MATHEMATICS SL TZ2

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 19	20 - 38	39 - 54	55 - 64	65 - 75	76 - 85	86 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2011 examination session the IB has produced time zone variants of the Mathematics SL papers.

General Comments

Many thanks to the teachers who provided feedback through the G2 forms about the examination. These were read by the senior examining team prior to setting the grade boundaries, and provided helpful and often insightful discussion points for consideration for this grade award and for future paper setting. Many of the issues raised regarding individual questions are contained within this report.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 13	14 - 19	20 - 23	24 - 28	29 - 33	34 - 40

The range and suitability of the work submitted

The vast majority of the work presented came from the set of tasks developed by the IB. Most schools appeared to be aware of the requirement to use these new tasks for this session. A few schools presented older tasks that are no longer usable for the portfolio and candidates suffered a penalty as a result. A very few teachers presented tasks of their own design. These varied in quality but included some very good ones. Others lacked the necessary depth for a portfolio task and did not allow for success against all levels of the criteria.

Candidate performance against each criterion

Criterion A

Overall, candidates and teachers are making a good effort to ensure the use of proper notation. However, despite many years of comments about the inappropriate use of computer and calculator notation, there persists a minority of schools that do not penalize these errors. Often moderators would note that a comment had been made on candidate work that these notations were inappropriate, yet no penalty was applied.

Moderators are also noting an increasing prevalence of informal language for mathematical terms and operations. One purpose of these tasks is to improve the standard of mathematical language and terminology use. Teachers should be on the lookout for confusion between “quadratic” and “exponential”, “curve” and “line”, “variable” and “parameter”, etc.

Criterion B

The vast majority of work presented is communicated well. Issues that persist include the inadequate or poor labelling of graphs, the use of a “question & answer” format, overly detailed descriptions of calculator steps, and the use of appendices for graphs and tables that should appear in the body of the work. In some tasks, for example the “Stellar Numbers” task, the use of suitable diagrams is not only recommended, but required. Often candidates made claims about how many dots appeared at a stage in the pattern without any supporting evidence in the form of a clear diagram.

Criterion C

Type I

While most candidates were successful in discovering appropriate patterns in these tasks, the resulting statements often came out of the blue, with little or no supporting analysis and examples. Teacher should take note that results presented without adequate support cannot be accepted. Once a statement is proposed the candidate must use new and further examples to validate the conjecture. Many use the same values that they used to develop the statement in the first place, which will obviously satisfy the statement.

Type II

There was an improvement noted in the quality of work presented regarding the definition and declaration of variables, constraints and parameters. However, many candidates give this point short shrift and leave much to be assumed. As with Type I tasks, there must be sufficient analysis present in order to accept the proposed model as a result. Teachers should be aware that the use of calculator regression techniques to develop a model function will limit the mark in criterion C to level 2. In some cases candidates would use regression to find a suitable model then work backwards to show some analysis that “leads” to this model. This is inappropriate and should be considered as if the regression were used alone.

In the “Population Trends in China” task many candidates used only a linear model. While the data certainly looks linear, candidates should realize that a linear model over the longer run is not likely to be appropriate. Other models should also be considered and developed.

A qualitative consideration of the fit of the function to the data is sufficient provided that there is some substance to the comments. Statements such as “it fits well” say little and are not enough to achieve level 4. There is no expectation that the error be measured in any way.

Applying the analytically developed model to a further set of data and commenting on how well the model fits this new data is sufficient for level 5. Candidates will make modifications to their model so that it does fit better and this is recognized under criterion D.

Criterion D

Type I

Many candidates obtain good results and present admirable arguments for allowing appropriate values or to explain the behaviour noted. However, the results obtained must come from some sound reasoning in order to achieve the higher levels of criterion D. A general statement that appears from nowhere cannot be considered more than an attempt at achieving what was desired. Some candidates continue to limit the discussion of scope and limitations to only the most superficial observations. While it may appear obvious that a certain value can only be, for example, a natural number, the candidate should check whether or not other values happen to work in the general statement and what this implies. Candidates also find it difficult to offer informal explanations for their statements. This may sometimes be an algebraic argument or it may simply be a clearly drawn series of diagrams indicating the progression of a geometric structure.

