MATHEMATICS SL TZ2

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 15	16 - 29	30 - 43	44 - 54	55 - 66	67 - 78	79 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 7	8 - 13	14 - 19	20 - 23	24 - 28	29 - 33	34 - 40

The range and suitability of the work submitted

Teachers should take note that new tasks for use in exam sessions from May 2009 to November 2010 are now available on the Online Curriculum Centre. Further, older tasks taken from Teacher Support Material documents (TSM) will **not** be accepted for submission as a part of the portfolio as of May 2009. The new tasks can be found at http://occ.ibo.org/ibis/documents/dp/gr5/mathematics_sl/d_5_matsl_tsm_0801_1_e.pdf

For this session moderators have noted that most schools chose to offer tasks selected from the TSM. Overall, students have achieved well, with approximately two-thirds attaining grades of 5 or above. These results hopefully reflect a greater confidence on the part of teachers in the application of the portfolio and its assessment. While some problems persist, there was evidence of greater understanding of the assessment criteria levels.

Tasks taken from other resources, or tasks that were teacher-designed, must be carefully reviewed to ensure that they adequately meet the requirements of the tasks as described in the subject guide, and that they offer students full opportunity to succeed at every level of each criteria. To not do so risks severe penalties that can have a serious impact on the success of all students in a school. It is critical that teachers work through any task they intend to set prior to assigning it to students, to ensure that the task provides sufficient opportunity for their students to address each of the criteria levels.

Candidate performance against each criterion

Criterion A (Notation and terminology):

Most candidates offered correct and appropriate notation in their work, yet the use of calculator notation (e.g. *, ^ , 10E4, etc.) is still an issue. Some candidates used 'y' repeatedly as the dependent variable for multiple model functions representing different quantities (e.g. in Stopping Distances). This can lead to an absurd relation for total stopping distance, '$y+y=y$', or such. Each model function should be identified distinctly, with subscripts or otherwise.

Criterion B (Communication):

Improvement was noted in the presentation of work and the labelling of graphs. Teachers are reminded that candidates must properly label all graphs, even if they must do so by hand if their computer software cannot accommodate this. While it is not required that work be word-processed, the use of word-processing is appreciated. In this case, students should be taught how to use the equation editor features of the given software, just as they should be trained in the resourceful use of graphing software.

Tasks are mistakenly viewed by some as homework exercises, and responses are offered in a "question & answer" format. The portfolio is intended to develop the skills of communicating mathematics in a smooth flow of mathematical writing. "Q&A" format is therefore inappropriate, and should be penalized. In general, tasks should be prescriptive enough to guide students, but not so prescriptive as to constitute a set of closed-ended exercises. Room to explore, modify, consider accuracy and reasonableness, and interpret should be provided in the task itself.

Criterion C (Process - Type I):

The performance here was generally good with many candidates scoring highly. It is important that sufficient evidence and analysis be evident. Candidates who arrive at a generalized statement without adequate supporting evidence cannot attain high marks in this criterion.

Many found it difficult to attain C5 as they did not understand how to validate their generalized statement. What is intended is that the students will consider the mathematical process and compare the results of test values against the results obtained through their general statement. Simple substitution of values of n into the statement to get a result does not constitute validation.

Criterion C (Process - Type II):

One of the most important aspects of modelling is to properly identify appropriate variables (those values that change due to the nature of the situation and/or the relationship between the quantities or measures). This has been emphasized in subject reports and in the supporting documents for Internal Assessment for many years. However, a great number of portfolios still do not address this issue adequately. While moderators will accept many implicit indications of variable declaration there is no substitute for a clear statement such as “Let t represent time in hours and A represent the amount in kg”. It is far better too that students use variables that make sense in the context of the problem. Using t for time, or A for an amount helps frame the model function and focus any discussion in the context of these quantities.

In the same way parameters (a parameter is a value that one can change, but once changed it stays at that value until changed again by the modeller) and constraints (the real or potential limitations on the variables and parameters) must be properly and explicitly defined. For example, in a function $A(t) = at^2$ the parameter a will impact on the rate of growth of the amount A, and given that the model function represents growth as time increases, a must have a value > 0 and t must be ≥ 0.

Another focus of criterion C is that of analysis of data to develop a model function. The expectation is that the analysis will involve the mathematical skills and knowledge students have learned in the course of study. Using a calculator or computer regression feature as the primary tool for development of the model circumvents the mathematical analysis. A maximum of C2 is possible in these cases. Regression may certainly be used to confirm or compare **after** the model has been developed “by hand”.

