MATHEMATICS SL TZ2

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 14	15 - 29	30 - 44	45 - 55	56 - 68	69 - 80	81 - 100

This was the second May examination session based on the revised program for mathematics SL.

Please note that as announced in Diploma Programme Co-ordinator Notes, March 2006, the format of the examination papers is changing from May 2008. There are no changes to the syllabus or to the internal assessment requirements. Each paper will consist of two sections, each section worth 45 marks. Section A will be consist of short questions, section B will consist of long questions. For Paper 1, **no calculators of any kind are allowed.** A graphic display calculator (GDC) will continue to be required for paper 2. This change is to enable the assessment of analytic skills to be more effective. It is not intended to assess arithmetic skills and only simple arithmetic will be required.

Given that several candidates in this session thought that 3^3 is 9, it may be advisable to ensure that students have practice in working questions without any calculator available.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 13	14 - 19	20 - 23	24 - 28	29 - 33	34 - 40

This session saw a consolidation of efforts in schools to implement the recently revised Internal Assessment requirements. Many teachers used material that they felt comfortable with, reinforcing their own understanding of the assessment criteria, and using the portfolio as a valuable teaching tool. Tasks were often drawn from the Teacher Support Material (TSM) and teachers appeared better equipped to work with these tasks after having gone through one examination session, or having attended teacher training.

The range and suitability of the work submitted

The majority of tasks selected came from the current TSM. Some schools still have copies of older editions of the TSM and submitted tasks from these. Many of the tasks in these older editions are no longer suitable as they were not designed to meet the current criteria. It is important that schools use the most current IB documents available. Please note that new tasks will be published early in 2008, for use in 2009 and 2010, and that the tasks from all other TSMs should not be submitted for final assessment after November 2008. Please see the Diploma Programme Co-ordinator Notes, November 2006 for further details.

Some teachers used tasks that they designed, or tasks taken from other resources. This is encouraging, and some of the tasks were interesting and effective. However, others, while wonderful mathematical projects, did not suit the assessment criteria well. It is essential that all tasks provide opportunity for candidates to achieve the highest levels in each of the criteria. Unfortunately some tasks submitted did not allow for this, and candidates suffered as a result. It is critical that teachers work through any task to ensure that its expectations will allow candidates to reach the highest levels of each criterion.

Teachers should be aware that only those resources available through the IB are official and that other text books do not have any official status. Tasks posted in the resource section on the Online Curriculum Centre are not vetted by the IB. Any IA tasks taken from sources other than the TSM may not satisfy all the criteria. As mentioned in the TSM, if using a task written by someone else, it will be necessary to work the task first to check suitability. Amendments will almost certainly be needed for the task to be incorporated into a particular course of study.

The work presented was of generally good quality, with some exceptional pieces. Many schools have taken to heart the aims of the portfolio and have used this form of internal assessment effectively.

Candidate performance against each criterion

Criterion A: Use of Notation and Terminology

Performance here was good with many candidates attaining level 2. The greatest problem is the use of non-mathematical notation, such as that from calculators or computers. Many schools do not insist on the use of an appropriate symbol for “approximately equals to”. Teachers are reminded to instruct their students that, if notation cannot be properly presented through word-processing (there are many good equation editors for use with most word processing packages), they should hand-write the notation.

Criterion B: Communication

The quality of communication generally improved over the last session. While there were still problems with the proper labelling of graphs and tables, candidates produced better mathematical *writing*. There was more of a flow to the work and more explanation was provided. That said, there are still quite a few cases where work was presented in a ‘question

& answer' format, as if the task were simply a series of homework questions. This is not good mathematical writing and any such work is limited to a maximum of B2.

The use of graphs in appendices is a problem. Graphs should appear in the body of the work, at the points where they are used to support the work at hand. To do otherwise interferes with the desired flow of the writing. If graphs cannot be labelled properly using the computer software available, then labelling by hand is expected.

Some candidates offered work that was extensive in its scope and length. It is not intended that a portfolio will be a long thesis. Rather, it should be in the order of 6 – 10 pages depending upon the task and additional graphs or other diagrams. Page after page of graphs, tables or calculations do not make for good communication.

Criterion C: Mathematical process

Type I

The portfolio is intended to address certain of the Assessment Objectives listed in the Subject Guide. A Type I investigation should meet the requirements as laid out there. The assessment criteria reflect the essential skills to be assessed, as also described in the guide. Tasks assigned for Type I must address these skills so that they can be marked appropriately against the criteria.

Where tasks provided for success, candidates generally performed well. They were able to generate and organize data, then use a suitable mathematical analysis to produce a general statement. Some candidates, however, did not recognize the need for sufficient data before making a conjecture. Problems were also encountered when they tried to "test the validity" of the general statement. This means checking other values by using the process they used to generate their data, verifying that the result matches that obtained from their general statement. Substituting other values into the statement and simplifying does not constitute validation. This was a common error at level C5.