Type II

The most obvious weakness in criterion D was the lack of consideration of the actual context. Many candidates do an admirable job of the mathematical work but neglect to relate the graphs and functions back to the context of the task. A task about G-force should be discussed in terms of the forces on the human body under different circumstances, not just increasing or decreasing values of variables or asymptotic behaviour of graphs. The best work often included thoughtful consideration of why there might be an asymptote in the G-force model, or why there was a fairly abrupt change in the population trend in China.

Criterion E

The access and quality of technology available has increased to a point where its use has become commonplace. Unfortunately moderators find that teachers do not properly inform them about the availability of technology in the school. High marks were often given in criterion E without any substantive evidence in the work, nor in any background information. Even in Type I tasks technology can often be used to produce results for many more and larger values of the variables, or for presenting graphs that support the conjecture. In Type II tasks multiple graphs can be used to provide evidence of an evolution of transformations that lead to a better fitting function, or for comparing multiple functions at one time.

Criterion F

This criterion was generally well assessed. Most of the marks were appropriately awarded at level F1. Teachers are reminded that 0 and F are reserved for work at either extreme; totally unacceptable or highly remarkable.

Recommendations for the teaching of future candidates

Teachers must work through the tasks beforehand so they have a good idea of what is possible and what to expect from their students. They are then better prepared to assist students in their understanding of the criteria and how the students can address the highest levels. Older tasks that are no longer allowed for final submission can be used for practice in this regard. Integrating small parts of these tasks into lessons can draw attention to the skills and concepts at work. This is especially important when teaching students how to develop models analytically or validate conjectures properly. Teachers should take time to teach the effective use of any software that might be useful.

Reading this subject report can give teachers and students a clearer idea of what is expected and what to watch for.

Further comments

Wherever tasks are adapted or self-designed teachers should try to avoid extensions that seriously increase the workload expected from students. The extra work is often too much for students.

Where there is more than one teacher in a school, it is essential that they standardise their marking so as to ensure a consistent and appropriate approach. Teachers are also encouraged to become IA moderators themselves. In this way they can be exposed to work of differing standard that is done by candidates all over the world, learning along the way how to improve their own teaching.

External assessment

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 18	19 - 36	37 - 51	52 - 60	61 - 70	71 - 79	80 - 90

The areas of the programme and examination that appeared difficult for the candidates

- understanding Venn diagrams
- working with rules of logarithms
- transformation of functions
- kinematics
- area between two functions with different boundaries
- finding the total range of two sets of values

- finding the parameters of a trigonometric function
- algebraic manipulation

The levels of knowledge, understanding and skill demonstrated

Candidates in this session seem to have been exposed to most areas of the syllabus, and most candidates were able to make some sort of attempt on each question. Some topics were very well done by the majority of candidates:

- arithmetic sequences and series
- operations with vectors
- using the quotient rule for derivatives
- finding and interpreting the roots of a function
- basic manipulation of a quadratic equation
- applying a cosine model to a real-life situation

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This question was answered correctly by the large majority of candidates. The few mistakes seen were due to either incorrect substitution into the formula or simple arithmetic errors. Even where candidates made mistakes, they were usually able to earn full follow-through marks in the subsequent parts of the question.

Question 2

Most candidates were able to find the correct values for the Venn diagram. Unfortunately, however, there were many candidates who did not understand what each region of the diagram represents. For example, a very common error was thinking that $\mathrm{P}(B) = p$, rather than the correct $\mathrm{P}(B) = p + n$. Candidates seemed to understand the idea of the complement in part (b), but some were not able to find the correct answer because of confusion over the separation of the different regions in the diagram.

Question 3

This question on two-dimensional vectors was generally very well done. A very small number of candidates had trouble with the "show that" in part (b) of the question. Nearly all candidates knew to use the scalar product in part (c) to show that the vectors are perpendicular.

Question 4

The majority of candidates were successful in using the quotient rule, and were able to earn most of the marks for this question. However, there were a large number of candidates who substituted correctly into the quotient rule, but then went on to make mistakes in simplifying this expression. These algebraic errors kept the candidates from earning the final mark for the correct answer. A few candidates tried to use the product rule to find the derivative, but they were generally not as successful as those who used the quotient rule. It was pleasing to note that most candidates did know the correct values for the sine and cosine of zero.

Question 5

This question was very poorly done by the majority of candidates. While candidates seemed to have a vague idea of how to apply the rules of logarithms in part (a), very few did so successfully. The most common error in part (a) was to begin incorrectly with $\ln 5x^3 = 3\ln 5x$. This error was often followed by other errors. In part (b), very few candidates were able to describe the transformation as a vertical translation (or shift). Many candidates attempted to describe numerous incorrect transformations, and some left part (b) entirely blank.

