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MATHEMATICS SL TZ1

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 15	16 – 31	32 – 46	47 – 57	58 – 69	70 – 81	82 – 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2010 examination session the IB has produced time zone variants of the Mathematics SL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 7	8 – 13	14 – 19	20 – 23	24 – 28	29 – 33	34 – 40

The range and suitability of the work submitted

The majority of work submitted came from the current set of tasks developed by the IB. Popular choices included "Matrix Binomials" and "Body Mass Index". Where teacher-designed tasks were used these varied significantly in quality. If the tasks did not allow for students to address all of the assessment levels the result was usually a significant downwards moderation of the marks. Some schools submitted atypical work without adding a substitute portfolio for these candidates. The quality of student work was generally good, with some outstanding examples.

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Candidate performance against each criterion

Overall there is much evidence that teachers and students understand the criteria well and make every effort to prepare quality work. Some areas of concern are noted below.

Criterion A: There continues to be a problem with the use of calculator notation and the lack of use of an appropriate "approximately equals" sign. In the modelling task, candidates often use the same dependent variable for different model functions.

Criterion B: The use of a "question and answer" style is a problem, with some teachers and students treating the tasks as a set of homework exercises. The work should be presented as a cohesive piece of mathematical writing with graphs and tables offered within the context, instead of as appendices. The proper labelling of graphs is an issue, especially where candidates have used graphing technology but don't know how to apply labels to axes.

Criterion C Type I: Candidates often do not present sufficient evidence or analysis to support their general statements. For example, in the "Matrix Binomials" task many announced that $(\boldsymbol{A}+\boldsymbol{B})\wedge n=\boldsymbol{A}\wedge n+\boldsymbol{B}\wedge n$ without any coherent support. Candidates would often "validate" their proposed general statement by using the same values they used to develop it. The process of validation involves using further values and comparing the results against the mathematical behaviour in the context of the task.

Criterion C Type II: Many candidates do not explicitly define variables, parameters, and constraints. As with the Type I tasks candidates are not providing sufficient and appropriate analysis to develop their model functions. In some cases teachers are still condoning the use of calculator or computer regression models without any supporting mathematics. There is some confusion as to the use of graphical transformations to develop a model. If students use their knowledge of these transformations appropriately and demonstrate a sequence of attempts to fit a model function using suitable modifications to an original basic function then they can access all of the marks available in criterion C. The comments on how well the models fit the data are generally superficial. These should include some specifics such as how the function fits the data in certain intervals, at the extremes, etc., and not simply something like "fits well". While a quantitative analysis is not expected for maths SL, the candidate should say more than "fits well". Intervals of good and poor fit should be identified and discussed.

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Criterion D Type I: The major issues here are the appropriate exploration of scope and limitations, as well as the quality of explanation offered. Given the availability of the graphic display calculator (GDC), it is expected that candidates will explore a wide variety of values for their general statements. Many focus only on positive integers with no thought to other possibilities. Most candidates found it very difficult to provide an explanation for the general statement.

Criterion D Type II: While most candidates proved themselves mathematically capable of matching a function to the data, many found it difficult to discuss the model in context, or simply ignored this aspect. The connection between reality and the mathematical attributes of variables and graphs seemed lost on the candidates. There was little application of critical thinking skills to the situations.

Criterion E: The use of graphing technologies is clearly growing. Many students presented high-quality graphs with some in colour to differentiate different models. While this is a positive development there were also cases where the candidates used the technology without thought. Graphs by themselves do not enhance the development of the tasks.

Criterion F: This criterion was well understood by most teachers. A reasonable effort to complete the task was awarded F1 in the majority of cases. Teachers seemed to appreciate that the award of F0 or F2 is justifiably rare.

Recommendation and guidance for the teaching of future candidates

Candidates should be reminded that they can check their work against the criteria to ensure that they are addressing all the important components of the assessment. However, it is necessary that the teachers take the time to help candidates understand the criteria. Given the clear expectations of levels C1 and C2 in a Type II task, there is no excuse for candidates not properly and explicitly identifying the variables, parameters and constraints, although teachers may need to explain the difference between these. Teachers might have candidates complete practice work and assess it themselves against the criteria.

