MATHEMATICS SL TZ1

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 18	19 - 35	36 - 49	50 - 60	61 - 73	74 - 84	85 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 7	8 – 13	14 - 19	20 – 23	24 - 28	29 - 33	34 - 40

The range and suitability of the work submitted

Teachers should take note that new tasks for use in exam sessions from May 2009 to November 2010 are now available on the Online Curriculum Centre. Further, older tasks taken from Teacher Support Material documents (TSM) will **not** be accepted for submission as a part of the portfolio as of May 2009. The new tasks can be found at http://occ.ibo.org/ibis/documents/dp/gr5/mathematics_sl/d_5_matsl_tsm_0801_1_e.pdf

For this session moderators have noted that most schools chose to offer tasks selected from the TSM. Overall, students have achieved well, with approximately two-thirds attaining grades of 5 or above. These results hopefully reflect a greater confidence on the part of teachers in the application of the portfolio and its assessment. While some problems persist, there was evidence of greater understanding of the assessment criteria levels.

Tasks taken from other resources, or tasks that were teacher-designed, must be carefully reviewed to ensure that they adequately meet the requirements of the tasks as described in the subject guide, and that they offer students full opportunity to succeed at every level of each criteria. To not do so risks severe penalties that can have a serious impact on the success of all students in a school. It is critical that teachers work through any task they intend to set prior to assigning it to students, to ensure that the task provides sufficient opportunity for their students to address each of the criteria levels.

Candidate performance against each criterion

Criterion A (Notation and terminology):

Most candidates offered correct and appropriate notation in their work, yet the use of calculator notation (e.g. *, ^ , 10E4, etc.) is still an issue. Some candidates used 'y' repeatedly as the dependent variable for multiple model functions representing different quantities (e.g. in Stopping Distances). This can lead to an absurd relation for total stopping distance, '$y + y = y$', or such. Each model function should be identified distinctly, with subscripts or otherwise.

Criterion B (Communication):

Improvement was noted in the presentation of work and the labelling of graphs. Teachers are reminded that candidates must properly label all graphs, even if they must do so by hand if their computer software cannot accommodate this. While it is not required that work be word-processed, the use of word-processing is appreciated. In this case, students should be taught how to use the equation editor features of the given software, just as they should be trained in the resourceful use of graphing software.

Tasks are mistakenly viewed by some as homework exercises, and responses are offered in a “question & answer” format. The portfolio is intended to develop the skills of communicating mathematics in a smooth flow of mathematical writing. “Q&A” format is therefore inappropriate, and should be penalized. In general, tasks should be prescriptive enough to guide students, but not so prescriptive as to constitute a set of closed-ended exercises. Room to explore, modify, consider accuracy and reasonableness, and interpret should be provided in the task itself.

Criterion C (Process - Type I):

The performance here was generally good with many candidates scoring highly. It is important that sufficient evidence and analysis be evident. Candidates who arrive at a generalized statement without adequate supporting evidence cannot attain high marks in this criterion.

Many found it difficult to attain C5 as they did not understand how to validate their generalized statement. What is intended is that the students will consider the mathematical process and compare the results of test values against the results obtained through their general statement. Simple substitution of values of n into the statement to get a result does not constitute validation.

Criterion C (Process - Type II):

One of the most important aspects of modelling is to properly identify appropriate variables (those values that change due to the nature of the situation and/or the relationship between the quantities or measures). This has been emphasized in subject reports and in the supporting documents for Internal Assessment for many years. However, a great number of portfolios still do not address this issue adequately. While moderators will accept many implicit indications of variable declaration there is no substitute for a clear statement such as "Let t represent time in hours and A represent the amount in kg". It is far better too that students use variables that make sense in the context of the problem. Using t for time, or A for an amount helps frame the model function and focus any discussion in the context of these quantities.

In the same way parameters (a parameter is a value that one can change, but once changed it stays at that value until changed again by the modeller) and constraints (the real or potential limitations on the variables and parameters) must be properly and explicitly defined. For example, in a function $A(t) = at^2$ the parameter a will impact on the rate of growth of the amount A, and given that the model function represents growth as time increases, a must have a value > 0 and t must be ≥ 0.