The criterion level C4 addresses the goodness of fit of the model function to the original data. Thus tasks that do not use data cannot achieve this level. While there exist many good problems involving the development of a model function through analytic methods, these are not appropriate as portfolio tasks.

Criterion D (Results - Type I):

The expected result of an exploration of a mathematical behaviour is a generalized statement that will allow one to determine a specific outcome at any particular point in the process. Most often this involves finding an expression for directly determining the general, n^{th}, term of the process. It may also involve a description of the general effect of changing parameters in a mathematical expression/function, or the end result of a process with a given starting value/shape/expression.

The higher levels of criterion D for Type I tasks require that students have appropriately explored the scope and limitations of the statement, and that they offer an informal explanation for their results. Teachers will have their own expectations of how far a student must go to adequately address scope or limitations, and this should be communicated to the moderator. Students may require some guidance as to what constitutes an informal explanation. This could be a logical, algebraic, or geometric presentation, or some other convincing argument. Examples, by themselves, do not constitute such an argument.

Criterion D (Results - Type II):

To achieve success in this criterion, students must consider the accuracy and reasonableness of **their** model function(s) in the context of the situation. Discussion of mathematical aspects such as intercepts, asymptotes, slopes, maxima or minima, etc, must be reframed into real considerations of things such as velocity, distance, time of day, greatest amount, long-term behaviour, etc. Many students offered a good **mathematical** discussion, but lost track of the real meaning of the task, scoring a maximum of D2. The interpretation should address the essential balance between accuracy (i.e. how good can I make it?) and reasonableness (i.e. what is good enough?). Further application(s) of the model function should involve appropriate modification(s) of the original.

Criterion E (Use of technology):

Moderators expressed concern that teachers are not informing them of the circumstances of the availability of technology, and the teachers' expectations of its use. Without such background information moderators may be unable to confirm the teachers' marks.

In Type I tasks it can be difficult to find resourceful ways to use technology. It may be appropriate to use spreadsheets or "sequential function" features, or graphs may be used to support analyses of patterns of mathematical behaviour. In all tasks, computer or calculator generated graphs do not in themselves constitute full and resourceful use of technology. Teachers should consider how many graphs or how multiple graphs on the same axes could improve the presentation of the solution.

Criterion F (Quality of work):

Most teachers recognized that students who completed a good majority of the task to a reasonable degree made a satisfactory effort and rightly awarded F1. However, students who complete all the requirements of the task without demonstrating any real insight or remarkable work should also receive F1. A mark of F2 should be awarded rarely, in those cases where the teacher stops to admire that the work presented reflects a greater insight or understanding. A mark of F0 should be reserved for a totally inadequate effort.

Recommendation and guidance for the teaching of future candidates

Teachers should review the assessment criteria with their students prior to assigning each task. Rather than outline the expectations for a specific task, the teacher can address general expectations of good use of notation, good communication, the essence of good analysis and interpretation, resourceful use of technology and expected quality of work.

Teachers should add comments on the work as they mark it, to provide feedback to the student and to inform the moderator as to why a given mark was awarded. Summary comments on form 5/PFCS or Form B (to be found in the TSM) also serve to inform moderators. The better the teacher can explain why a given mark was awarded the more likely their marks will be confirmed in moderation.

Teachers are reminded that specific instructions regarding the assessment of portfolios, including annotations to the criteria to help explain their application, are available on the Online Curriculum Centre in a variety of documents. Teachers may find the following links useful.

http://occ.ibo.org/ibis/documents/dp/gr5/mathematics_sl/d_5_matsl_gui_0805_1_e.pdf

http://occ.ibo.org/ibis/documents/dp/gr5/mathematics_sl/d_5_matsl_int-ass_0611_1_e.pdf

http://occ.ibo.org/ibis/documents/dp/gr5/mathematics_sl/d_5_matsl_tsm_0509_1_e.pdf

External assessment

Component grade boundaries

Paper 1

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 11	12 – 22	23 - 33	34 – 44	45 - 55	56 - 66	67 - 90

Paper 2

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 – 29	30 - 42	43 – 52	53 - 63	64 - 73	74 - 90

This was the first session with the new assessment model, where paper 1 allows no calculator and paper 2 requires use of a graphic display calculator (GDC). Students did not appear to encounter any undue difficulties working without the calculator on paper 1, except possibly in Question 4.

However it appears that many students are still not clear what “working” to write in the examination when using the GDC, so candidates often spent precious time writing analytic methods to problems most efficiently solved using the GDC. To “show working” does not mean to perform algebraic steps or manipulations. Rather, what is important is to show the mathematical thinking, the setup, before reaching for the GDC, and then to let the GDC do the work of calculation. Whatever supports the solution, making the problem “calculator-ready,” is what students need to show as working.