Candidates should be taught to treat the work as an essay in mathematics, requiring a cohesive and complete written presentation that flows smoothly.

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Candidates would benefit from discussions about the purposes of the different tasks. The processes of mathematical investigation and mathematical modelling may be foreign to their experience. The necessity of proper evidence and analysis, as well as the appreciation for critical consideration of the implications of the work are important skills that teachers can explain. Again, practice work would be helpful here. The use of technology must go beyond simply producing graphs. Candidates should better understand the power of the technology available to them as a tool to explain and explore.

Teachers are reminded to offer written comments on the student work that help explain why certain marks were awarded. It is also expected that teachers will provide solutions to the tasks that describe their own expectations as to how the levels of the criteria can be attained. Tasks designed by teachers must address all the levels of the criteria. They should also focus on one problem rather than branch out into multi-part questions as these confuse the marking.

All teachers would benefit from a careful reading of current and past subject reports, as well as participation in discussion forums on the Online Curriculum Centre.

External Assessment

Paper 1

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 15	16 – 31	32 – 45	46 – 55	56 – 66	67 – 76	77 – 90

The areas of the programme which proved difficult for candidates

- Right-angled trigonometry
- Conditional probability
- Volume of revolution
- Composition of functions involving logarithms
- Transformation of a graph under a horizontal stretch
- Vector geometry

The levels of knowledge, understanding and skill demonstrated

The levels of knowledge and understanding varied widely. Basic algebraic skills (*e.g.* factorization, solving systems, substituting values) proved to be a strength of this group. Somewhat more sophisticated algebraic techniques (*e.g.* finding the term in x^2, composite involving logarithmic function) proved challenging. Students were adept at simple tasks (*e.g.* finding derivative, applying quotient rule, matrix algebra, scalar product) while less so with higher-order skills (*e.g.* chain rule, conditions for point of inflexion).

Candidates were able to start most of the questions and gain some marks. Still, there was evidence that some areas of the syllabus need greater emphasis, such as vector equations of lines and conditional probability.

Students were generally well prepared for writing the paper without a calculator. Working, whether done correctly or incorrectly, was generally shown, although there were some cases where arithmetic calculations proved troublesome. This was particularly evident when calculating with fractions.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

This question was answered well by most candidates. Some did not give an equation for their axis of symmetry.

Question 2

This question was also answered very well by most candidates. Many earned follow-through marks when an error was made in part (a).

Question 3

Surprisingly few candidates employed the binomial theorem, choosing instead to expand by repeated use of the distributive property. This earned full marks if done correctly, but often proved prone to error. Candidates often expanded the entire expression in part (b). Few

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recognized that only two distributions are required to answer the question. Some gave the coefficient as the final answer.

Question 4

Many candidates drew a diagram to correctly find $\tan\theta$, although few recognized that a line through the origin can be expressed as $y = x\tan\theta$, with gradient $\tan\theta$, which is explicit in the syllabus. Many went on to complete the question correctly, however a surprising number were unable to find the ratios for $\sin\theta$ and $\cos\theta$ from $\tan\theta$. It was not uncommon for candidates to use unreasonable values, such as $\sin\theta = 3$ and $\cos\theta = 4$, or to write nonsense such as $2\sin\frac{3}{5}\cos\frac{4}{5}$.

Question 5

As the definitions of *p* and *q* were not clear to candidates, both responses of $p = 0.2, q = 0.4$ and $p = 0.5, q = 0.7$ were accepted for full marks. However, finding *r* eluded many. Few candidates answered the conditional probability correctly. Many attempted to use the formula in the booklet without considering the complement, and there was little evidence of the Venn diagram being utilized as a helpful aid. To show the events are not independent, many correctly reasoned that $0.3 \neq 0.35$. A handful recognized that $\mathrm{P}(A \mid B') \neq \mathrm{P}(A)$ is an alternative approach that uses the answer in part (b). Some candidates do not know the difference between *independent* and *mutually exclusive*.

