MATHEMATICS SL TZ2

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 14	15 – 28	29 – 45	46 – 58	59 – 70	71 – 83	84 – 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2010 examination session the IB has produced time zone variants of the Mathematics SL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 7	8 – 13	14 – 19	20 – 23	24 – 28	29 – 33	34 – 40

The range and suitability of the work submitted

The majority of work submitted came from the current set of tasks developed by the IB. Popular choices included "Matrix Binomials" and "Body Mass Index". Where teacher-designed tasks were used these varied significantly in quality. If the tasks did not allow for students to address all of the assessment levels the result was usually a significant downwards moderation of the marks. Some schools submitted atypical work without adding a substitute portfolio for these candidates. The quality of student work was generally good, with some outstanding examples.

Candidate performance against each criterion

Overall there is much evidence that teachers and students understand the criteria well and make every effort to prepare quality work. Some areas of concern are noted below.

Criterion A: There continues to be a problem with the use of calculator notation and the lack of use of an appropriate "approximately equals" sign. In the modelling task, candidates often use the same dependent variable for different model functions.

Criterion B: The use of a "question and answer" style is a problem, with some teachers and students treating the tasks as a set of homework exercises. The work should be presented as a cohesive piece of mathematical writing with graphs and tables offered within the context, instead of as appendices. The proper labelling of graphs is an issue, especially where candidates have used graphing technology but don't know how to apply labels to axes.

Criterion C Type I: Candidates often do not present sufficient evidence or analysis to support their general statements. For example, in the "Matrix Binomials" task many announced that $(\boldsymbol{A}+\boldsymbol{B})\wedge n=\boldsymbol{A}\wedge n+\boldsymbol{B}\wedge n$ without any coherent support. Candidates would often "validate" their proposed general statement by using the same values they used to develop it. The process of validation involves using further values and comparing the results against the mathematical behaviour in the context of the task.

Criterion C Type II: Many candidates do not explicitly define variables, parameters, and constraints. As with the Type I tasks candidates are not providing sufficient and appropriate analysis to develop their model functions. In some cases teachers are still condoning the use of calculator or computer regression models without any supporting mathematics. There is some confusion as to the use of graphical transformations to develop a model. If students use their knowledge of these transformations appropriately and demonstrate a sequence of attempts to fit a model function using suitable modifications to an original basic function then they can access all of the marks available in criterion C. The comments on how well the models fit the data are generally superficial. These should include some specifics such as how the function fits the data in certain intervals, at the extremes, etc., and not simply something like "fits well". While a quantitative analysis is not expected for maths SL, the candidate should say more than "fits well". Intervals of good and poor fit should be identified and discussed.

Criterion D Type I: The major issues here are the appropriate exploration of scope and limitations, as well as the quality of explanation offered. Given the availability of the graphic display calculator (GDC), it is expected that candidates will explore a wide variety of values for their general statements. Many focus only on positive integers with no thought to other possibilities. Most candidates found it very difficult to provide an explanation for the general statement.

Criterion D Type II: While most candidates proved themselves mathematically capable of matching a function to the data, many found it difficult to discuss the model in context, or simply ignored this aspect. The connection between reality and the mathematical attributes of variables and graphs seemed lost on the candidates. There was little application of critical thinking skills to the situations.

Criterion E: The use of graphing technologies is clearly growing. Many students presented high-quality graphs with some in colour to differentiate different models. While this is a positive development there were also cases where the candidates used the technology without thought. Graphs by themselves do not enhance the development of the tasks.

Criterion F: This criterion was well understood by most teachers. A reasonable effort to complete the task was awarded F1 in the majority of cases. Teachers seemed to appreciate that the award of F0 or F2 is justifiably rare.

Recommendation and guidance for the teaching of future candidates

Candidates should be reminded that they can check their work against the criteria to ensure that they are addressing all the important components of the assessment. However, it is necessary that the teachers take the time to help candidates understand the criteria. Given the clear expectations of levels C1 and C2 in a Type II task, there is no excuse for candidates not properly and explicitly identifying the variables, parameters and constraints, although teachers may need to explain the difference between these. Teachers might have candidates complete practice work and assess it themselves against the criteria.

Candidates should be taught to treat the work as an essay in mathematics, requiring a cohesive and complete written presentation that flows smoothly.

Candidates would benefit from discussions about the purposes of the different tasks. The processes of mathematical investigation and mathematical modelling may be foreign to their experience. The necessity of proper evidence and analysis, as well as the appreciation for critical consideration of the implications of the work are important skills that teachers can explain. Again, practice work would be helpful here. The use of technology must go beyond simply producing graphs. Candidates should better understand the power of the technology available to them as a tool to explain and explore.