To help teachers and students to understand more clearly what this means in practice, model solutions for paper 2 are attached to this report. When looking at the mark-scheme for paper 2, please bear in mind that any analytical approaches given there are to inform examiners how to award marks to such attempts. It is not intended to imply that these are the preferred or expected approaches.

A number of candidates did not present their work correctly. In Section A, all working should be done on the question paper. However, for Section B all the working is to be done on the lined paper which is then attached to the back of the question booklet. A large number of candidates also did working on the question paper for Section B and this caused the examiners difficulty in knowing which work to mark. Note that candidates are required to use pen when writing examinations

Paper 1

The areas of the programme which proved difficult for candidates

- Trigonometric functions in general and specifically use of identities
- Providing sufficient mathematical evidence to support a conclusion
- Properties of definite integrals
- Probabilities from a contingency table
- Correct use of derivatives in extremum problems

The levels of knowledge, understanding and skills demonstrated

Students seemed to cope quite well without their calculators and seemed particularly adept in the following areas:

- Quadratic functions and their graphs
- Elementary statistics and use of frequency table
- Finding matrix determinants and inverses
- Basic vector algebra and use of scalar product

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Frequencies, median and quartiles

Frequencies and median seemed well-understood, but quartiles and inter-quartile range less so. A few students, probably based on past papers, drew cumulative frequency diagrams, generating slightly different answers for median and quartiles.

Question 2: Graphs of quadratic functions

This question was consistently the best handled one on the entire paper.

Question 3: Matrix determinant and inverse with application to equation solving

Most candidates could find the determinant and inverse of a $2x2$ matrix, though quite a few believed the determinant to be $\frac{1}{ad-bc}$. The stronger candidates knew how to use the inverse to solve a system of equations, but weaker ones placed the inverse on the right or resorted to an algebraic solution, both approaches resulting in no marks for part (c).

Question 4: Values of trigonometric functions of obtuse angles using identities

This question was very poorly done, and knowledge of basic trigonometric identities and values of trigonometric functions of obtuse angles seemed distinctly lacking. Candidates who recognized the need of an identity for finding $\cos 2A$ given $\cos A$ seldom chose the most appropriate of the three and even when they did often used it incorrectly with expressions such as $2\cos^2 \frac{1}{9} - 1$.

Question 5: Transformations of graphs of functions

This question was reasonably well done. Many recognized the graph of $-f(x)$ as a reflection in a horizontal line, but fewer recognized the x-axis as the mirror line. A fair number gave $g(-3) = f(0)$, but did not carry through to $f(0) = -1.5$. The majority of candidates recognized that moving the graph of $f(x)$ by 3 units to the left results in the graph of $g(x)$, but the language used to describe the transformation was often far from precise mathematically.

Question 6: Probability from a contingency table

Many candidates had difficulty with this question, usually as a result of seeking to solve the problem by formula instead of looking carefully at the table frequencies. A very common error in part (b) was to assume identical probabilities for each selection instead of dependent probabilities where there is no replacement.

Question 7: Properties of definite integrals

This question was very poorly done. Very few candidates provided proper justification for part (a), a common error being to write $\int_1^5 f(x)dx = f(5) - f(1)$. What was being looked for was that $\int_1^5 3f(x)dx = 3\int_1^5 f(x)dx$ and $\int_5^1 f(x)dx = -\int_1^5 f(x)dx$.

Part (b) had similar problems with neither the combining of limits nor the splitting of integrals being done very often. A common error was to treat $f(x)$ as 1 in order to make $\int_1^5 f(x)dx = 4$ and then write $\int_1^5 (x + f(x))dx = [x+1]_1^5$.

Question 8: Vectors

This question was well done by many candidates. Most found $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AD}}$ correctly and the majority of candidates correctly used the scalar product to show $k = 7$. Some confusion arose in substituting $k = 7$ into $\overrightarrow{\mathrm{AD}}$, but otherwise part (c) was well done, though finding the position vector of C presented greater difficulty. Owing to $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{BC}}$ being perpendicular, no problems were created by using these two vectors to find $\cos \mathrm{A\hat{B}C} = 0$, and the majority of candidates answering part (d) did exactly that.

Question 9: Trigonometric functions and Volume of Revolution

This question was not done well by most candidates. No more than one-third of them could correctly give the range of $f(x) = \sin^3 x$ and few could provide adequate justification for there being exactly one solution to $f(x) = 1$ in the interval $[0, 2\pi]$. Finding the derivative of this function also presented major problems, thus making part (c) of the question much more difficult. In spite of the formula for volume of revolution being given in the Information Booklet, fewer than half of the candidates could correctly put the necessary function and limits into $\pi\int_a^b y^2 dx$ and fewer still could square $\sqrt{3}\sin x \cos^{1/2} x$ correctly. From those who did square correctly, the correct antiderivative was not often recognized. All manner of antiderivatives were suggested instead.