Another focus of criterion C is that of analysis of data to develop a model function. The expectation is that the analysis will involve the mathematical skills and knowledge students have learned in the course of study. Using a calculator or computer regression feature as the primary tool for development of the model circumvents the mathematical analysis. A maximum of C2 is possible in these cases. Regression may certainly be used to confirm or compare **after** the model has been developed "by hand".

The criterion level C4 addresses the goodness of fit of the model function to the original data. Thus tasks that do not use data cannot achieve this level. While there exist many good problems involving the development of a model function through analytic methods, these are not appropriate as portfolio tasks.

Criterion D (Results - Type I):

The expected result of an exploration of a mathematical behaviour is a generalized statement that will allow one to determine a specific outcome at any particular point in the process. Most often this involves finding an expression for directly determining the general, n^{th}, term of the process. It may also involve a description of the general effect of changing parameters in a mathematical expression/function, or the end result of a process with a given starting value/shape/expression.

The higher levels of criterion D for Type I tasks require that students have appropriately explored the scope and limitations of the statement, and that they offer an informal explanation for their results. Teachers will have their own expectations of how far a student must go to adequately address scope or limitations, and this should be communicated to the moderator. Students may require some guidance as to what constitutes an informal explanation. This could be a logical, algebraic, or geometric presentation, or some other convincing argument. Examples, by themselves, do not constitute such an argument.

Criterion D (Results - Type II):

To achieve success in this criterion, students must consider the accuracy and reasonableness of **their** model function(s) in the context of the situation. Discussion of mathematical aspects such as intercepts, asymptotes, slopes, maxima or minima, etc, must be reframed into real considerations of things such as velocity, distance, time of day, greatest amount, long-term behaviour, etc. Many students offered a good **mathematical** discussion, but lost track of the real meaning of the task, scoring a maximum of D2. The interpretation should address the essential balance between accuracy (i.e. how good can I make it?) and reasonableness (i.e. what is good enough?). Further application(s) of the model function should involve appropriate modification(s) of the original.

Criterion E (Use of technology):

Moderators expressed concern that teachers are not informing them of the circumstances of the availability of technology, and the teachers' expectations of its use. Without such background information moderators may be unable to confirm the teachers' marks.

In Type I tasks it can be difficult to find resourceful ways to use technology. It may be appropriate to use spreadsheets or "sequential function" features, or graphs may be used to support analyses of patterns of mathematical behaviour. In all tasks, computer or calculator generated graphs do not in themselves constitute full and resourceful use of technology. Teachers should consider how many graphs or how multiple graphs on the same axes could improve the presentation of the solution.

Criterion F (Quality of work):

Most teachers recognized that students who completed a good majority of the task to a reasonable degree made a satisfactory effort and rightly awarded F1. However, students who complete all the requirements of the task without demonstrating any real insight or remarkable work should also receive F1. A mark of F2 should be awarded rarely, in those cases where the teacher stops to admire that the work presented reflects a greater insight or understanding. A mark of F0 should be reserved for a totally inadequate effort.

Recommendation and guidance for the teaching of future candidates

Teachers should review the assessment criteria with their students prior to assigning each task. Rather than outline the expectations for a specific task, the teacher can address general expectations of good use of notation, good communication, the essence of good analysis and interpretation, resourceful use of technology and expected quality of work.

Teachers should add comments on the work as they mark it, to provide feedback to the student and to inform the moderator as to why a given mark was awarded. Summary comments on form 5/PFCS or Form B (to be found in the TSM) also serve to inform moderators. The better the teacher can explain why a given mark was awarded the more likely their marks will be confirmed in moderation.