Question 6

Many candidates correctly integrated using $f(x)$, although some neglected to square the function and mired themselves in awkward integration attempts. Upon substituting the limits, many were unable to carry the calculation to completion. Occasionally the π was neglected in a final answer. Weaker candidates considered the solid formed to be a sphere and did not use integration.

Question 7

Candidates were generally skilled at finding the inverse of a logarithmic function. Few correctly gave the range of this function, often stating "all real numbers" or "$y \geq 0$", missing

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the idea that the range of an inverse is the domain of the original function. Some candidates answered part (c) correctly, although many did not get beyond $3^{2\log_3 2}$. Some attempted to form the composite in the incorrect order. Others interpreted $\left(f^{-1}\circ g\right)(2)$ as multiplication by 2.

Question 8

A majority of candidates answered part (a) completely, and were generally successful in finding images after single transformations in part (b). Common incorrect answers for (biii) included $\left(\frac{3}{2},\frac{9}{2}\right)$, $(6,9)$ and $(6,18)$, demonstrating difficulty with images from horizontal stretches.

Question 9

Many candidates comfortably applied the quotient rule, although some did not completely show that the Pythagorean identity achieves the numerator of the answer given. Whether changing to $-(\sin x)^{-2}$, or applying the quotient rule a second time, most candidates neglected the chain rule in finding the second derivative. Those who knew the trigonometric ratios at $\frac{\pi}{2}$ typically found the values of *p* and of *q*, sometimes in follow-through from an incorrect $f''(x)$. Few candidates gave two reasons from the table that supported the existence of a point of inflexion. Most stated that the second derivative is zero and neglected to consider the sign change to the left and right of *q*. Some discussed a change of concavity, but without supporting this statement by referencing the change of sign in $f''(x)$, so no marks were earned.

Question 10

Many candidates gave a correct vector equation for the line. A common error was to misplace the initial position and direction vectors. Those who set the scalar product of the direction vectors to zero typically solved for *k* successfully. Those who substituted $k=-2$ earned fewer marks for working backwards in a *show that* question. Many went on to find the coordinates of point A, however some used the same letter, say *p*, for each parameter and thus could not solve the system. Part (d) proved challenging as many candidates did not consider that

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$\vec{AB} + \vec{BC} = \vec{AC}$. Rather, many attempted to find the coordinates of point C, which became a more arduous and error-prone task.

The type of assistance and guidance the teachers should provide for future candidates

The ability to sustain a longer Section B question is a skill for which students need practice and guidance. For example, the final question on vectors proved challenging for candidates, not necessarily for its content, but for the need to keep a sense of coherence while working through it. Drawing a simple diagram of lines may have been a helpful aid in thinking about the geometry, but few candidates employed such a technique.

Although candidates had clearly been exposed to conditional probability, it seemed most reached for the formula in the information booklet before thinking about the question in relation to the diagram. It may be helpful to emphasize conceptual understanding prior to introducing the conditional probability formula, so that the formula supports the mathematical understanding and does not drive it.

As a calculator is not permitted on this paper, opportunity to use basic arithmetic skills needs to be a regular and natural part of classroom activity and instruction.

Students not exposed to the entire syllabus experience a serious disadvantage in the paper. Candidate performance in questions 6 and 10 suggests that the topics of vectors and volumes of revolution are not being fully treated.

Paper 2

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 12	13 – 25	26 – 37	38 – 48	49 – 59	60 – 70	71 – 90

The areas of the programme which proved difficult for candidates

It was pleasing to see a large number of candidates demonstrate a comprehensive knowledge

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and understanding of the syllabus. The following areas proved difficult for candidates:

- Graphical solutions of equations
- Trigonometric functions and their transformations
- Normal distribution
- Finding the mean from a frequency table
- Conditional probability
- Integration with a boundary condition
- Using a calculator to find the standard deviation
- Using a calculator to find key features of a function

The levels of knowledge, understanding and skill demonstrated

The levels of knowledge and understanding varied widely. A large number of students seemed to be well prepared while others were not. In many instances, students did a fine job of showing their work. In fact, the overall quality of the scripts, in terms of showing relevant work, was impressive, and an improvement on past sessions.