Teachers are reminded to offer written comments on the student work that help explain why certain marks were awarded. It is also expected that teachers will provide solutions to the tasks that describe their own expectations as to how the levels of the criteria can be attained. Tasks designed by teachers must address all the levels of the criteria. They should also focus on one problem rather than branch out into multi-part questions as these confuse the marking.

All teachers would benefit from a careful reading of current and past subject reports, as well as participation in discussion forums on the Online Curriculum Centre.

External assessment

Paper 1

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 11	12 – 22	23 – 40	41 – 52	53 – 64	65 – 76	77 – 90

The areas of the programme which proved difficult for candidates

- Finding a unit vector in the direction of another vector
- Working with trigonometric functions of certain angles (0, $\pi/2$, π, and $3\pi/2$)
- Relating the derivative to the gradient of a curve
- Applying logarithm properties
- Interpreting second derivative from the concavity of a graph
- Concept of the constant of integration
- Conditional and combined probability
- Algebraic manipulation and arithmetic with fractions

The levels of knowledge, understanding and skills demonstrated

As is to be expected, candidates' levels of knowledge and understanding varied widely. A large number of candidates seemed to be well prepared for taking Paper 1 without a calculator. In most cases the candidates did a nice job showing their work. The following areas were handled well by candidates:

- using scalar product for perpendicularity
- composition of functions
- multiplying matrices
- differentiation and integration of polynomial functions
- recognizing that areas can be found with integration

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

The majority of candidates were successful on some or all parts of this question, with some candidates using a mix of algebra and graphical reasoning and others ignoring the graph and working only algebraically. Some did not recognize that p and q are the roots of the quadratic function and hence gave the answers as 2 and –4. A common error in part (b) was the absence of an equation. Some candidates wrote down the equation $x = \frac{-b}{2a}$ but were not able to substitute correctly. Those students did not realize that the axis of symmetry is always halfway between the x-intercepts. More candidates had trouble with part (c) with erroneous substitutions and simplification mistakes commonplace.

Question 2

Part (a) was generally done well with candidates employing different correct methods to find the vector $\overrightarrow{\mathrm{BC}}$. Some candidates subtracted the given vectors in the wrong order and others simply added them. Calculation errors were seen with some frequency.

Many candidates did not appear to know how to find a unit vector in part (b). Some tried to write down the vector equation of a line, indicating no familiarity with the concept of unit vectors while others gave the vector $(1,-1,1)$ or wrote the same vector $\overrightarrow{\mathrm{AB}}$ as a linear combination of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$. A number of candidates correctly found the magnitude but did not continue on to write the unit vector.

Candidates were generally successful in showing that the vectors in part (c) were perpendicular. Many used the efficient approach of showing that the scalar product equaled zero, while others worked a little harder than necessary and used the cosine rule to find the angle between the two vectors.

Question 3

Most candidates were able to multiply the matrices in part (a) although some made minor arithmetic mistakes. There were a few candidates who employed creative albeit incorrect methods of multiplication.

Part (b) caused many candidates more difficulty. Stronger candidates realized that the matrix equation solution was the calculation they had just done in the previous part and finished, but many candidates felt it necessary to find the inverse matrix. Some then realized that this was unnecessary and went back to the simpler solution while others either stopped or soldiered on and created a system of equations. A few were able to solve their system correctly but many made errors or used up valuable time pursuing a lengthy solution to a two-mark question. Some candidates who employed a matrix algebra solution did not appreciate the non-commutativity of matrix multiplication, or attempted to divide by the inverse matrix, indicating a fundamental misunderstanding of operations with matrices.

Question 4: Function Composition and Trigonometry

In part (a), a number of candidates were not able to evaluate $\cos \pi$, either leaving it or evaluating it incorrectly.

Almost all candidates evaluated the composite function in part (b) in the given order, many earning follow-through marks for incorrect answers from part (a). On both parts (a) and (b), there were candidates who correctly used double-angle formulas to come up with correct answers; while this is a valid method, it required unnecessary additional work.

Candidates were not as successful in part (c). Many tried to use double-angle formulas, but either used the formula incorrectly or used it to write the expression in terms of $\cos x$ and went no further. There were a number of cases in which the candidates "accidentally" came up with the correct answer based on errors or lucky guesses and did not earn credit for their final answer. Only a few candidates recognized the correct method of solution.

Question 5

Candidates' success with this question was mixed. Those who understood the relationship between the derivative and the gradient of the normal line were not bothered by the lack of structure in the question, solving clearly with only a few steps, earning full marks. Those who were unclear often either gained a few marks for finding the derivative and substituting $x=1$, or no marks for working that did not employ the derivative. Misunderstandings included simply finding the equation of the tangent or normal line, setting the derivative equal to the gradient of the normal, and equating the function with the normal or tangent line equation. Among the candidates who demonstrated greater understanding, more used the gradient of the normal (the equation $-1/4k=-1/8$) than the gradient of the tangent $(4k=8)$; this led to more algebraic errors in obtaining the final answer of $k=2$. A number of unsuccessful candidates wrote down a lot of irrelevant mathematics with no plan in mind and earned no marks.