Teachers are reminded that specific instructions regarding the assessment of portfolios, including annotations to the criteria to help explain their application, are available on the Online Curriculum Centre in a variety of documents. Teachers may find the following links useful.

http://occ.ibo.org/ibis/documents/dp/gr5/mathematics_sl/d_5_matsl_gui_0805_1_e.pdf

http://occ.ibo.org/ibis/documents/dp/gr5/mathematics_sl/d_5_matsl_int-ass_0611_1_e.pdf

http://occ.ibo.org/ibis/documents/dp/gr5/mathematics_sl/d_5_matsl_tsm_0509_1_e.pdf

External assessment

Component grade boundaries

Paper 1

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 18	19 - 36	37 - 48	49 – 58	59 - 69	70 - 79	80 - 90

Paper 2

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 28	29 - 41	42 – 52	53 - 63	64 - 74	75 - 90

This was the first session with the new assessment model, where paper 1 allows no calculator and paper 2 requires use of a graphic display calculator (GDC). Students did not appear to encounter any undue difficulties working without the calculator on paper 1, except possibly in Question 2.

However it appears that many students are still not clear what “working” to write in the examination when using the GDC, so candidates often spent precious time writing analytic methods to problems most efficiently solved using the GDC. To “show working” does not mean to perform algebraic steps or manipulations. Rather, what is important is to show the mathematical thinking, the setup, before reaching for the GDC, and then to let the GDC do the work of calculation. Whatever supports the solution, making the problem “calculator-ready,” is what students need to show as working.

To help teachers and students to understand more clearly what this means in practice, model solutions for paper 2 are attached to this report. When looking at the mark scheme for paper 2, please bear in mind that any analytical approaches given there are to inform examiners how to award marks to such attempts. It is not intended to imply that these are the preferred or expected approaches.

A number of candidates did not present their work correctly. In Section A, all working should be done on the question paper. However, for Section B all the working is to be done on the lined paper which is then attached to the back of the question booklet. A large number of candidates also did working on the question paper for Section B and this caused the examiners difficulty in knowing which work to mark.

Note that candidates are required to use pen when writing examinations.

Paper 1

The areas of the programme which proved difficult for candidates

- using the laws of logarithms and the laws of exponents
- reference angles, using trigonometric identities and graphing of trig functions
- inverse relationship between e^x and $\ln x$
- writing a probability distribution, finding expected value and multiple event probability
- integration involving a chain rule

The levels of knowledge, understanding and skill demonstrated

The following areas were handled well by candidates:

- matrix algebra

- arithmetic sequences
- simple kinematics problems
- calculus of cubic polynomials
- quadratic functions and their graphs

The levels of knowledge and understanding varied widely. A large number of candidates seemed to be well prepared for taking the Mathematics SL paper 1 without a calculator. In most cases the candidates did a nice job showing their work. In fact, the overall quality of the scripts, in terms of showing relevant work, was impressive.

Most candidates also did a well on the “show that” problems, although quite a few candidates incorrectly worked backwards from the given information.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

This problem was very well done by most candidates. Part (c) on multiplication of matrices gave some candidates difficulties. There were a number of candidates who found the determinants of some matrix for all three parts instead of performing the operation.

Question 2

This was one of the most difficult problems for the candidates. Even the strongest candidates had a hard time with this one and only a few received any marks at all. Many did not appear to know the relationships between trigonometric functions of supplementary angles and that the use of $\sin^2 x + \cos^2 x = 1$ results in a $\pm$ value. The application of a double angle formula also seemed weak.

Question 3

Candidates probably had the most success with this question with many good solutions which were written with the working clearly shown. Many used the alternate approach of $u_n = 3n - 1$

Question 4

Many candidates were unable to write down the period of the function. However, they were often then able to go and correctly sketch the graph with the correct period. The final part was poorly done with many candidates finding the number of zeros instead of the intersection with the line $y = 2$.

Question 5

Many candidates were unable to correctly integrate but did recognize that the integral involved the natural log function; they most often missed the factor ½ or replaced it with 2. Part (b) proved difficult as many were unable to use the basic rules of logarithms.

Question 6

There were a number of completely correct solutions to this question. However, there were many who did not know the relationship between velocity and position. Many students differentiated rather than integrated and those who did integrate often had difficulty with the term involving e. Many who integrated correctly neglected the C or made C=7.