Question 10: Triangles in a semi-circle

Most candidates could obtain the area of triangle OPB as equal to $2\sin\theta$, though 2θ was given quite often as the area. Justifying why the area of the two triangles was the same was done very poorly. A minority recognized the equality of the sines of supplementary angles and the term complementary was frequently used instead of supplementary. Only a handful of candidates used the simple equal base and altitude argument. Many candidates seemed to see why $S = 2(\pi - 2\sin\theta)$ but the arguments presented for showing why this result was true were not very convincing in many cases. Explicit evidence of why the area of the semicircle was 2π was often missing as was an explanation for $2(2\sin\theta)$ and for subtraction.

Only a small number of candidates recognized the fact S would be minimum when $\sin\theta$ was maximum, leading to a simple non-calculus solution. Those who chose the calculus route often had difficulty finding the derivative of S, failing in a significant number of cases to recognize that the derivative of a constant is 0, and also going through painstaking application of the product rule to find the simple derivative. When it came to justify a minimum, there was evidence in some cases of using some form of valid test, but explanation of the test being used was generally poor. Candidates who answered part (d) correctly generally did well in part (e) as well, though answers outside the domain of θ were frequently seen.

The type of assistance and guidance teachers should provide for future candidates

- It would appear that all aspects of trigonometry could benefit from greater emphasis.
- More practice with problems demanding sufficient mathematical evidence to support a conclusion also is advisable.
- Students need to understand that they should not “cut corners” in their responses to “show that” questions.
- Greater understanding of the properties of definite integrals and the logic behind various tests for maxima, minima and points of inflexion is also to be encouraged.

Paper 2

The areas of the programme which proved difficult for candidates

Candidates showed difficulty answering questions on:

- Normal distributions
- Binomial probability

- Trigonometric equations
- Integration and area

The levels of knowledge, understanding and skill demonstrated

Candidates demonstrated a good level of knowledge and understanding in

- Geometric sequences
- Binomial expansion
- Differentiation
- Problem-solving involving exponential functions which model a practical situation

Overall the GDC was not used effectively by a majority of candidates, yet knowing when to choose the GDC is an essential feature of this paper. Often analytic approaches were chosen and either went nowhere or bogged down the candidate in unnecessary algebra.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1 (Geometric Sequences and Series)

This question was generally well done by most candidates, although quite a few showed difficulty answering part (b) exactly or to three significant figures. Some candidates reversed the division of terms to obtain a ratio of -5/3. Of these, most did not recognize this ratio as an inappropriate value when finding the sum in part (c).

Question 2 (Binomial Expansion)

Candidates produced mixed results in this question. Many showed a binomial expansion in some form, although simply writing rows of Pascal's triangle is insufficient evidence. A common error was to answer with the coefficient of the term, and many neglected the use of brackets when showing working. Although sloppy notation was not penalized if candidates achieved a correct result, for some the missing brackets led to a wrong answer.

Question 3 (Graph of a Function, numerical gradient)

When finding x-intercepts, candidates commonly attempted a fruitless algebraic approach to solve $f(x)=0$ using logarithms. Using the graphing features of the GDC, these values can be found readily, which also prepares the candidate for the sketching in part (b). Most candidates then sketched an approximately correct shape through all three intercepts. However, few candidates considered the domain and range of the function with any precision. Although the "write down" instruction in part (c) clearly indicates that no working is required, many spent unnecessary time finding the derivative function. Done correctly this earns full marks, but those who used the numerical derivative feature of their GDC also obtained the gradient without the risk of an analytic error.

Question 4 (Probability Distribution and Expected Value)

A good number of candidates answered this question well, although some incorrectly set the sum of the probabilities to zero instead of one, suggesting rote recognition of a quadratic equal to zero. Many candidates recognized that only the positive value for k was appropriate and correctly indicated this in their working. Many went on to find the correct expected value as well, although at times candidates wrote the formula from the information booklet without making use of it, thus earning no marks.

Question 5 (Normal Distribution)

Although many candidates shaded or otherwise correctly labelled the appropriate regions in the normal curve, far fewer could apply techniques of normal probabilities to achieve correct results in part (b). Many set the standardized formula equal to the probabilities instead of the appropriate z-scores, which can be found either by the use of tables or the GDC. Others simply left this part blank, which suggests a lack of preparation for such “inverse” types of questions in a normal distribution.