Question 6

Candidates secure in their understanding of logarithm properties usually had success with this problem, solving the resulting quadratic either by factoring or using the quadratic formula. The majority of successful candidates correctly rejected the solution that was not in the domain. A number of candidates, however, were unclear on logarithm properties. Some unsuccessful candidates were able to demonstrate understanding of one property but without both were not able to make much progress. A few candidates employed a "guess and check" strategy, but this did not earn full marks.

Question 7

Candidates had mixed success with parts (a) and (b). Weaker candidates either incorrectly used the *x*-intercepts of *f* or left this question blank. Some wrote down only two of the three values in part (a). Candidates who answered part (a) correctly often had trouble writing the set of values in part (b); difficulties included poor notation and incorrectly including the endpoints. Other candidates listed individual *x*-values here rather than a range of values.

Many candidates had difficulty explaining why the second derivative is negative in part (c). A number claimed that since the point D was "close" to a maximum value, the second derivative must be negative; this incorrect appeal to the second derivative test indicates a lack of understanding of how the test works and the relative concept of closeness. Some candidates claimed D was a point of inflexion, again demonstrating poor understanding of the second derivative. Among candidates who answered part (c) correctly, some stated that f was concave down while others gave well-formed arguments for why the first derivative was decreasing. A few candidates provided nicely sketched graphs of f' and f'' and used them in their explanations.

Question 8

Many candidates were successful with this question. In part (a), some candidates found $f''(-4/3)$ and were unclear how to conclude, but most demonstrated a good understanding of the second derivative test.

A large percentage of candidates were successful in showing that $p=-4$ but there were still some who worked backwards from the answer. Others did not use the given information and worked from the second derivative, integrated, and then realized that p was the constant of integration. Candidates who evaluated the derivative at $x=2$ but set the result equal to 4 clearly did not understand the concept being assessed. Few candidates used the point B with fractional coordinates.

Candidates often did well on the first part of (c), knowing to integrate and successfully finding some or all terms. Some had trouble with the fractions or made careless errors with the signs; others did not use the value of $p=-4$ and so could not find the third term when integrating. It was very common for candidates to either forget the constant of integration or to leave it in without finding its value.

Question 9

Candidates generally handled some or all of parts (a) and (b) well. Errors included adding probabilities along branches and trying to use the union formula from the information booklet. On part (b)(ii), many candidates knew that they were supposed to use some type of conditional probability but did not know how to find $\mathrm{P}(E|F)$. Many candidates made errors working with fractions. Some candidates who missed part (a)(ii) were able to earn follow-through credit on part (b)(ii).

Many candidates had difficulty completing the probability distribution table. While the common error of finding the probability for $x = 3$ as $2/9$ was understandable as the candidate did not appreciate that there were two ways of paying three Euros, it was disappointing that these candidates often correctly found $\mathrm{P}(X = 4)$ as $4/9$ and did not note that the probabilities failed to sum to one. These candidates could not earn full follow-through marks on their expected value calculation in part (d). Some candidates did use the probabilities summing to one with incorrect probabilities in part (c); these candidates often earned full follow-through marks in part (d), as a majority of candidates knew the method for finding expected value.

Question 10

Many candidates again had difficulty finding the common angles in the trigonometric equations. In part (a), some did not show sufficient working in solving the equations. Others obtained a single solution in (a)(i) and did not find another. Some candidates worked in degrees; the majority worked in radians.

While some candidates appeared to use their understanding of the graph of the original function to find the *x*-intercept in part (b), most used their working from part (a)(ii) sometimes with follow-through on an incorrect answer.

Most candidates recognized the need for integration in part (c) but far fewer were able to see the solution through correctly to the end. Some did not show the full substitution of the limits, having incorrectly assumed that evaluating the integral at 0 would be 0; without this working, the mark for evaluating at the limits could not be earned. Again, many candidates had trouble working with the common trigonometric values.

While there was an issue in the wording of the question with the given domains, this did not appear to bother candidates in part (d). This part was often well completed with candidates using a variety of language to describe the horizontal translation to the right by $\pi/2$.

Most candidates who attempted part (e) realized that the integral was equal to the value that they had found in part (c), but a majority tried to integrate the function *g* without success.

Some candidates used sketches to find one or both values for *p*. The problem in the wording of the question did not appear to have been noticed by candidates in this part either.