Question 7

This was one of the more difficult problems for the candidates. Knowledge of the laws of logarithms appeared weak as did the inverse nature of the exponential and logarithmic functions. There were a number of candidates who mistook the notation for the inverse to mean either the derivative or the reciprocal. The order of composition seemed well understood by most candidates but they were unable to simplify by the rules of indices to obtain the correct final answer.

Question 8

This question was very well done with most candidates showing their work in an orderly manner. There were a number of candidates, however, who were a bit sloppy in indicating when a function was being equated to zero and they "solved" an expression rather than an equation. Many candidates went through first and second derivative tests to verify that the point they found was a maximum or an inflexion point; this was unnecessary since the graph was given. Many also found the y-coordinate which was unnecessary and used up valuable time on the exam.

Question 9

This problem was generally well done. The "show that" question in part (a) was done correctly by most candidates, with a few attempting to show it by working backwards, which earned no marks. Most candidates were able to identify the vertex but were unable to write the equation for the axis of symmetry. There was a great deal of success with the x and y intercepts. Some of the sketches of the graph left much to be desired even if they were technically correct; many were v-shaped. The final part was poorly done with indicated that defining a graph in terms of stretch and translation was unfamiliar to many candidates.

Question 10

This was the most difficult of the extended response questions for the candidates. Finding *s* and *t* correctly in part (b) was difficult, with many confused between writing appropriate probabilities on a single branch compared to at the final end of a multiple branch. Many candidates had no idea what to write for a probability distribution and those who did often had probabilities that did not sum to 1. Candidates who wrote a probability distribution often could correctly compute the expected value. The final part was the most challenging, but some good answers were seen. The most common error was not recognizing that there were two different ways of winning.

The type of assistance and guidance the teachers should provide for future candidates

Candidates need to be exposed to the following skills and concepts:

- the use of trigonometric relationships
- the function $y = a + b\sin kx$ without reliance on the gdc
- the inverse relationship between exponential and logarithmic functions
- the laws of logarithms and laws of exponents
- writing a probability distribution and practice with multiple event probability
- sketching a graph without copying from the gdc

Teachers should continue to work with candidates on how to correctly work a “show that” question. Candidates need to understand that they should not work in reverse and that reasoning must be shown in considerable detail.

Teachers should continue to work with candidates to help them work problems without a calculator.

Paper 2

The areas of the programme which proved difficult for candidates

Overall, candidates had difficulty in the following areas:

- Binomial probability
- Normal distributions
- Vectors

Some candidates seemed to not understand how to use their GDC to solve equations, locate intersection points, and find the area between curves.

The levels of knowledge, understanding and skill demonstrated

There seemed to be a wide range of understanding and skill demonstrated with most questions being attempted with some success. Questions that proved the most difficult were those that were not straightforward but required a conceptual knowledge.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

Part (a) of this question was well done. Finding the median seemed to be the most difficult for the candidates. Most had the idea that it was in the middle but did not know how to find the value. When calculating the mean, many ignored the frequencies.

Question 2

This question was well done with most students using the law of sines to find the angle. In part (b), the most common error occurred when angle *R* or 75 degrees was used to find the area. This particular question was the most common place to incur an accuracy penalty.

Question 3

Few errors were made in this question. Those that were made were usually arithmetical in nature.

Question 4

In part (a), some did not realize that they should copy the curve from their GDC, paying attention to domain and range. Not using their GDC, and trying to solve the equation analytically in part (b) proved to be very difficult for many. A common error was to substitute

$x = 1$.

Question 5

Several candidates had a correct sketch in part (a). The majority of the errors occurred in parts (b) and (c). In part (b), some seemed to just guess while others left it blank. In part (c), justification lacked completeness. For example, many stated that the second derivative must equal zero but said nothing of its change in sign.

Question 6

Part (a) was handled well by most students. Although this question was a rather straightforward question on binomial distribution, parts (b) and(c) seemed to cause much difficulty. In part (c), finding at least one defective switch, many forgot to take the complement.