Candidates were comfortable with basic skills such as simple differentiation and integration processes as well as the use of and substitution into formulas. Many candidates did very well on matrices, arithmetic progressions and calculus but continue to have difficulties knowing when and how to use their GDCs appropriately.

Candidates continue to fall short when they refuse to spot the implied uses of a GDC. For example, when candidate work leads to an equation, many are still opting for an analytical approach rather than opting for their GDCs first. More explicit expectations of GDC use, such as finding the inverse of a matrix, are generally handled well although it is rather distressing that finding a standard deviation with the GDC, or graphing a function within a specified domain (both syllabus expectations) escapes a great majority of candidates.

Those candidates demonstrating skill in using both analytical and geometrical techniques, had little difficulty with this paper.

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The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

This question was generally well done. In part (a), some candidates were not careful when copying the inverse from their calculator or did not use the correct number of significant figures when giving decimal values. In part (b), there were a number of candidates who reversed the matrices when solving the matrix equation $\boldsymbol{AX} = \boldsymbol{B}$. Most recovered and found the correct answer but were not awarded full marks.

Question 2

Most candidates did well on this question. Any errors were usually arithmetic in nature but candidates were able to obtain follow through marks on errors made in earlier parts.

Question 3

This problem was well done by most candidates. There were some candidates that struggled to apply the product rule in part (a) and often wrote nonsense like $-x\sin x = -\sin x^2$. In part (b), few candidates were able to sketch the function within the required domain and a large number of candidates did not have their calculator in the correct mode.

Question 4

Surprisingly, this question was not well done by many candidates. A good number of candidates understood the importance of the frequencies in calculating mean. Some neglected to sum the frequencies for the denominator, which often led to a negative value for a frequency. Unfortunately, candidates did not appreciate the unreasonableness of this result. In part (b), many candidates could not find the standard deviation in their GDC, often trying to calculate it by hand with no success. Further, many could not distinguish between the sample and the population standard deviation given in the GDC.

Question 5

Many candidates did not recognize that the value of p was negative. The value of q was often interpreted incorrectly as the period but most candidates could find the value of r, the vertical translation. In part (b), candidates either could not find a solution or found too many.

Question 6

This problem was not well done. A large number of students failed to recognize that they needed to integrate the acceleration function. Even among those who integrated the function, there were many who integrated incorrectly. A great number of candidates were not able to handle the given initial condition to find the integration constant but incorrectly substituted $t = 5$ directly into their expression.

Question 7

Parts of this question were handled very well by a great many candidates. Most were able to recognize the binomial condition and had little difficulty with part (a). However, more than a few reported the answer as 0.23, thus incurring the accuracy penalty. Those candidates that were successful in part (a) could easily write the required expression for part (b).

In part (c), many candidates set up the question correctly or set their expression from (b) equal to 0.15, however few candidates considered the GDC as a method to solve the equation. Rather, those who attempted usually tried to expand the polynomial, and *still* did not use the GDC to solve *this* equation. A graphical approach to the solution would reveal that there are two solutions for p, but few caught this subtlety.

Question 8

Many candidates worked comfortably with the sine and cosine rules in part (a) and (b). Equally as many did not take the cue from the word "hence" and used an alternate method to solve the problem and thus did not receive full marks. Those who managed to set up an equation, again did not go directly to their GDC but rather engaged in a long, laborious analytical approach that was usually unsuccessful. No matter what values were found in (c) (i) most candidates recovered and earned follow through marks for the remainder of the question.

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A large number of candidates worked in the wrong mode and rounded prematurely throughout this question often resulting in accuracy penalties.

Question 9

This question was quite well done by a great number of candidates indicating that calculus is a topic that is covered well by most centres. Parts (a) and (b) proved very accessible to many candidates. The chain rule in part (c) was also carried out well. Few however, recognized the command term "hence" and that $f'(x) < 0$ guarantees a decreasing function. A common answer for the equation of the asymptote was to give $y = 0$ or $x = 3$. In part (d), it was again surprising and somewhat disappointing to see how few candidates were able to use their GDC effectively to find the area between curves, often not finding correct limits, and often trying to evaluate the definite integral without the GDC, which led nowhere.