The type of assistance and guidance the teachers should provide for future candidates

- Candidates need plenty of practice justifying, explaining, and showing given results, as these caused many of them difficulty on the paper.
- Some candidates need more work interpreting derivatives.
- Teachers should emphasize that candidates should look for links where one part of a question is following on from another. This is particularly true when given information can be used to earn marks on a later question part, even if the given information cannot be shown by the candidate.
- Candidates should be aware of the command terms used in questions; i.e. “write down” means that the answer can be found without showing working while “find” indicates that there is working to be shown.
- Candidates need to know the trig ratios of the angles 0, $\pi/2$, π etc
- Candidates should practice addition and multiplication of fractions without a calculator and be able to use the most efficient method of calculation (e.g. not finding common denominators when multiplying). Work without the calculator should be given periodically to maintain arithmetic and algebraic skills for the Paper 1.

Areas of the examination that gave candidates particular difficulty as noted in part A of this report need special attention.

Paper 2

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 13	14 – 26	27 – 41	42 - 52	53 – 63	64 – 74	75 – 90

The areas of the programme which proved difficult for candidates

- A considerable number of candidates still find challenging the use of the GDC as a tool to find information (i.e. standard deviation, local maximum and minimum points, solution of an equation, finding the value of a definite integral, etc.)

- Finding a specific term in the expansion of a binomial, in which both terms are functions of x.
- Identifying rates of change with derivatives.
- When to use radians and degrees.
- Determining the number of solutions of an equation such as $f(x)=k$ from the graph.
- Recognize which are the vectors needed to find the angle between two lines given in vector form.
- Accuracy.

The levels of knowledge, understanding and skill demonstrated

There was good overall syllabus coverage by schools and use of time by candidates. Most of them made reasonable attempts at all questions.

Clear working was seen in routine problems like Q2.

The following areas were well handled:

- Understanding of arithmetic sequences
- Understanding of basic vector algebra
- Sketching the graph of a function using the graphing calculator
- The trigonometry of sectors, arcs and triangles.
- Calculating the mean and missing values from a data table.
- Use of product rule.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

The majority of candidates had little trouble finding the missing values in the frequency distribution table, but many did not seem comfortable calculating the mean and standard deviation using their GDCs.

The correct mean was often found without the use of the statistical functions on the graphing calculator, but a large number of candidates were unable to find the standard deviation.

Question 2

This problem was done well by the vast majority of candidates. Most students set out their working very neatly and logically and gained full marks.

Question 3

While many candidates were successful at part (a), far fewer recognized the binomial distribution in the second part of the problem.

Those who did not obtain the correct answer at part (a) often scored partial credit by either drawing a table to represent the sample space or by noting relevant pairs.

Question 4

Although a great number of students recognized they could use the binomial theorem, fewer were successful in finding the term in x^4.

Candidates showed various difficulties when trying to solve this problem:

- choosing the incorrect term
- attempting to expand $\left(3x^2 - \frac{2}{x}\right)^5$ by hand
- finding only the coefficient of the term
- not being able to determine which term would yield an x^4
- errors in the calculations of the coefficient

Question 5

This question was answered well by a pleasing number of candidates.

For part (a), many good graphs were seen, though a significant number of candidates either used degrees or a function such as $\sin e^{x}$. There were students who lost marks for poor diagrams. For example, the shape was correct but the maximum and minimum were not accurate enough.

There were candidates who struggled in vain to solve the equation in part (b) algebraically instead of using a GDC. Those that did use their GDCs to solve the equation frequently gave their answers inaccurately, suggesting that they did not know how to use the 'zero' function on their calculator but found a rough solution using the 'trace' function.

In part (c) they often gave the correct solution, or obtained follow through marks on their incorrect results to part (b).

Question 6

Parts (a) and (b) were generally well answered, the main problem being the accuracy.

Many students lacked the calculator skills to successfully complete (6)(c) in that they could not find the value of the definite integral. Some tried to find it by hand.

When trying to explain why the integral was not the area, most knew the region under the *x*-axis was the cause of the integral not giving the total area, but the explanations were not sufficiently clear. It was often stated that the area below the axis was negative rather than the integral was negative.

Question 7

This question seemed to be challenging for the great majority of the candidates.

Part (a) was generally well answered but in parts (b) and (c) they did not consider that rates of change meant they needed to use differentiation. Most students completely missed or did not understand that the question was asking about the instantaneous rate of change, which resulted in the fact that most of them used the original equation. Some did attempt to find an average rate of change over the time interval, but even fewer attempted to use the derivative.

Of those who did realize to use the derivative in (b), a vast majority calculated it by hand instead of using their GDC feature to evaluate it.

The inequality for part (c) was sometimes well solved using the original function but many failed to round their answers to the nearest integer.

Question 8

Most candidates demonstrated understanding of trigonometry on this question. They generally did well in parts (a) and (c), and even many of them on part (b). Fewer candidates could do part (d).

Many opted to work in degrees rather than in radians, which often introduced multiple inaccuracies. Some worked with an incorrect radius of 6 or 10.