Question 7

This question was very poorly done with many leaving it blank. Of those that did attempt it, most were able to find $\boldsymbol{v} + p\boldsymbol{w}$ but really did not know how to proceed from there. They tried many approaches, such as, finding magnitudes, using negative reciprocals, or calculating the angle between two vectors. A few had the idea that the scalar product should equal zero but had trouble trying to set it up.

Question 8

Those that understood the normal distribution did well on parts (a) and (bi). Parts (bii) and (c) proved to be a little more difficult. In particular, in part (bii) the *z*-score was incorrectly set equal to 0.05 and in part (c), 0.2 was used instead of the *z*-score. For those who had a good grasp of the concept of normal distributions the entire question was quite accessible and full marks were gained.

Question 9

Part (ai) was done well by most students. Most knew how to approach finding the angle in part (aii). The problems occurred when the incorrect vectors were chosen. If the vectors being used were stated, then follow through marks could be given. Part (b) was well done. In part (ci), the error that occurred most often was the incorrect choice for the direction vector. Those that were able to find the coordinates in part (cii) were also able to be successful in part (d).

Question 10

Many candidates did not make good use of the GDC in this problem. Most had the correct expression but incorrect limits. Some tried to integrate to find the area without using their GDC. This became extremely complicated and time consuming. In part (b), the chain rule was not used by some. Most candidates realized the relationship between the gradient and the first derivative and set the two derivatives equal to one another. Once again many did not realize that the intersection could be easily found on their GDC.

The type of assistance and guidance the teachers should provide for future candidates

1. Students need to be more aware of what they can do with the GDC (and how to use it) and to recognize when analytic approaches are not appropriate.

2. Practice past exam papers so candidates understand where to put their working.
3. Teach the whole course.
4. Teachers should advise students about the structure of the questions, for example, in question 4 (a), a sketch was required so this could be used in part (b).
5. Students should show all working but calculator notation is not acceptable.
6. Emphasise the 3 s.f. rule

Teachers should check the information given in the subject guide and make sure candidates are aware of the meaning of the command terms and notation that may be used in questions.

Give students practice in showing that certain results are true. Each step of working/reasoning must be clearly shown. It is also important that candidates do not work in reverse and simply verify that the answer is correct.

Give students practice in giving explanations for results and be tough in the marking of such explanations, demanding accuracy and clarity.

M08/5/MATME/SP2/ENG/TZ1/XX

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GDC solutions

MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 8 May 2008 (morning)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer ***all*** *the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

In a school with 125 girls, each student is tested to see how many sit-up exercises (sit-ups) she can do in one minute. The results are given in the table below.

Number of sit-ups	Number of students	Cumulative number of students
15	11	11
16	21	32
17	33	p
18	q	99
19	18	117
20	8	125

(This question continues on the following page)

(Question 1 continued)

(a) (i) Write down the value of p.

(ii) Find the value of q. *[3 marks]*

(b) Find the median number of sit-ups. *[2 marks]*

(c) Find the mean number of sit-ups. *[2 marks]*

(a) (i) $p = 65$

(ii) $q = 99 - p$
$= 34$

(b) median at the 63rd student
∴ median is 17 situps.

(c) $\bar{x} = \frac{15(11) + 16(21) + \ldots + 20(8)}{125}$

$= 17.4$

2. *[Maximum mark: 6]*

The diagram below shows triangle PQR. The length of [PQ] is 7 cm, the length of [PR] is 10 cm, and $P\hat{Q}R$ is 75°.

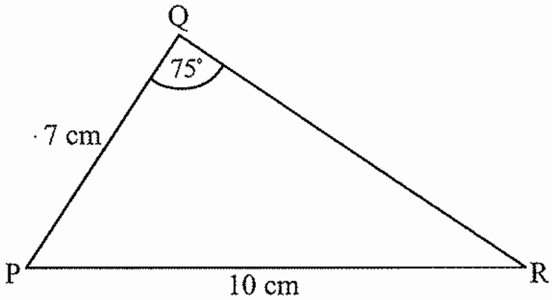


diagram not to scale

(a) Find $P\hat{R}Q$. *[3 marks]*

(b) Find the area of triangle PQR. *[3 marks]*

(a) $\frac{\sin R}{7} = \frac{\sin 75^\circ}{10}$

$P\hat{R}Q = 42.5^\circ$

(b) $Q\hat{P}R = 62.5^\circ$

$A = \frac{1}{2}(7)(10)\sin 62.5^\circ$

$= 31.0 \text{ cm}^2$

3. *[Maximum mark: 6]*

The diagram below shows a circle centre O, with radius r. The length of arc ABC is 3π cm and $\hat{AOC} = \frac{2\pi}{9}$.