Question 6

While there were a large number of candidates who answered both parts of this question correctly, a surprising number did not know how to find the range of all 200 fish in part (a). Common errors included finding the ranges of the male and female fish separately, or averaging the separate ranges of the male and female fish.

Some candidates did not interpret the cumulative frequency graphs correctly, or just seemed to guess which graph was correct. The most common incorrect "guess" was graph 4, likely because this graph had a more familiar cumulative shape.

Question 7

Most candidates did a good job using the determinant and finding the correct *x*-value in part (a). In part (b), many candidates were successful using a number of different methods. There were some who were not able to earn full marks due to errors in their inverse matrices.

Question 8

The majority of candidates seemed to know what was meant by the tangent to the graph in part (a), but there were many who did not fully show their work, which is of course necessary on a "show that" question. While many candidates knew they needed to find the derivative of f, some failed to substitute the given value of x in order to find the gradient of the tangent.

Part(b), finding the *x*-intercept, was answered correctly by nearly every candidate.

In part (c), most candidates struggled with writing an expression for the area of R. Many tried to use the difference of the two functions over the entire interval 0-1, not noticing that the area from 0-0.5 only required the use of function f. Many of these candidates were able to earn follow-through marks in the second part of (c) for their correct integration. There were a few candidates who successfully found the area under the line as the area of a triangle.

Question 9

Parts (a) and (c) of this question were very well done by most candidates.

In part (b), many candidates attempted to use the method of completing the square, but were unsuccessful dealing with the coefficient of -10. Candidates who recognized that the *x*-coordinate of the vertex was 1, then substituted this value into the function from part (a), were generally able to earn full marks here.

In part (d), it was clear that many candidates were not familiar with the relationship between velocity and acceleration, and did not understand how those concepts were related to the graph which was given. A large number of candidates used time $t=1$ in part b(ii), rather than $t=6$. To find the acceleration, some candidates tried to integrate the velocity function, rather than taking the derivative of velocity. Still others found the derivative in part b(i), but did not realize they needed to use it in part b(ii), as well.

Question 10

Nearly all candidates answered part (a) correctly, finding the height of the wheel at ½ and ¾ of a revolution.

While many candidates were successful in part (b), there were many who tried to use right-angled triangles or find a function for height, rather than recognizing the symmetry of the wheel in its different positions and using the values given in the table.

In part (c), most candidates were able to sketch a somewhat accurate representation of the height of the wheel over two full cycles. However, it seems that many candidates are not familiar with the shape of a sinusoidal wave, as many of the candidates' graphs were constructed of line segments, rather than a curve.

For part (d), candidates were less successful in finding the parameters of the cosine function. Even candidates who drew accurate sketches were not always able to relate their sketch to the function. These candidates understood the context of the problem, that the position on the wheel goes up and down, but they did not relate this to a trigonometric function. Only a small number of candidates recognized that the value of a would be negative. Candidates should be aware that while working in degrees may be acceptable, the expectation is that radians will be used in these types of questions.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 50	51 - 59	60 - 67	68 - 76	77 - 90

The areas of the programme and examination that appeared difficult for the candidates

- normal distribution
- direction vectors
- recognizing binomial distribution
- using the graphic display calculator (GDC) to solve algebraically complicated equations
- show that questions

The levels of knowledge, understanding and skill demonstrated

Candidates demonstrated a good level of knowledge and understanding with most topics. Strengths included:

- functions
- graph sketching using the correct domain
- including sketches of the graphs to support GDC solutions
- binomial theorem
- matrices
- triangle trigonometry
- including full working for each question

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Composite and inverse functions

Most candidates handled this question with ease. Some were not familiar with the notation of composite functions assuming that $(f \circ g)(x)$ implied finding the composition and then multiplying this by x. Others misunderstood part (b) and found the reciprocal function or the derivative, indicating they were not familiar with the notation for an inverse function.

Question 2: Graph of a function and finding intersection

This question was well done by the majority of candidates. Most sketched an approximately correct shape in the given domain, though some candidates did not realize they had to set their GDC to radians, producing a meaningless sketch. Candidates need to be aware that unless otherwise specified, questions will expect radians to be used. The most confident candidates used a table to aid their graphing. Although most recognized the need of the GDC to answer part (b), some used the trace function, hence obtaining an inaccurate result, while others attempted a fruitless analytical approach. Merely stating “using GDC” is insufficient evidence of method; a sketch or an equation set equal to zero are both examples of appropriate evidence.