A pleasing number knew how to find the area of the shaded region.

Inability to work in radians and misunderstanding of significant figures were common problems, though. Weaker candidates often made the mistake of using triangle formulae for sectors or used degrees in the formulas instead of radians.

For some candidates there were many instances of confusion between lines and arcs. In (a) some treated 6 as the length of AC. In (d) some found the length of arc EF rather than the length of the segment.

Several students seemed to confuse the area of sector in (b) with the shaded region.

Question 9

Many candidates demonstrated a good understanding of the vector equation of a line and its application to a kinematics problem by correctly answering the first two parts of this question.

Some knew that speed and distance were magnitudes of vectors but chose the wrong vectors to calculate magnitudes.

Very few candidates were able to get the two correct answers in (c) even if they set up the equation correctly. Much contorted algebra was seen and little evidence of using the GDC to solve the equation. Many made simple algebraic errors by combining unlike terms in working with the scalar product (often writing $8a$ rather than $8+a$) or the magnitude (often writing $5a^2$ rather than $5+a^2$).

Question 10

Many candidates correctly found the x-coordinates of P and Q in (a) (i) with their GDC. In (a)(ii) some candidates incorrectly interpreted the words “exactly two solutions” as an indication that the discriminant of a quadratic was required. Many failed to realise that the values of k they were looking for in this question were the y-coordinates of the points found in (a)(i).

Many candidates were unclear in their application of the product formula in the verifying the given derivative of g. Showing that the derivative was the given expression often received full marks though it was not easy to tell in some cases if that demonstration came through understanding of the product and chain rules or from reasoning backwards from the given result.

Some candidates drew their graphs of the derivative in (c) on their examination papers despite clear instructions to do their work on separate sheets. Most who tried to plot the graph in (c) did so successfully, but correct solutions to 10(d) were not often seen.

The type of assistance and guidance the teachers should provide for future candidates

Paper 2 is a GDC active paper, where candidates are expected to use them as their first resource, yet this is still not the case for a considerable amount of candidates. It is fundamental that students are able to decide when and why a GDC is a useful tool. In this paper they should use them at least when:

- Working with statistics
- Solving definite integrals
- Solving equations. The GDC does not only give the solutions but it also helps the candidate visualize the amount of solutions.
- Sketching a graph. In this case teachers should emphasize that key features such as zeros, maximum and minimum points, and domain and end points need to be quite accurate.
- Using a graph to obtain accurate values for the maximums, minimums and zeros of a function. It is important to show the students that the trace features of their GDCs only give approximate results for these values as opposed to using the built-in features to

more accurately find them.

Unless they are given advice, candidates will continue to choose analytical approaches, which sometimes are fruitless.

It is important to emphasize the need to presenting their work clearly; Section A on the question paper and Section B on lined paper.

The connection between derivatives and rates of change needs greater emphasis. Much more familiarity with the use of radian measure is needed and that can only be obtained through more practice with radians and their relationship with degrees.

Most candidates lose accuracy marks. The difference between 3 significant figures and 3 decimal places needs more attention as does the avoidance of premature rounding in calculations.

It appears that many students are still not clear what "working" to write in the examination when using the GDC, so candidates often spent precious time writing analytic methods to problems most efficiently solved using the GDC. To "show working" does not mean to perform algebraic steps or manipulations. Rather, what is important is to show the mathematical thinking, the setup, before reaching for the GDC, and then to let the GDC do the work of calculation. Whatever supports the solution, making the problem "calculator-ready," is what students need to show as working.

To help teachers and students to understand more clearly what this means in practice, model solutions for paper 2 are attached to this report. When looking at the markscheme for paper 2 please bear in mind that any analytical approaches given there are to inform examiners how to award marks to such attempts. It is not intended to imply that these are the preferred or expected approaches.

M10/5/MATME/SP2/ENG/TZ2/XX

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International Baccalaureate®
Baccalauréat International
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MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 6 May 2010 (morning)

1 hour 30 minutes

Candidate session number

0	0								

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

GDC solutions.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

The following table gives the examination grades for 120 students.

Grade	Number of students	Cumulative frequency
1	9	9
2	25	34
3	35	p
4	q	109
5	11	120

(a) Find the value of

(i) p;

(ii) q. *[4 marks]*

(b) Find the mean grade. *[2 marks]*

(c) Write down the standard deviation. *[1 mark]*

(a) (i) $p = 34 + 35$
$p = 69$

(ii) $69 + q = 109$
$q = 40$

(b) Mean $= \dfrac{\Sigma fx}{120} = 3.16$

(c) Standard Dev. $= 1.09$

2. *[Maximum mark: 6]*

An arithmetic sequence, $u_1, u_2, u_3 \ldots$, has $d = 11$ and $u_{27} = 263$.