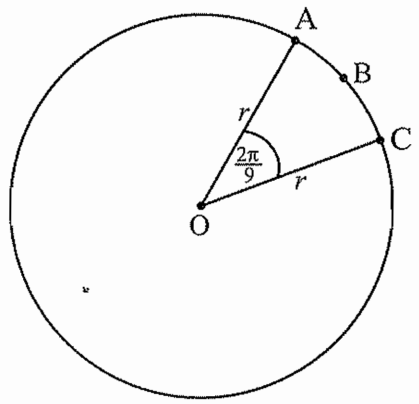


diagram not to scale

(a) Find the value of r. *[2 marks]*

(b) Find the perimeter of sector OABC. *[2 marks]*

(c) Find the area of sector OABC. *[2 marks]*

(a) $3\pi = r\left(\frac{2\pi}{9}\right)$

$r = 13.5$ cm.

(b) perimeter $= 2(13.5) + 3\pi$

$= 36.4$ cm

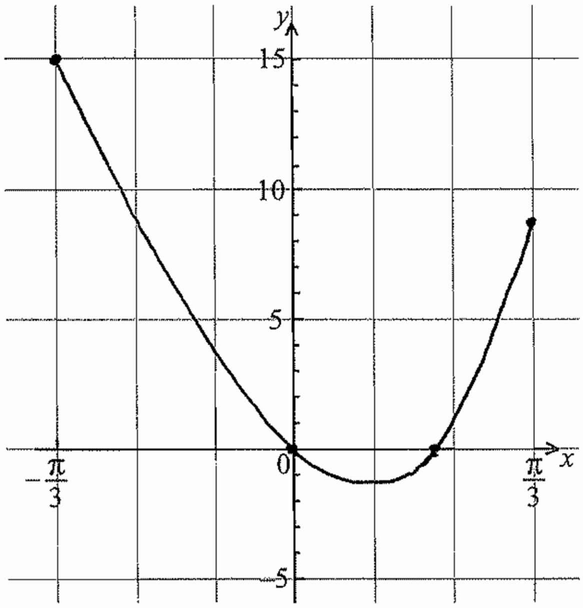
(c) Area $= \frac{1}{2}(13.5)^2\left(\frac{2\pi}{9}\right)$

$= 63.6 \text{ cm}^2$

4. *[Maximum mark: 6]*

Let $f(x) = 4\tan^2 x - 4\sin x$, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$.

(a) On the grid below, sketch the graph of $y = f(x)$. *[3 marks]*



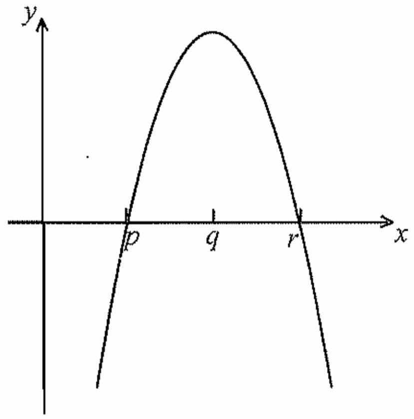
(b) Solve the equation $f(x) = 1$. *[3 marks]*

$4\tan^2 x - 4\sin x = 1$

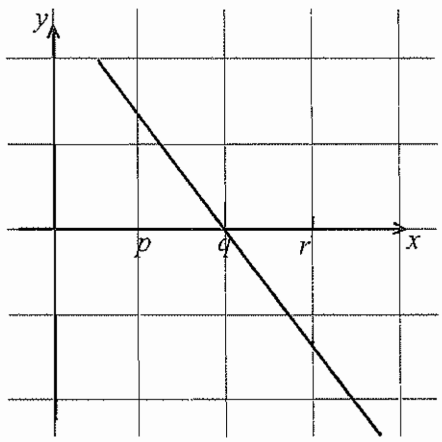
$x = -0.207$, $x = 0.772$

5. *[Maximum mark: 6]*

The diagram below shows part of the graph of the **gradient** function, $y = f'(x)$.



