute to F0.

Recommendations for the teaching of future candidates

Students should be taught appropriate mathematical notation and encouraged to use it consistently in their work. Teachers can model this with good use of notation on their assignments and tests. Students should also be required to provide full written answers to short problems so that they can learn to write more in the style expected of the portfolio tasks. Questions that focus on the development of a general statement and how to test its validity are encouraged. Such questions can promote discussion of scope and limitations, as well as allow for explanations to support the statement. For modelling tasks students should be reminded that certain functions fit certain types of behaviour in data plots, and certain scenarios in real life. It is not useful considering model functions that are inappropriate. Once models are developed a thorough discussion on possible interpretations and modifications would be useful. Resourceful use of technology must be explored in the classroom and not left to the students' own devices. The production of pages and pages of printed output does not usually enhance the work. Teachers may also wish to teach students how to use mathematical templates for word processing. Above all teachers must explain each of the assessment criteria to students.

Further comments

Teachers are reminded that solutions to tasks are essential to the moderators so that they can better understand the teacher's assessment. Comments written directly on the work can also clarify why marks were awarded or where penalties were applied. Summary comments on the form 5/PFCS will also help. Teachers should read the subject reports and feedback forms from past years to get a better idea of what to watch for in the presentation of portfolio tasks. The best professional development in this regard is for teachers to become moderators themselves.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-15	16-30	31-41	42-53	54-64	65-76	77-90

The areas of the programme and examination that appeared difficult for the candidates

- reading values from cumulative frequency graphs
- conditional probability

- combining a stretch with a translation of a sinusoidal curve
- integration leading to natural logarithm
- solving double angle trigonometric equation
- applying calculus results

The areas of the programme and examination in which candidates appeared well prepared

Candidates demonstrated a good level of knowledge and understanding with most topics. Strengths included:

- matrices
- vectors
- manipulating logs
- basic probability
- differentiation of the exponential function
- equation of tangent line
- manipulating logarithms

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1:

Part (a) was generally answered correctly, with most candidates showing a good grasp of cumulative frequency from a table. A surprising number of candidates had difficulty reading values off the cumulative frequency curve. A common incorrect answer for (b)(i) was 29, indicating carelessness with the given scale. Too many candidates gave 40 and 120 for the quartile values.

Question 2:

This question was generally well answered with candidates showing a good understanding of matrix dimensions and multiplication. Many candidates clearly showed their method of multiplication. Common errors included reversing row and column order and careless errors in one or more multiplication of elements.

Question 3:

On the whole, candidates handled this question quite well with most candidates correctly applying the chain rule to an exponential function and successfully finding the equation of the tangent line. Some candidates lost a mark in (b)(i) for not showing sufficient working leading to the given answer.

Question 4:

Parts (a), (b), and (c)(i) of this Venn diagram probability question were answered quite well with candidates consistently earning full marks. Only a few candidates worked backwards from the given $r = 0.2$ in the "show that" portion of part (b). Many candidates struggled on part (c)(ii), either not recognizing conditional probability or multiplying probabilities to find the numerator as if the events were independent. A number of candidates who successfully found the probability in part (c)(ii) left their incomplete answer of $0.1/0.4$

Question 5:

A pleasing number of candidates correctly found the values of *a*, *b*, and *c* for this sinusoidal graph. Some candidates had trouble showing that the period was π, either incorrectly adding the given $\pi/4$ and $3\pi/4$ or using the value of b that they found first for part (b)(ii).

Question 6:

Knowing that the answer to this integration led to a natural logarithm function helped many candidates make progress on this more challenging question, although some candidates simply substituted the limits straight away without integrating. Although some candidates incorrectly simplified $\ln 15 - \ln 5$ as $\ln 10$ or $\ln 15 / \ln 5 = \ln 3$, a pleasing number applied the logarithm property correctly. Some candidates had difficulty with missing brackets which typically led to $\ln 0$ in their answer.

Question 7:

Simplifying a trigonometric expression and applying identities was generally well answered in part (a), although some candidates were certainly helped by the fact that it was a "show that" question. More candidates had difficulty with part (b) with many assuming the first graph was $1 + \sin(x)$ and hence sketching a horizontal translation of $\pi/2$ for the graph of *g*; some attempts were not even sinusoidal. While some candidates found the stretch factor *p* correctly or from follow-through on their own graph, very few successfully found the value and direction for the translation. Part (c) certainly served as a discriminator between the grade 6 and 7 candidates.

Question 8:

A pleasing number of candidates were successful on this straightforward vector and line question. Part (a) was generally well answered, although a few candidates still labeled their line $L =$ or used a position vector for the direction vector. Follow-through marking allowed full recovery from the latter error.

Few candidates wrote down their direction vector in part (b) which led to lost follow-through marks, and a common error was finding an incorrect scalar product due to difficulty multiplying by zero.