(a) Find u_1. *[2 marks]*

(b) (i) Given that $u_n = 516$, find the value of n.

(ii) For this value of n, find S_n. *[4 marks]*

(a) $263 = u_1 + 26 \cdot 11$

$u_1 = -23$

(b) (i) $516 = -23 + 11(n-1)$

$n = 50$

(ii) $S_{50} = \frac{50}{2}(-23 + 516)$

$= 12\,325$

3. *[Maximum mark: 5]*

Jan plays a game where she tosses two fair six-sided dice. She wins a prize if the sum of her scores is 5.

(a) Jan tosses the two dice once. Find the probability that she wins a prize. *[3 marks]*

(b) Jan tosses the two dice 8 times. Find the probability that she wins 3 prizes. *[2 marks]*

(a) Sum 5: (1,4) (4,1) (2,3) (3,2)

$P(\text{prize}) = \frac{4}{36} = \frac{1}{9}$

(b) $X \sim Bi\left(8, \frac{1}{9}\right)$

$P(x=3) = 0.0426$

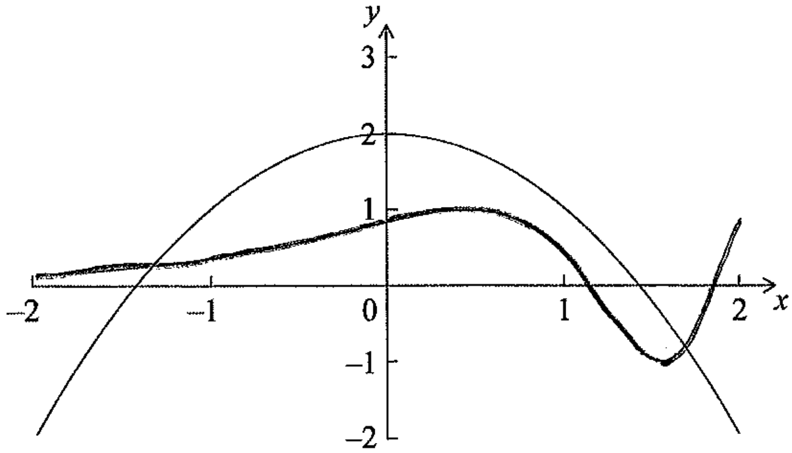
4. *[Maximum mark: 6]*

Find the term in x^4 in the expansion of $\left(3x^2 - \frac{2}{x}\right)^5$.

$\binom{5}{2}(3x^2)^3 \cdot \left(-\frac{2}{x}\right)^2 = 1080x^4$

5. *[Maximum mark: 7]*

Consider $f(x) = 2 - x^2$, for $-2 \le x \le 2$ and $g(x) = \sin e^x$, for $-2 \le x \le 2$. The graph of f is given below.



(a) On the diagram above, sketch the graph of g. *[3 marks]*

(b) Solve $f(x) = g(x)$. *[2 marks]*

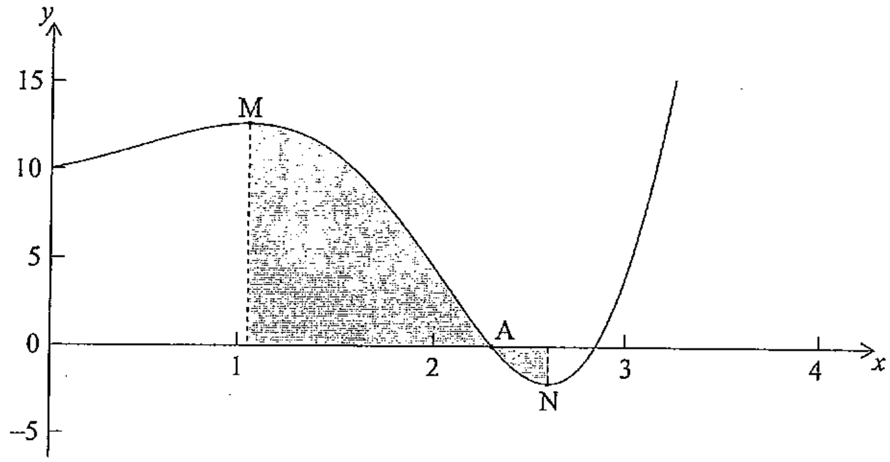
(c) Write down the set of values of x such that $f(x) > g(x)$. *[2 marks]*

(b) $x = -1.32$, $x = 1.68$

(c) $-1.32 < x < 1.68$

6. *[Maximum mark: 6]*

Let $f(x) = e^x \sin 2x + 10$, for $0 \le x \le 4$. Part of the graph of f is given below.



There is an x-intercept at the point A, a local maximum point at M, where $x = p$ and a local minimum point at N, where $x = q$.