Question 10

This question was quite accessible to those candidates in centres where this topic is given the attention that it deserves. Most candidates handled part (a) well using the basic properties of a normal distribution. In part (b) (i), candidates often confused the *z*-score with the area in the table which led to a standard deviation that was less than zero in part (b) (ii). At this point, candidates "fudged" results in order to continue with the remaining parts of the question. In (b) (ii), the "hence" command was used expecting candidates to use the results of (b) (i) to find a standard deviation of 4.86. Unfortunately, many decided to use their answers and the information from part (a) resulting in quite a different standard deviation of 5.79. Recognizing the inconsistency in the question, full marks were awarded for this approach, as well as full follow-through in subsequent parts of the question.

Candidates could obtain full marks easily in part (c) with little understanding of a normal distribution but they often confused *z*-scores with data values, adding and subtracting 1.5 from the mean of 76.

In part (d), few recognized the conditional nature of the question and only determined the probability that a woman qualifies AND takes part in the tournament.

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The type of assistance and guidance the teachers should provide for future candidates

Teachers must instruct candidates on how to set up their work on Paper 2 and how to use their GDCs effectively and efficiently to find solutions. Far too many candidates are losing valuable time engaging in long, fruitless analytical approaches. Analytical approaches are largely assessed on paper 1. Teachers would do well in preparing students to choose the GDC as the primary approach to solving equations, calculating intersections, and finding areas under and between curves. The more familiar students are with these practices, the better prepared they are to tackle the time constraints of a paper designed with the GDC in mind. Candidates should also ensure that the GDC is in the correct mode e.g. radians for calculus problems.

Candidates should be instructed to consider both analytical and geometric approaches to solving problems to facilitate understanding. When preparing candidates for future examinations, emphasizing a graphical understanding in conjunction with analytical techniques may be helpful.

Teachers should continue to stress the meaning of the command terms, particularly the "hence" command and have students look at the number of marks allocated to each question part to determine how much "work" they should show. In addition, teachers should continue to work with students on "show that" type questions. Although each problem is a little different, students need to show enough steps, show the general and not the specific case, and not work "backwards." Although the command to "sketch" does not require decimal point precision, candidates need to be clearly aware that their sketches must be attentive to the domain and locations of important points and features. It may be helpful to teach students to locate and plot any relevant points before attempting to draw the curve.

Teachers should continue to work with students on writing their answers to the correct number of significant figures. Many candidates incur the accuracy penalty because they work with fewer than three significant figures.

Teachers should not only spend more time on the normal distribution but also stress the need for correct notation as well as the usefulness of drawing a diagram.

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Candidates should be encouraged to use plenty of space to communicate their work and draw figures where appropriate. Work must be clearly numbered. Many candidates attempt to complete problems in a minimal amount of space, which makes it difficult for them to see the flow of their work and to check results later.

It appears that many students are still not clear what "working" to write in the examination when using the GDC, so candidates often spent precious time writing analytic methods to problems most efficiently solved using the GDC. To "show working" does not mean to perform algebraic steps or manipulations. Rather, what is important is to show the mathematical thinking, the setup, before reaching for the GDC, and then to let the GDC do the work of calculation. Whatever supports the solution, making the problem "calculator-ready," is what students need to show as working.

To help teachers and students to understand more clearly what this means in practice, model solutions for paper 2 are attached to this report. When looking at the markscheme for paper 2 please bear in mind that any analytical approaches given there are to inform examiners how to award marks to such attempts. It is not intended to imply that these are the preferred or expected approaches.

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 6 May 2010 (morning)

1 hour 30 minutes

Candidate session number

0	0								

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

Let $A = \begin{pmatrix} 1 & 2 & -3 \\ -1 & -1 & 4 \\ 2 & 4 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$.