Question 6 (Binomial Probability)

Many candidates did not recognize the binomial nature of this question, suggesting an overall lack of preparation with this topic. Many used 7 days instead of 3 but could still earn marks in follow-through if working was shown. Those who could use their GDC effectively often answered correctly, although in part (c) some candidates misinterpreted the meaning of “at least one” and found either $P(X \le 1)$ or $1 - P(X \le 1)$.

Question 7 (Vectors)

Those candidates prepared in this topic area answered the question particularly well, often only making some calculation error when solving the resulting system of equations. Curiously, a few candidates found correct values for s and t, but when substituting back into one of the vector equations, neglected to find the z-coordinate of T.

Question 8 (Trigonometric Equations)

For part (a), most candidates correctly used the graph to identify the times of maximum and minimum depth. Most failed to consider that the depth of water is increasing most rapidly at a point of inflexion and often answered with the interval $t = 9$ to $t = 11$. A few candidates answered with the depth instead of time, misinterpreting which axis to consider.

A substantial number of candidates showed difficulty finding parameters of a trigonometric function with many only making superficial attempts at part (b), often leaving it blank entirely.

Some divided 2π by the period of 12, while others substituted an ordered pair such as $(4,10)$ and solved for *B*, often correctly. Many found that $c = 17$, thus confusing the vertical translation with a y-intercept.

For (c), many candidates simply read approximate values from the graph where $y = 12$ and thus answered with $t = 3.5$ and $t = 10.5$. Although the latter value is correct to three significant figures, $t = 3.5$ incurs the accuracy penalty as it was expected that candidates calculate this value in their GDC to achieve a result of $t = 3.52$. Those who attempted an analytic approach rarely achieved correct results.

Question 9 (Calculus)

Many candidates clearly applied the product rule to correctly show the given derivative. Some candidates missed the multiplicative nature of the function and attempted to apply a chain rule instead.

For part (b), the equation of the horizontal asymptote was commonly written as $x = 0$.

Although part (c) was a “write down” question where no working is required, a good number of candidates used an algebraic method of solving for *r* and *s* which sometimes returned incorrect answers. Those who used their GDC usually found correct values, although not always to three significant figures.

In part (d), many candidates showed some skill showing the equation of a normal, although some tried to work with the gradient of the tangent.

Surprisingly few candidates set up a completely correct expression for the area between curves that considered both integration and the correct subtraction of functions. Using limits of -6 and 2 was a common error, as was integrating on $f(x)$ alone. Where candidates did write a correct expression, many attempted to perform analytic techniques to calculate the area instead of using their GDC.

Question 10 (Exponential Functions)

A number of candidates found this question very accessible. In part (a), many correctly solved for *n*, but often incorrectly answered with the year 2006, thus misinterpreting that 6.12 years after the end of 2000 is in the year 2007.

Many found correct values in part (b) and often justified their result by simply noting the value after seven years is less than 51200. A common alternative was to divide 46807 by 25600 and note that this ratio is less than two. There were still a good number of candidates who failed to provide any justification as instructed.

Part (c) proved more challenging to candidates. Many found the correct ratio for R, however few candidates then created a proper equation or inequality by dividing the function for P by the function for T and setting this equal (or less) than 70. Such a function, although unfamiliar, can be solved using the graphing or solving features of the GDC. Many candidates chose a tabular approach but often only wrote down one value of the table, such as $n = 10, R = 68.3$. What is essential is to include the two values between which the correct answer falls. Sufficient evidence would include $n = 9, R = 70.8$ so that it is clear the value of $R = 70$ has been surpassed.

The type of assistance and guidance the teachers should provide for future candidates

Complete coverage of the syllabus is essential for student success on this exam. Judging by the number of candidates who did not attempt or showed little appreciation for some topics, notably in probability and integration, it is clear more emphasis could be placed in these areas when preparing candidates.

It seems in this session more than in the past that candidates had little appreciation for answering to three significant figures. Many answered to three decimal places, or rounded to the nearest whole number, while others wrote down whatever their calculator screen showed as a result. In a two-year course, it may be helpful to emphasize this significant figure rule from the onset and on a daily basis, whether using the GDC or not.

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GDC solutions

MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 8 May 2008 (morning)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer ***all*** *the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 6]*

Consider the infinite geometric sequence $3000, -1800, 1080, -648, \ldots$.