(a) Write down the x-coordinate of A. *[1 mark]*

(b) Find the value of

(i) p;

(ii) q. *[2 marks]*

(c) Find $\int_p^q f(x)\,dx$. Explain why this is not the area of the shaded region. *[3 marks]*

(a) 2.31

(b) (i) 1.02

(ii) 2.59

(c) $\int_p^q f(x)\,dx = 9.96$

It is not the area of the shaded region because part of the graph is below the x-axis.

7. *[Maximum mark: 8]*

The number of bacteria, n, in a dish, after t minutes is given by $n = 800e^{0.13t}$.

(a) Find the value of n when $t = 0$. *[2 marks]*

(b) Find the rate at which n is increasing when $t = 15$. *[2 marks]*

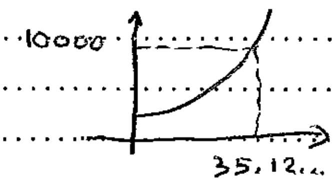
(c) After k minutes, the rate of increase in n is greater than 10 000 bacteria per minute. Find the least value of k, where $k \in \mathbb{Z}$. *[4 marks]*

(a) $n = 800 \cdot e^{0.13 \cdot 0}$

$= 800$

(b) $n'(15) = 731$

(c) $n'(t) > 10000$



$k > 35.12...$

The least value of k is 36.

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The diagram below shows a circle with centre O and radius 8 cm.

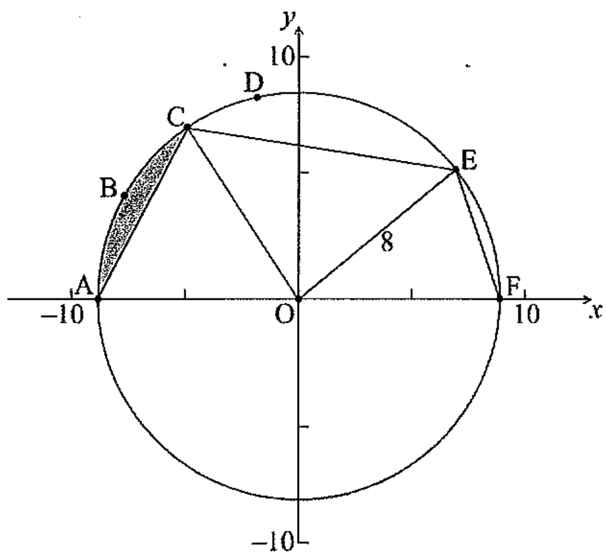

diagram not to scale

The points A, B, C, D, E and F are on the circle, and [AF] is a diameter. The length of arc ABC is 6 cm.

(a) Find the size of angle AOC. *[2 marks]*

(b) Hence find the area of the shaded region. *[6 marks]*

The area of sector OCDE is 45 cm^2.

(c) Find the size of angle COE. *[2 marks]*

(d) Find EF. *[5 marks]*

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ANSWER SHEET
FEUILLE DE RÉPONSES
HOJA DE RESPUESTAS

Please complete the boxes/Veuillez remplir les cases/Llene los recuadros

Question / Question / Pregunta

Examiner / Examinateur / Examinador

(8) (a) $6 = 8\theta$

$\hat{AOC} = 0.75$

(b) Area of sector $= \frac{1}{2} \cdot 0.75 \cdot 8^2 = 24 \text{ cm}^2$

Area of triangle $= \frac{1}{2} \cdot 8 \cdot 8 \cdot \sin 0.75$

$= 21.8\ldots \text{ cm}^2$

Shaded region $= 2.19 \text{ cm}^2$

(c) $45 = \frac{1}{2} \cdot 8^2 \cdot \theta$

$\hat{COE} = 1.40625$

(d) $\hat{EOF} = \pi - 1.40625 - 0.75 = 0.985\ldots$

$EF = \sqrt{8^2 + 8^2 - 2 \cdot 8 \cdot 8 \cdot \cos \hat{EOF}}$

$= 7.57 \text{ cm}$

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9. *[Maximum mark: 16]*

In this question, distance is in metres.

Toy airplanes fly in a straight line at a constant speed. Airplane 1 passes through a point A.

Its position, p seconds after it has passed through A, is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + p \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$.

(a) (i) Write down the coordinates of A.

(ii) Find the speed of the airplane in ms^{-1}. *[4 marks]*

(b) After seven seconds the airplane passes through a point B.

(i) Find the coordinates of B.

(ii) Find the distance the airplane has travelled during the seven seconds. *[5 marks]*

(c) Airplane 2 passes through a point C. Its position q seconds after it passes through C is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 8 \end{pmatrix} + q \begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}, a \in \mathbb{R}$.