(a) Write down A^{-1}. *[2 marks]*

(b) Solve $AX = B$. *[3 marks]*

(a) $A^{-1} = \begin{pmatrix} -4.33 & -2 & 1.67 \\ 1.67 & 1 & -0.333 \\ -0.667 & 0 & 0.333 \end{pmatrix}$

(b) $X = A^{-1}B$

$= \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$

2. *[Maximum mark: 6]*

Consider the arithmetic sequence 3, 9, 15, …, 1353.

(a) Write down the common difference. *[1 mark]*

(b) Find the number of terms in the sequence. *[3 marks]*

(c) Find the sum of the sequence. *[2 marks]*

(a) $d = 6$

(b) $1353 = 3 + (n-1)6$

$n = 226$

(c) $S_{226} = \dfrac{226(3+1353)}{2}$

$= 153228$

3. *[Maximum mark: 7]*

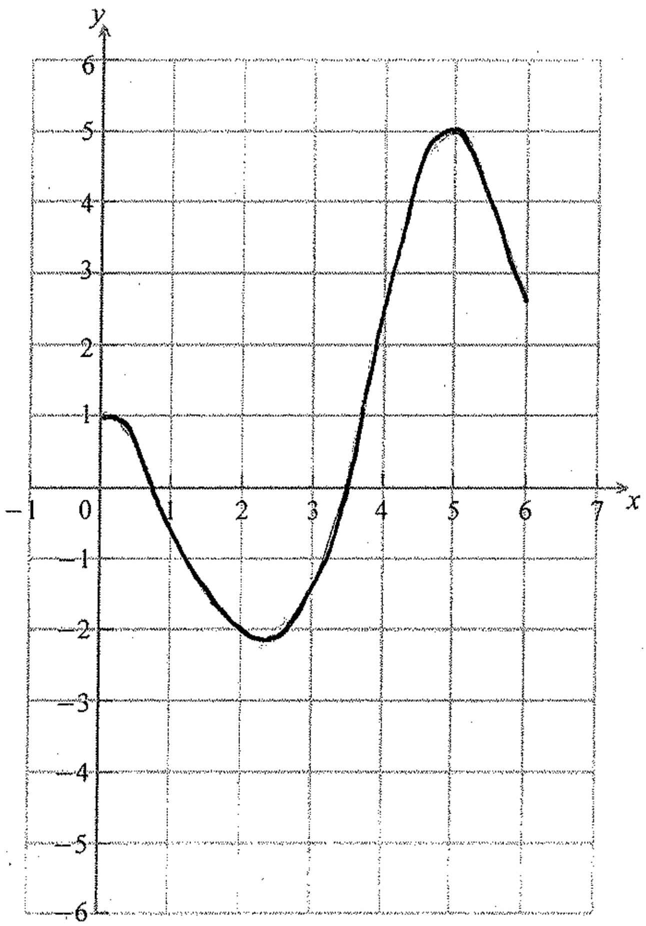
Let $f(x) = x\cos x$, for $0 \le x \le 6$.

(a) Find $f'(x)$. *[3 marks]*

(a) $f'(x) = x \times (-\sin x) + 1 \times \cos x$

$= \cos x - x\sin x$

(b) On the grid below, sketch the graph of $y = f'(x)$. *[4 marks]*



4. *[Maximum mark: 6]*

The following frequency distribution of marks has mean 4.5.

Mark	1	2	3	4	5	6	7
Frequency	2	4	6	9	x	9	4

(a) Find the value of x. *[4 marks]*

(b) Write down the standard deviation. *[2 marks]*

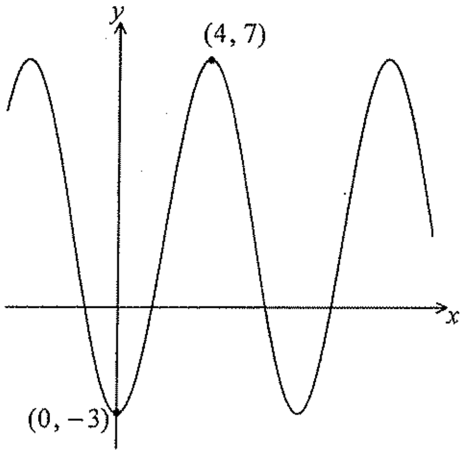
(a) $\frac{146+5x}{34+x} = 4.5$

$x = 14$

(b) $\sigma = 1.54$

5. *[Maximum mark: 7]*

The graph of $y = p\cos qx + r$, for $-5 \le x \le 14$, is shown below.