(a) Find the common ratio. *[2 marks]*

(b) Find the 10^{th} term. *[2 marks]*

(c) Find the **exact** sum of the infinite sequence. *[2 marks]*

(a) $r = \frac{-1800}{3000} = -0.6$

(b) $u_{10} = 3000(-.6)^9$

$= -30.2$

(c) $S_\infty = \frac{3000}{1+.6}$

$= 1875$

2. *[Maximum mark: 5]*

Find the term in x^3 in the expansion of $\left(\frac{2}{3}x-3\right)^8$.

$\binom{8}{5}\left(\frac{2}{3}x\right)^3(-3)^5$

$= -4032x^3$

3. *[Maximum mark: 7]*

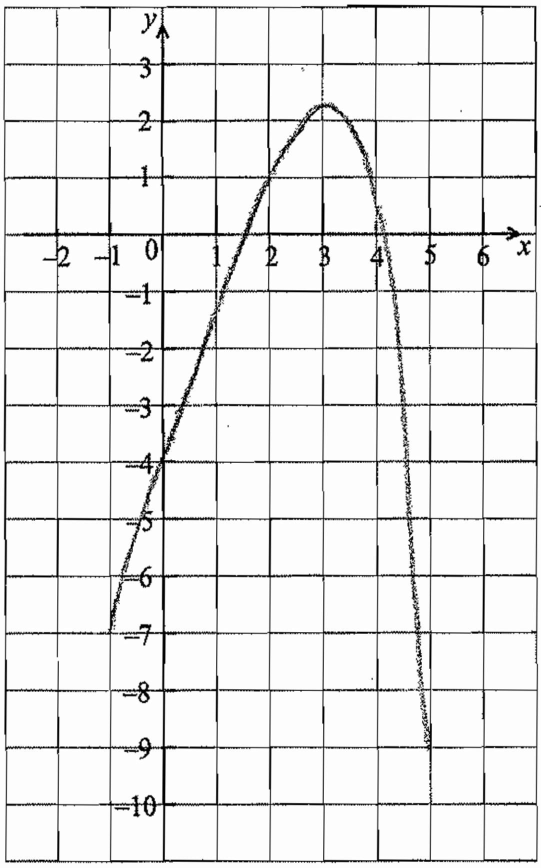
Let $f(x) = 3x - e^{x-2} - 4$, for $-1 \le x \le 5$.

(a) Find the x-intercepts of the graph of f. *[3 marks]*

$3x - e^{x-2} - 4 = 0$ (or reference to graph below)

x-intercepts at $(1.54, 0)$ and $(4.13, 0)$

(b) On the grid below, sketch the graph of f. *[3 marks]*



(This question continues on the following page)

(Question 3 continued)

(c) Write down the gradient of the graph of f at $x = 2$. *[1 mark]*

$f'(2) = 2$

4. *[Maximum mark: 7]*

The following table shows the probability distribution of a discrete random variable X.

x	-1	0	2	3
$P(X = x)$	0.2	$10k^2$	0.4	$3k$

(a) Find the value of k. *[4 marks]*

(b) Find the expected value of X. *[3 marks]*

(a) $0.2 + 10k^2 + 0.4 + 3k = 1$

$k = 0.1$

(b) $E(x) = -1(0.2) + 2(0.4) + 3(0.3)$

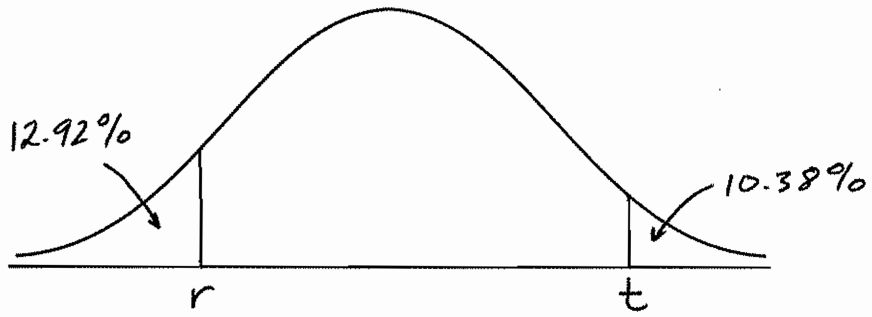
$= 1.5$

5. *[Maximum mark: 7]*

The heights of certain plants are normally distributed. The plants are classified into three categories.

The shortest 12.92 % are in category A.
The tallest 10.38 % are in category C.
All the other plants are in category B with heights between *r* cm and *t* cm.

(a) Complete the following diagram to represent this information. *[2 marks]*



(b) Given that the mean height is 6.84 cm and the standard deviation 0.25 cm, find the value of *r* and of *t*. *[5 marks]*

(b) $P(x<r) = 0.1292$
$r = 6.56.$

$P(x<t) = 1-0.1038$
$t = 7.16$

6. *[Maximum mark: 7]*

Paula goes to work three days a week. On any day, the probability that she goes on a red bus is $\frac{1}{4}$.