Part (c) was generally well understood with some candidates realizing that the equation in just one variable led to the correct parameter more quickly than solving a system of two equations to find both parameters. Some candidates gave the answer as (s,t) instead of substituting those parameters, indicating a more rote understanding of the problem. Another common error was using the same parameter for both lines.

There were an alarming number of misreads of negative signs from the question or from the candidate working.

Question 9:

In part (a), many candidates successfully substituted the point A to find the base of the logarithm, although some candidates lost a mark for not showing their manipulation of the logarithm equation into the exponential equation.

A number of candidates who correctly stated the *y*-intercept was –2 had difficulty sketching the graph of the reflection in the line $y=x$. A number of candidates graphed directly on the question paper rather than sketching their own graph; candidates should be reminded to show all working for Section B on separate paper. Some correct sketches did not have the position of A indicated. Many candidates had difficulty reflecting the asymptote.

Part (c) was often well done, with candidates showing clear and correct working.

The most successful candidates clearly appreciated the linkage between the question parts.

Question 10:

The derivative in part (a) was reasonably well done, but errors here often caused trouble in later parts. Candidates occasionally attempted to use the double angle identity for $\sin 2t$ before differentiating, but they rarely were successful in then applying the product rule.

In part (b), most candidates understood that they needed to set their derivative equal to zero, but fewer were able to take the next step to solve the resulting double angle equation. Again, some candidates over-complicated the equation by using the double angle identity. Few ended up with the correct answer $5\pi/6$.

In part (c), many candidates knew they needed to test a value between $\pi/6$ and their value from part (b), but fewer were able to successfully complete that calculation. Some candidates simply tested their boundary values while others unsuccessfully attempted to make use of the second derivative.

Although many candidates did not attempt part (d), those who did often demonstrated a good understanding of how to use the displacement function *s* or the integral of their derivative from part (a). Candidates who had made an error in part (b) often could not finish, as $\sin(2t)$ could not be evaluated at their value without a calculator. Of those who had successfully found the other boundary of $5\pi/6$, a common error was giving the incorrect sign of the value of $\sin(5\pi/3)$. Again, this part was a good discriminator between the grade 6 and 7 candidates.

Recommendations and guidance for the teaching of future candidates for both papers

- Candidates should practice sketching graphs by hand, as some had great difficulty with those parts of the examination paper.
- Candidates need to practice carefully reading information from graphs as in Question 1.
- While many candidates show good working, all need to be continually reminded to show their method clearly. Sloppy presentation often leads to arithmetical errors and difficulty in correctly applying a chosen method.
- Conditional probability should be explored with diagrams as well as the formula.
- Candidates need to be counseled not to work backwards from a given answer.
- Candidates should be reminded not to write their answers on Section B of the question paper and to only use graph paper for graphs.
- Candidates should be reminded that papers are scanned in black and white. Referring to an answer in color is meaningless to the examiner.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-15	16-30	31-39	40-49	50-59	60-69	70-90

The areas of the programme and examination that appeared difficult for the candidates

Candidates in this session had difficulties in the following areas of the programme:

- Finding a term in a binomial expansion
- Binomial distributions
- Appropriate and timely use of a graphic display calculator (GDC) to find roots, probabilities and volumes.
- Using matrices to solve systems of equations
- Presumed knowledge – geometry of simple plane figures

The areas of the programme and examination in which candidates appeared well prepared

For students who were well prepared, there was ample opportunity to demonstrate a high level of knowledge and understanding on this paper. The following areas of the programme were handled well by most students.

- Arithmetic sequences and series
- Quadratic functions
- 2×2 and 3×3 determinants
- Binomial probability
- Volumes of revolution
- Determining summary statistics for grouped data.
- Basic use of sine and cosine rules

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1 Arithmetic sequences and series

The majority of candidates were successful with this question. Most had little difficulty with part (a) although some candidates were unable to show the required result in part (b), often substituting values for *n* rather than working with the formula for the sum of an arithmetic series.

Question 2 Quadratic functions

This question was well done by the majority of candidates. There were still many however who opted for an analytical approach in part (b), which often led to errors in sign and accuracy. Some candidates used the trace feature on their GDC to find the vertex which often resulted in accuracy errors.

Question 3 Determinants

Candidates could easily apply the formula for 2×2 determinants in part (a) and most could use their calculators successfully to find the 3×3 determinant in part (b). Analytical approaches in part (c) were always unsuccessful as candidates attempted to solve the resulting cubic with the quadratic formula.

Question 4 Volumes of revolution

Candidates showed marked improvement in writing fully correct expressions for a volume of revolution. Common errors of course included the omission of $\mathrm{d}x$, using the given domain as the upper and lower bounds of integration, forgetting to square their function and/or the omission of π. There were still many who were unable to use their calculator successfully to find the required volume.