There is a minimum point at $(0, -3)$ and a maximum point at $(4, 7)$.

(a) Find the value of

(i) p;

(ii) q;

(iii) r. *[6 marks]*

(b) The equation $y = k$ has exactly **two** solutions. Write down the value of k. *[1 mark]*

(a) (i) $\frac{7+3}{2} = 5$

$p = -5$

(ii) period $= 8$

$q = 0.785$

(iii) $r = \frac{7-3}{2}$

$= 2$

(b) $k = -3$

6. *[Maximum mark: 7]*

The acceleration, $a\,\text{m}\,\text{s}^{-2}$, of a particle at time t seconds is given by

$$a = \frac{1}{t} + 3\sin 2t, \text{ for } t \geq 1.$$

The particle is at rest when $t = 1$.

Find the velocity of the particle when $t = 5$.

a) $v(t) = \int a(t)\,dt$

$= \ln t - \frac{3}{2}\cos 2t + C$

$0 = \ln 1 - \frac{3}{2}\cos 2 + C$

$C = -0.624$

$v(5) = 2.24$

7. *[Maximum mark: 7]*

Evan likes to play two games of chance, A and B.

For game A, the probability that Evan wins is 0.9. He plays game A seven times.

(a) Find the probability that he wins exactly four games. *[2 marks]*

For game B, the probability that Evan wins is p. He plays game B seven times.

(b) Write down an expression, in terms of p, for the probability that he wins exactly four games. *[2 marks]*

(c) Hence, find the values of p such that the probability that he wins exactly four games is 0.15. *[3 marks]*

(a) $X \sim B(7, 0.9)$

$P(X=4) = 0.0230$

(b) $\binom{7}{4} p^4 (1-p)^3$

(c) $\binom{7}{4} p^4 (1-p)^3 = 0.15$

$p = 0.356, \ 0.770$

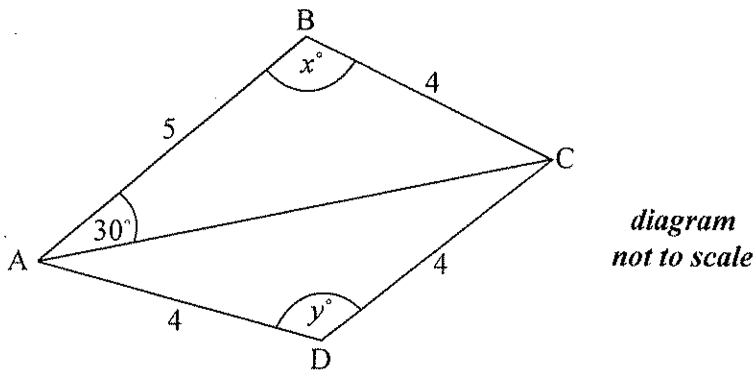
*Do **NOT** write on this page.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

The diagram below shows a quadrilateral ABCD with obtuse angles $A\hat{B}C$ and $A\hat{D}C$.



$AB = 5$ cm, $BC = 4$ cm, $CD = 4$ cm, $AD = 4$ cm, $B\hat{A}C = 30°$, $A\hat{B}C = x°$, $A\hat{D}C = y°$.

(a) Use the cosine rule to show that $AC = \sqrt{41 - 40\cos x}$. *[1 mark]*

(b) Use the sine rule in triangle ABC to find another expression for AC. *[2 marks]*

(c) (i) Hence, find x, giving your answer to two decimal places.