(a) On the grid below, sketch a graph of $y = f''(x)$, clearly indicating the x-intercept. *[2 marks]*



(b) Complete the table, for the graph of $y = f(x)$. *[2 marks]*

	x-coordinate
(i) Maximum point on f	r
(ii) Inflexion point on f	q

(c) Justify your answer to part (b)(ii). *[2 marks]*

At q, f''(x) = 0 and f''(x) changes sign

from positive to negative.

6. *[Maximum mark: 7]*

A factory makes switches. The probability that a switch is defective is 0.04. The factory tests a random sample of 100 switches.

(a) Find the mean number of defective switches in the sample. *[2 marks]*

(b) Find the probability that there are exactly six defective switches in the sample. *[2 marks]*

(c) Find the probability that there is at least one defective switch in the sample. *[3 marks]*

(a) $E(x) = 100 \times 0.04$
$= 4$

(b) $X \sim B(100, 0.04)$

$P(x=6) = 0.105$

(c) $P(X \geq 1) = 1 - P(x=0)$
$= 0.983$

7. *[Maximum mark: 7]*

Let $v = 3i + 4j + k$ and $w = i + 2j - 3k$. The vector $v + pw$ is perpendicular to w. Find the value of p.

$$\vec{v} + p\vec{w} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + p\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 3+p \\ 4+2p \\ 1-3p \end{pmatrix}$$

$$(3+p)(1) + (4+2p)(2) + (1-3p)(-3) = 0$$

$$8 + 14p = 0$$

$$p = \frac{-8}{14}$$

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7 g and standard deviation 0.5 g.

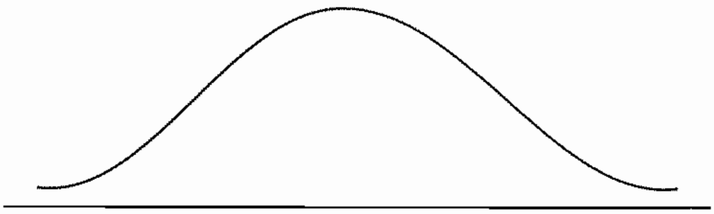
(a) One biscuit is chosen at random from the box. Find the probability that this biscuit

(i) weighs less than 8 g;

(ii) weighs between 6 g and 8 g. *[4 marks]*

(b) Five percent of the biscuits in the box weigh less than *d* grams.

(i) Copy and complete the following normal distribution diagram, to represent this information, by indicating *d*, and shading the appropriate region.



(ii) Find the value of *d*. *[5 marks]*

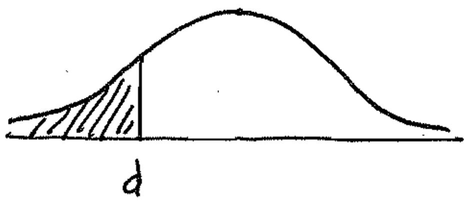
(c) The weights of biscuits in another box are normally distributed with mean μ and standard deviation 0.5 g. It is known that 20 % of the biscuits in this second box weigh less than 5 g.

Find the value of μ. *[4 marks]*

8. (a) (i) $P(X<8) = 0.977$

(ii) $P(6<X<8) = 0.954$

(b) (i)



(ii) $P(X<d) = 0.05$

$d = 6.18$

(c) $\frac{5-\mu}{0.5} = -0.8416$

$\mu = 5.42$

9. *[Maximum mark: 18]*

The point O has coordinates $(0, 0, 0)$, point A has coordinates $(1, -2, 3)$ and point B has coordinates $(-3, 4, 2)$.