Question 3: Finding a specific term in a binomial expansion

Most candidates attempted this question, and many made good progress. A number of candidates spent time writing out Pascal’s triangle in full. Common errors included 11 for part (a) and not writing out the simplified form of the term for part (b). Another common error was adding instead of multiplying the parts of the term in part (b).

Question 4: Inverse of a 3×3 matrix and matrix equation

Most candidates answered part (a) without difficulties, although some candidates wrote the transpose of matrix ***M***. Well-prepared candidates clearly understood the requirements of the GDC in part (b), finding the inverse of the matrix without any problems and solving the matrix equation to obtain the correct 3×1 matrix. In some cases, the correct answer followed from working where the matrices were reversed. Those who tried to solve the system analytically usually struggled with algebraic errors. Some candidates did not understand what was required in part (c), and substituted the solutions of the matrix into the linear system but did not specify the coordinates.

Question 5: Trigonometry in non-right angled triangles

This question was attempted in a satisfactory manner. Even the weakest candidates earned some marks here, showing some clear working. In part (a) the diagram was completed fairly well, with some candidates incorrectly labeling the angle with the vertical as 4^0. The cosine rule was applied satisfactory in part (b), although some candidates incorrectly used their calculators in radian mode. Approaches using a combination of the sine rule and/or right-angled triangle trigonometry were seen, especially when candidates incorrectly labeled the 25m path as being the distance from the horizontal to U.

Question 6: Normal distribution

This question proved challenging for many candidates. A surprising number did not use the symmetry of the normal curve to find the probability required in (a). While many students were able to set up a standardized equation in (b), far fewer were able to use the complement to find the correct *z*-score. Others used 0.8 as the *z*-score. A common confusion when approaching parts (a) and (b) was whether to use a probability or a *z*-score. Additionally, many candidates seemed unsure of appropriate notation on this problem which would have allowed them to better demonstrate their method.

Question 7: Integration

Although a pleasing number of candidates recognized the requirement of integration, many did not correctly apply the reverse of the chain rule to integration. While some candidates did not write the constant of integration, many did, earning additional follow-through marks even with an incorrect integral. Weaker candidates sometimes substituted $x=1$ into $\frac{\mathrm{d}y}{\mathrm{d}x}$ or attempted some work with a tangent line equation, earning no marks.

Question 8: Vector equation of a line, angle and intersection

Finding $\overrightarrow{\mathrm{AB}}$ was generally well done, although some candidates reversed the subtraction. However, in part (b) not all the candidates recognized that $\overrightarrow{\mathrm{AB}}$ was the direction vector of the line, as some used the position vector of point B as the direction vector. Many candidates successfully used scalar product and magnitudes in part (c), although a large number did choose vectors other than the direction vectors and many did not state clearly which vectors they were using. Candidates who were comfortable on the first three parts often had little difficulty with the final part. While the resulting systems were easily solved algebraically, a surprising number of candidates did not check their solutions either manually or with technology. An occasionally seen error in the final part was using a midpoint to find C. Some candidates found the point of intersection in part (c) rather than in part (d), indicating a familiarity with the type of question but a lack of understanding of the concepts involved.

Question 9: Binomial probability

All but the weakest candidates managed to score full marks for parts (a) and (b). An occasional error in part (a) was including additional pair(s) or listing (3,3) twice. Many candidates found part (c) challenging, as they failed to recognize the binomial probability. Successful candidates generally used either the binomial CDF function or the sum of two binomial probabilities. Some used approaches like multiplying probabilities or tree diagrams, but these were less successful.

Question 10: Trigonometry

As the final question of the paper, this question was understandably challenging for the majority of the candidates. Part (a) was generally attempted, but often with a lack of method or correct reasoning. Many candidates had difficulty presenting their ideas in a clear and organized manner. Some tried a “working backwards” approach, earning no marks. In part (b), most candidates understood what was required and set up an equation, but many did not make use of the GDC and instead attempted to solve this equation algebraically which did not result in the correct solution. A common error was finding a second solution outside the domain. A pleasing number of stronger candidates made progress on part (c), recognizing the need for the end point of the domain and/or the maximum value of the area function (found graphically, analytically, or on occasion, geometrically). However, it was evident from candidate work and teacher comments that some candidates did not understand the wording of the question. This has been taken into consideration for future paper writing.