(a) Write down the expected number of times that Paula goes to work on a red bus in one week. *[2 marks]*

In one week, find the probability that she goes to work on a red bus

(b) on exactly two days; *[2 marks]*

(c) on at least one day. *[3 marks]*

(a) $E(x) = 3 \times \frac{1}{4}$

$= \frac{3}{4}$

(b) $X \sim B\left(3, \frac{1}{4}\right)$

$P(x=2) = 0.141$

(c) $P(x \geq 1) = 1 - P(x=0)$

$= 0.578$

7. *[Maximum mark: 6]*

The line L_1 is represented by $r_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line L_2 by $r_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$.

The lines L_1 and L_2 intersect at point T. Find the coordinates of T.

at T, $\vec{r}_1 = \vec{r}_2$

$2+s = 3-t$
$5+2s = -3+3t$
$s=-1,\ t=2$

at T, $\vec{r}_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$

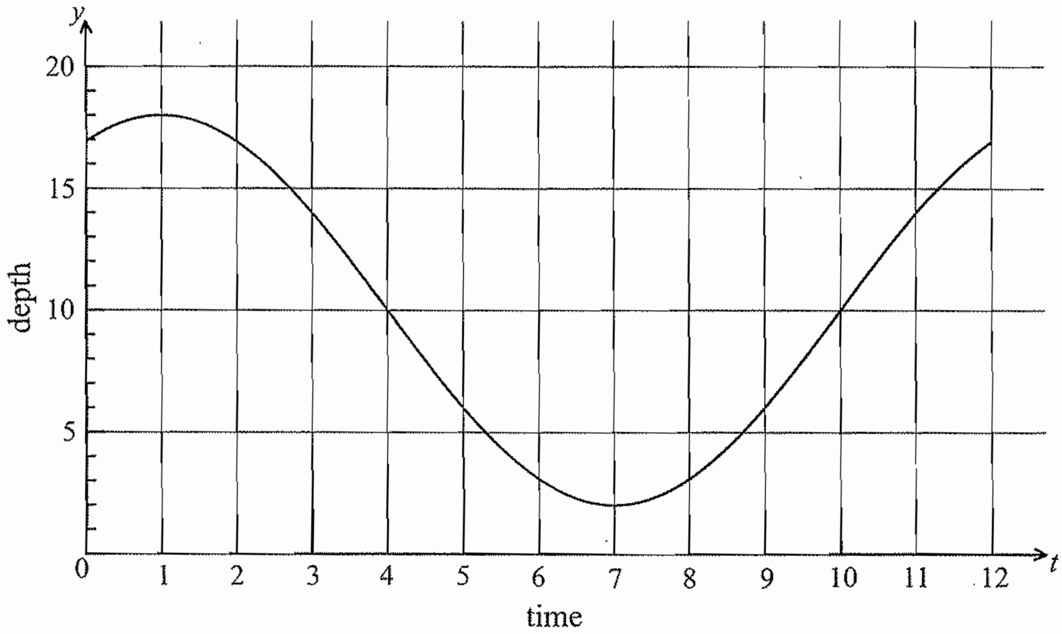
$= \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$ $\therefore$ T$(1,3,0)$

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 11]*

The following graph shows the depth of water, y metres, at a point P, during one day. The time t is given in hours, from midnight to noon.



(a) Use the graph to write down an estimate of the value of t when

(i) the depth of water is minimum;

(ii) the depth of water is maximum;

(iii) the depth of the water is increasing most rapidly. *[3 marks]*

(b) The depth of water can be modelled by the function $y = A\cos(B(t-1)) + C$.

(i) Show that $A = 8$.

(ii) Write down the value of C.

(iii) Find the value of B. *[6 marks]*

(c) A sailor knows that he cannot sail past P when the depth of the water is less than 12 m. Calculate the values of t between which he cannot sail past P. *[2 marks]*