(a) (i) Show that $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

(ii) Find $B\hat{A}O$. *[7 marks]*

(b) The line L_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s\begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of two points on L_1. *[2 marks]*

(c) The line L_2 passes through A and is parallel to $\overrightarrow{OB}$.

(i) Find a vector equation for L_2, giving your answer in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

(ii) Point $C(k, -k, 5)$ is on L_2. Find the coordinates of C. *[7 marks]*

(d) The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, and passes through the point C.

Find the value of p at C. *[2 marks]*

9 (a) (i) $\vec{AB} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$

(ii) using vectors $\vec{AO}$ and $\vec{AB}$

$$\vec{AO} \cdot \vec{AB} = (-1)(-4) + (2)(6) + (-3)(-1)$$

$$|\vec{AO}| = \sqrt{(-1)^2 + (2)^2 + (-3)^2}$$

$$|\vec{AB}| = \sqrt{(-4)^2 + (6)^2 + (-1)^2}$$

$$\therefore \cos B\hat{A}O = \frac{19}{\sqrt{14} \cdot \sqrt{53}}$$

$$\therefore B\hat{A}O = 0.799 \text{ radians}$$

(b) $(-3, 4, 2)$; $(-7, 10, 1)$

(c) (i) $\vec{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$

(ii) $\begin{pmatrix} k \\ -k \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$

so $5 = 3 + 2t$

$t = 1 \quad \therefore k = -2.$

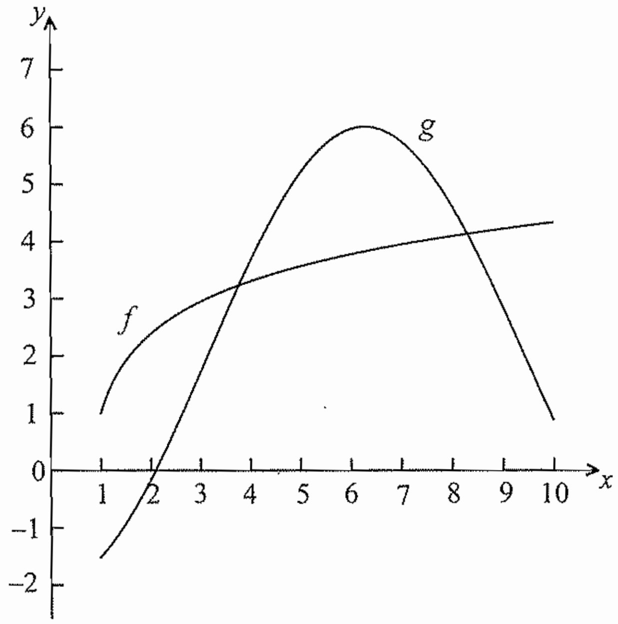
$\therefore$ C has coordinates $(-2, 2, 5)$

(d) $\begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ (or $-2 = 3 + p$)

$\therefore p = -5$

10. *[Maximum mark: 14]*

The following diagram shows the graphs of $f(x) = \ln(3x - 2) + 1$ and $g(x) = -4\cos(0.5x) + 2$, for $1 \le x \le 10$.



(a) Let A be the area of the region **enclosed** by the curves of f and g.

(i) Find an expression for A.

(ii) Calculate the value of A. *[6 marks]*

(b) (i) Find $f'(x)$.

(ii) Find $g'(x)$. *[4 marks]*

(c) There are two values of x for which the gradient of f is equal to the gradient of g. Find both these values of x. *[4 marks]*

10. (a) (i) limits when $f(x) = g(x)$

$\therefore \; x = 3.7667... \; ; \; x = 8.3009...$

$$\therefore \text{Area} = \int_{3.7667...}^{8.3009...} \left(\left(-4\cos(0.5x)+2\right) - \left(\ln(3x-2)+1\right) \right) dx$$

(ii) $A = 6.46$.

(b) (i) $f'(x) = \dfrac{3}{3x-2}$

$g'(x) = 2\sin(0.5x)$

(c) $f'(x) = g'(x)$

$x = 1.43, \; 6.10$