Recommendations and guidance for the teaching of future candidates for both papers

Teachers need to be sure their students are exposed to all areas of the syllabus. It was apparent that this is not always the case, as some candidates left questions blank or gave answers which made no sense. Too often it is clear that candidates are not given complete preparation in the areas of vectors and probability. It should be noted that the recommended teaching hours for probability and statistics is substantial and near equal to that of calculus.

It is also helpful to candidates if they can be familiar with the information booklet. However, it is not enough to simply know these formulas. Candidates need to know what kind of situations these formulas are used for. Then they also need to know what the values they are using represent, and how to manipulate and work with these formulas correctly.

Practicing exam-style questions under timed conditions can be helpful to candidates. While most candidates seemed to be able to finish the exam, there were many who seemed to be rushed at the end, and some who left the last parts blank, presumably because they ran out of time. Candidates need to understand that they should not need to spend a lot of time on a 1 or 2 mark question, and that a 9-mark question generally takes more time and requires more working to be shown. It would also be helpful if candidates could work on practice exam papers, then reflect on what they had done by looking at the requirements of the different command terms and at their use of time relative to the amount of marks for each question.

Some teachers expressed concern that some questions seemed to have too many marks allocated. During paper setting, marks are carefully allocated to questions based upon the amount of work needed for solution. Students should be encouraged to show full working, as an incorrect answer with complete working can earn the majority of the marks

Candidates should be familiar with the command terms, and understand what is required. The command term “show that” is not well understood by many candidates. As this is not an obvious instruction, it is helpful if they are exposed to the terminology throughout the two years of the course, so as to become accustomed to its meaning.

Some candidates do not appear aware of the three significant figure requirement; this requires continued emphasis during the course.

Teachers should remind candidates that it is important to use proper notation throughout their working, as this makes their working easier to understand. It has often been noted by examiners that the stronger candidates tend to work through questions in a more organized manner. Poor mathematical communication can cause problems for candidates at this level. Teachers are encouraged to persevere with candidates emphasizing appropriate language and set up of solutions. Avoid calculator language and notation when communicating solutions and encourage candidates to label questions and their parts.

Teachers should be encouraged to provide more opportunities for students to develop the quality of their explanations and justifications of important mathematical results. Design the course in such a way as to provide adequate time for students to develop conceptual understanding in conjunction with good technique. Encourage understanding through reading and communicating appropriate mathematical language. Expose students to more mathematics set in both familiar and unfamiliar contexts particularly in the areas of trigonometry and calculus.

In vector problems, candidates should develop an understanding of the techniques and should be encouraged to clearly indicate which vectors they are using when finding the angle between two lines.

Teachers are encouraged to ensure that candidates are familiar with all the GDC skills and techniques that are found in the guide and the GDC TSM. This can be achieved by incorporating the GDC into daily lessons to augment understanding of most syllabus topics. Candidates should be taught not simply to transcribe graphs from their GDC without considering their intrinsic knowledge of key features and behaviors of functions.

Candidates need to be aware that not all equations can be solved using algebra; they will be expected to use their GDC to solve equations on Paper 2. They also need to be aware that a graph sketch or setting the equation equal to zero is suitable working for a GDC solution. Candidates should understand how to sketch an accurate graph from a GDC screen by using key graph features and/or the table function.

Unless otherwise specified, trigonometry questions are in radians. It was clear from teacher comments that some candidates were not aware of the importance of checking the mode of their calculator.

Many students seem to be formula-driven, and consequently, they have difficulty interpreting or explaining a situation. If teachers focus on concepts to develop methods of solution, then students will have greater success interpreting questions that introduce different situations.

With regards to e-marking, candidates and teachers need to be aware that **everything** on a scanned script will show up as dark black. This means that stray marks, ink from pens that bleed through the paper, and even items which have been partially erased will all show up as black when they are scanned. This often makes it difficult to decipher the candidates' intended working and answers. Candidates are reminded that graph paper should not be used for anything except the drawing of graphs, and that when a question uses the command term "sketch", it is generally not necessary to use graph paper.

Finally, many teachers are doing a very good job of preparing their students, and are encouraged to continue doing so. It is hoped that the comments here will help identify where there are weaknesses, and provide advice for future improvement.