8(a) (i) $t=7$

(ii) $t=1$

(iii) $t=10$

(b) (i) $A = \frac{18-2}{2}$

$= 8$

(ii) $C=10$

(iii) period $= 12 = \frac{2\pi}{B}$

$B = \frac{\pi}{6}$

(c) $y = 8\cos\left[\frac{\pi}{6}(t-1)\right]+10 = 12$

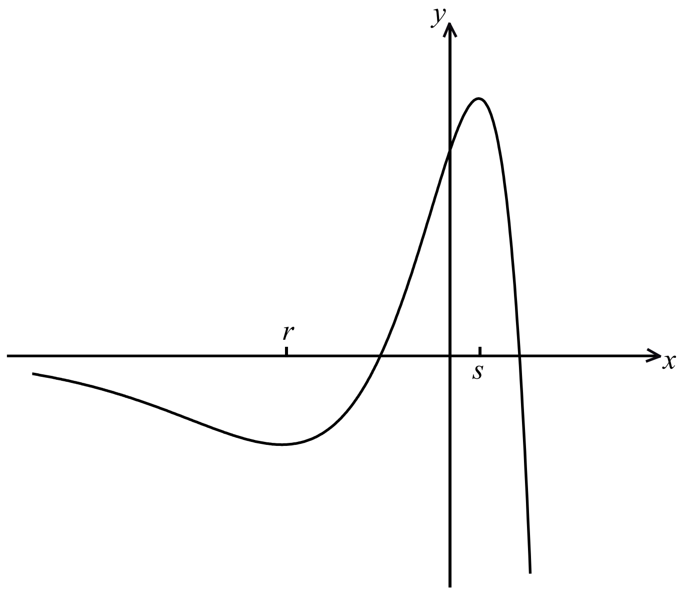
$t = 3.52, \ t = 10.5$

9. *[Maximum mark: 17]*

Let $f(x) = e^x(1 - x^2)$.

(a) Show that $f'(x) = e^x(1 - 2x - x^2)$. *[3 marks]*

Part of the graph of $y = f(x)$, for $-6 \le x \le 2$, is shown below. The *x*-coordinates of the local minimum and maximum points are *r* and *s* respectively.



(b) Write down the **equation** of the horizontal asymptote. *[1 mark]*

(c) Write down the value of *r* and of *s*. *[4 marks]*

(d) Let L be the normal to the curve of f at $P(0, 1)$. Show that L has equation $x + y = 1$. *[4 marks]*

(e) Let *R* be the region enclosed by the curve $y = f(x)$ and the line *L*.

(i) Find an expression for the area of *R*.

(ii) Calculate the area of *R*. *[5 marks]*

9. (a) $f'(x) = e^x(1-x^2) + e^x(-2x)$
$= e^x(1-2x-x^2)$

(b) $y=0$

(c) $r = -2.41$, $s = 0.414$

(d) $f'(0) = 1$
gradient of normal $= -1$
$y-1 = -1(x-0)$
$x+y=1$

(e) (i) limits when $1-x = e^x(1-x^2)$
$x = 0, 1$

$\therefore$ Area $= \int_0^1 \left[e^x(1-x^2) - (1-x)\right] dx$

(ii) $R = 0.5$

10. *[Maximum mark: 17]*

A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After n years the number of taxis, T, in the city is given by

$$T = 280 \times 1.12^n .$$

(a) (i) Find the number of taxis in the city at the end of 2005.

(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000. *[6 marks]*

(b) At the end of 2000 there were 25 600 people in the city who used taxis. After n years the number of people, P, in the city who used taxis is given by

$$P = \frac{2\,560\,000}{10 + 90\mathrm{e}^{-0.1n}} .$$

(i) Find the value of P at the end of 2005, giving your answer to the nearest whole number.

(ii) After seven complete years, will the value of P be double its value at the end of 2000? Justify your answer. *[6 marks]*

(c) Let R be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$.

(i) Find the value of R at the end of 2000.

(ii) After how many complete years will the city first reduce the number of taxis? *[5 marks]*

10. (a) (i) $T = 280 \times 1.12^5$

$= 493$

(ii) $280 \times 1.12^n = 560$ or $1.12^n = 2$

$n = 6.116\ldots$

$\therefore$ in the year 2007

(b) (i) $P(5) = 39636$

(ii) $P(7) = 46806.997$

$\therefore$ not double as $46807 < 51200$

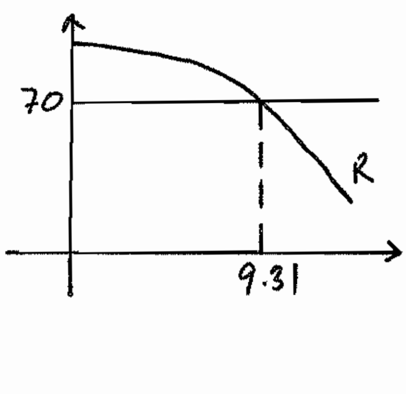
(c) (i) $R = \frac{P}{T} = 91.4$

(ii) $\frac{P}{T} < 70$

$n = 9.31\ldots$

$\therefore$ after 10 years.

GDC #1



GDC #2

n	R
8	73.209
9	70.764
10	68.286