The angle between the flight paths of Airplane 1 and Airplane 2 is $40°$. Find the two values of a. *[7 marks]*

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ANSWER SHEET
FEUILLE DE RÉPONSES
HOJA DE RESPUESTAS

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(9) (a) (i) $(3, -4, 0)$

(ii) velocity vector $\begin{pmatrix}-2\\3\\1\end{pmatrix}$

So, speed $= \sqrt{(-2)^2+3^2+1^2}$

$= 3.74$

(b) (i) $B = (-3, -4, 0) + 7\cdot(-2, 3, 1)$

$B = (-11, 17, 7)$

(ii) $\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix}-14\\21\\7\end{pmatrix}$

$AB^2 = (-14)^2 + 21^2 + 7^2$

distance $= 26.2$

(c) Angle between vectors $\begin{pmatrix}-2\\3\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\2\\a\end{pmatrix}$

$a.b = ab\cos\theta$

$\left|\begin{matrix}-2\\3\\1\end{matrix}\right| = \sqrt{14}$, $\left|\begin{matrix}-1\\2\\a\end{matrix}\right| = \sqrt{a^2+5}$, $\begin{pmatrix}-2\\3\\1\end{pmatrix}\circ\begin{pmatrix}-1\\2\\a\end{pmatrix} = a+8$

$\cos 40° = \dfrac{a+8}{\sqrt{14}\cdot\sqrt{a^2+5}}$

$a = 3.21$, $a = -0.990$

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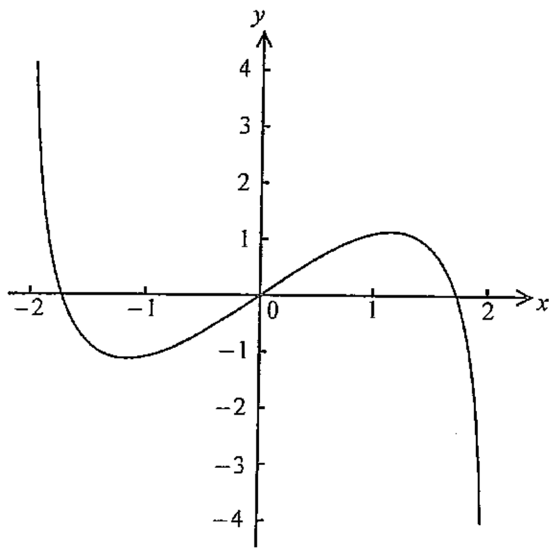
10. *[Maximum mark: 14]*

Consider $f(x) = x\ln(4-x^2)$, for $-2 < x < 2$. The graph of f is given below.



(a) Let P and Q be points on the curve of f where the tangent to the graph of f is parallel to the x-axis.

(i) Find the x-coordinate of P and of Q.

(ii) Consider $f(x) = k$. Write down all values of k for which there are exactly two solutions. *[5 marks]*

Let $g(x) = x^3\ln(4-x^2)$, for $-2 < x < 2$.

(b) Show that $g'(x) = \dfrac{-2x^4}{4-x^2} + 3x^2\ln(4-x^2)$. *[4 marks]*

(c) Sketch the graph of g'. *[2 marks]*

(d) Consider $g'(x) = w$. Write down all values of w for which there are exactly two solutions. *[3 marks]*

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ANSWER SHEET
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HOJA DE RESPUESTAS

Please complete the boxes/Veuillez remplir les cases/Llene los recuadros

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(10) (a) (i) $-1.15, \quad 1.15$

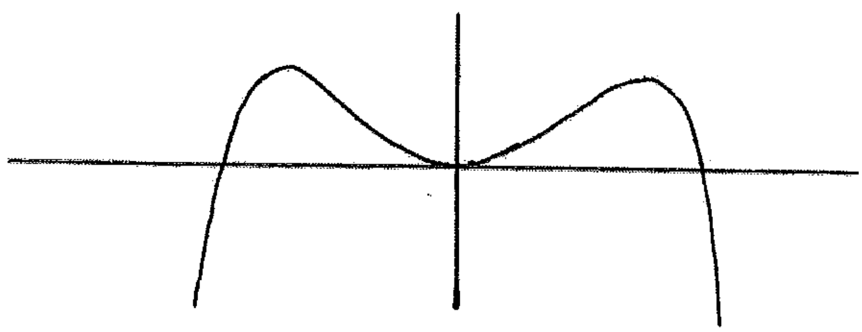
(ii) $k = -1.13, \quad k = 1.13$

(b) $g'(x) = (x^3)' \cdot \ln(4-x^2) + x^3 [\ln(4-x^2)]'$

$= 3x^2 \cdot \ln(4-x^2) + x^3 \cdot \frac{1}{4-x^2} \cdot (-2x)$

$= 3x^2 \ln(4-x^2) - \frac{2x^4}{4-x^2}$

(c)



(d) $\omega = 2.69, \quad \omega < 0$

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