Question 5 Inverse matrices and linear systems

Those who knew that the product of a matrix and its inverse yields the identity matrix had little difficulty with this question. Others attempted to set up a determinant expression for ***M*** and set it equal to the denominator of the multiplier of $\boldsymbol{M}^{-1}$ (an incorrect but fortunate outcome). This resulted in a rather complex equation in *p* and *q* that required some sophisticated algebra to solve. Few if any, were successful. In part (b), the solution to the linear system was often attempted analytically rather than simply using the tools available on the calculator. Of course analytical approaches were time consuming and often resulted in algebraic errors.

Question 6 Binomial theorem

An unfamiliar presentation confused a number of candidates who attempted to set up an equation with the wrong term in part (a). Time and again, candidates omitted the binomial coefficient in their set up leading to an incorrect result. In part (b) it was common to see the constant term treated as the last term of the expansion rather than the 7^{th} term.

Question 7 Binomial Distributions

This was an accessible problem that created some difficulties for candidates. Most were able to recognize the binomial nature of the problem but were confused by the phrase “at least four tails” which was often interpreted as the complement of four or less. Poor algebraic manipulation also led to unnecessary errors that the calculator approach would have avoided.

Question 8 Statistics

Parts (a) and (b) were generally well done. The terms “median” and “median class” were often confused. In part (c) some candidates had problems with the term “interval width” and there were some rather interesting mid – interval values noted. In part (d), candidates often ignored the “hence” command and estimated values from the graph rather than from the information in part (c). Those who correctly obtained the mean and standard deviation had little difficulty with part (e) although candidates often used unfamiliar calculator notation as their working or used the mid - interval value as the mean of the distribution.

Question 9 Triangle trigonometry

There were mixed results with this question. Most candidates could access part (a) and made the correct choice with the cosine rule but sloppy notation often led to candidates not being able to show the desired result. In part (c), candidates again correctly identified an appropriate method but failed to recognize that their result of 0.925 was acute and not obtuse as required. In (d) (i), many attempted to use the sine rule under the incorrect assumption that DC was equal to 5*p*, rather than rely on some basic isosceles triangle geometry. Consequently, the result of 1.29 for $\mathrm{C\hat{B}D}$ was not easy to show. There was a great deal of success with (d) (ii) with candidates using appropriate techniques to find the area of the shaded region although some stopped after finding the area of the sector.

Question 10 Optimization

Part (a) was generally well done although some candidates incorrectly used the function given in part (b) to find the required values. There was evidence that some candidates are not comfortable with a 24-hour clock. Candidates had difficulty generalizing the problem and therefore, were unable to show how the function $s(t)$ was obtained in part (b). Surprisingly, the graph in part (c) was not well done. Candidates often ignored the given domain, provided no indication of scale, and drew "V" shapes or parabolas. In part (d), candidates simply regurgitated the question without providing any significant evidence for their statements that the two ships must have been more than 8 km apart

Recommendations and guidance for the teaching of future candidates for both papers

Candidates need to be encouraged to follow instructions, especially those on giving answers exactly or correct to three significant figures. Marks may be lost if answers are not given to three significant figures. There are many who still interpret three significant figures as three decimal places. Candidates should further be encouraged to show their working, as answers left to 1 or 2 significant figures with no working may achieve no marks. They should also be encouraged to avoid premature rounding, as this may lead to incorrect answers.

Candidates should be taught not simply to transcribe graphs from their GDC without considering their intrinsic knowledge of key features and behaviors of functions. The command term "sketch" is not well understood although its definition is clearly stated in the guide. Candidate should be encouraged to use the appropriate GDC tools to find key features of graphs rather than estimate them using "trace" functions.

Poor mathematical communication continues to plague candidates at this level. Stronger candidates have work that is presented clearly and concisely. Teachers are encouraged to persevere with candidates emphasizing appropriate language and set up of solutions. Avoid calculator language and notation when communicating solutions and encourage candidates to label questions and their parts.

Although GDC use was much improved this session, there are still candidate who are opting for an analytical approach rather than a more efficient GDC approach. This is leading to significant errors and consumes valuable time. The use of a GDC to solve equations and find intersections should be stressed.

For "show that" questions, stress that students must approach this problem from the beginning and strive to reach the conclusion indicated. Often, students are substituting in values and working backwards, thinking that this is the evidence that is required.

Familiarize students with an IB mark scheme so that they are made aware of where method and answer marks are obtained thereby emphasizing the importance of showing relevant working.

Design the course in such a way as to provide adequate time for students to develop conceptual understanding in conjunction with good technique and timely use of a GDC. Encourage understanding through reading and communicating appropriate mathematical language. Expose students to more mathematics set in both familiar and unfamiliar contexts particularly in the areas of trigonometry and calculus.