(ii) Find AC. *[6 marks]*

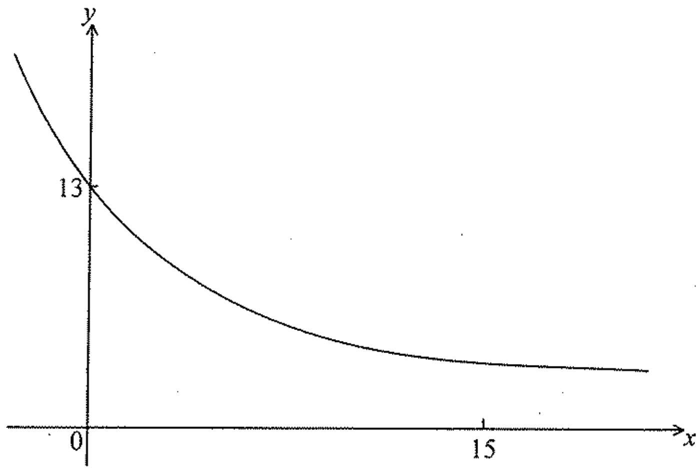
(d) (i) Find y.

(ii) Hence, or otherwise, find the area of triangle ACD. *[5 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 16]*

Let $f(x) = Ae^{kx} + 3$. Part of the graph of f is shown below.



The y-intercept is at $(0, 13)$.

(a) Show that $A = 10$. *[2 marks]*

(b) Given that $f(15) = 3.49$ (correct to 3 significant figures), find the value of k. *[3 marks]*

(c) (i) Using your value of k, find $f'(x)$.

(ii) Hence, explain why f is a decreasing function.

(iii) Write down the equation of the horizontal asymptote of the graph f. *[5 marks]*

Let $g(x) = -x^2 + 12x - 24$.

(d) Find the area enclosed by the graphs of f and g. *[6 marks]*

*Do **NOT** write on this page.*

10. *[Maximum mark: 15]*

The weights of players in a sports league are normally distributed with a mean of 76 kg. It is known that 80 % of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

(a) Find the probability that a player weighs more than 82 kg. *[2 marks]*

(b) (i) Write down the standardized value, z, for 68 kg.

(ii) Hence, find the standard deviation of weights. *[4 marks]*

To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.

(c) (i) Find the set of all possible weights of players that take part in the tournament.

(ii) A player is selected at random. Find the probability that the player takes part in the tournament. *[5 marks]*

Of the players in the league, 25 % are women. Of the women, 70 % take part in the tournament.

(d) Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman. *[4 marks]*

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ANSWER SHEET
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8(a) $(AC)^2 = 5^2 + 4^2 - 2 \times 4 \times 5 \cos x$

$AC = \sqrt{41 - 40\cos x}$

(b) $\frac{AC}{\sin x} = \frac{4}{\sin 30}$

$AC = 8 \sin x$

(c)(i) $8 \sin x = \sqrt{41 - 40 \cos x}$

$x = 111.32$ (obtuse angle)

(ii) $AC = 8 \sin 111.32$

$= 7.45$

(d)(i) $\cos y = \frac{4^2 + 4^2 - 7.45^2}{2 \times 4 \times 4}$

$y = 137$

(d)(ii) $\text{Area} = \frac{1}{2} \times 4 \times 4 \times \sin 137$

$= 5.42$

Question
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9(a) $13 = Ae^0 + 3$

$13 = A + 3$

$A = 10$

(b) $3.49 = 10e^{15k} + 3$

$k = -0.201$

(c)(i) $f'(x) = 10e^{-0.201x} \times -0.201$

$= -2.01e^{-0.201x}$

(ii) $f'(x) < 0$

(iii) $y = 3$

(d) $\text{Area} = \int_{3.90}^{8.69} [g(x) - f(x)]\,dx$

$= 19.5$

AB0102

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10(a) $P(w \geq 82) = 1 - 0.85$

$= 0.15$

(b)(i) $Z = -1.64$

(ii) $-1.64 = \dfrac{68 - 76.6}{\sigma}$

$\sigma = 5.23$

(c)(i) $68.8 \leq \text{weight} \leq 84.4$

(ii) $P(-1.5 \leq z \leq 1.5) = 0.866$

(d) $P(\text{woman} \mid \text{qualify}) = \dfrac{0.25 \times 0.7}{0.866}$

$= 0.202$

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