In some cases it was clear that the result of the investigation was already known to students. This undermines the whole purpose of an investigation, and teachers should try to assign tasks within the course of study in a way that maintains the integrity of the intended purpose.

Type II

The process expected in a Type II task, and reflected in the criteria, is that the student will consider data provided or generated from a real-life scenario. They will then use their mathematical skills to apply an appropriate strategy to develop a suitable mathematical model. They must check how well the function developed fits the data, and make any appropriate modifications. They will also apply their model to another set of data, or another scenario, either obtained through their own research, or provided for in the task itself.

Where teachers and candidates adhered to this process they were generally successful. Candidates used their knowledge of periodic, exponential and other functions to good effect. They demonstrated visually how well the graph of the model function fitted the data, and described the fit qualitatively, and sometimes quantitatively. One problem was that candidates immediately graphed both the data and the model function. Ordinarily one would expect to see a plot of the data points followed by a discussion of why one function or another would be a suitable model, followed by the presentation of the model function graph and a comparison to the data.

Given the power of technology it is tempting to let the calculator or computer do the work and create a "best-fit" model by way of regression analysis. While this is a useful feature, regression must not be used as the primary tool for developing the function.

Candidates and teachers have not properly recognized the need to explicitly identify the variables to be used. Where multiple functions are involved in a model, for example in the Stopping Distances task, they should not all be identified as 'y' since each function represents a distinct aspect of the situation. The variables used were very often x and y, the old standards. While this is not incorrect, it tends to lead the student away from the modelling aspects of the task and towards a strictly mathematical discussion. Candidates rarely outlined in an explicit manner the real-life constraints of the quantities involved.

Criterion D: Results

Type I

While criterion C considers the process of obtaining and verifying a general statement, criterion D considers the results. Candidates may arrive at <u>a</u> general statement, incorrect or incomplete, and still score up to D3. Some teachers did not appreciate this nuance and applied severe penalties where the correct general statement did not appear. Candidates were not always able to produce <u>the</u> correct (at times this would be the final correct statement after a series of preliminary ones) statement, yet made a good attempt at doing so. More difficulty was apparent in levels D4 and D5. Candidates often offered only a limited exploration of scope and limitations to their general statement. Fewer still were able to properly explain why their statement worked.

Type II

For a modelling task the assessment under criterion D addresses the results and the interpretation of them in the real life scenario of the task. Candidates may arrive at some results, even if those results don't properly model the data, and be awarded up to D2. Beyond this the quality of interpretation will determine the marks awarded. However, no marks are available beyond D2 if the context is not addressed. This is a common problem, with many candidates losing track of the true purpose of the task, and focusing instead on the *mathematical* behaviour of graphs and variables. This was highlighted by the many cases where candidates would simply create a graph of a continuous function without any consideration of the discrete nature of some data.

Any model is a balance between convenience and accuracy. Consideration of the reasonableness of the model, including an appropriate degree of accuracy, highlights the notion that models should work reasonably well, yet not be bogged down with parameters of six or more significant figures. Discussion as to limitations and possible modifications to make the model function fit better, or adjust to new circumstances, is also expected at levels D4 and D5. Many candidates found it difficult to achieve these higher levels in criterion D.

Criterion E: Use of technology

Candidates were generally more sophisticated in their use of technology. In the best cases graphing software allowed candidates to offer various versions of their efforts in modelling tasks, and show how a given general statement matched the pattern of behaviour in an investigation. Some students used spreadsheets effectively to show numerical patterns. In other cases little or no evidence of the use of technology was offered, nor did the candidate or teacher identify its use. Some tasks were not well suited to the use of technology and this made the assessment under criterion E difficult.

Teachers have widely varying expectations of the use of the technology available to their students. It is extremely important that teachers provide information that gives a rationale for their assessment. Without comment, the simple inclusion of a few printouts of graphs may not constitute effective use of technology.

Criterion F: Quality of work

This holistic criterion assesses the level of understanding, insight, and mathematical sophistication shown in the work. While it is expected that candidates should score well elsewhere, it is not required that every other criteria receive maximum marks before a mark of F2 can be awarded. However, simply answering the questions in the task is generally not sufficient for F2. A mark of F0 should be given only where a candidate has made little effort whatsoever.

Recommendations for the teaching of future candidates

Teachers and students should pay closer attention to the proper use of appropriate notation, especially in cases where approximations are involved.

Candidates must consider their work as a cohesive piece of mathematical writing, not a homework exercise. Graphs and diagrams should be placed in context, and there should be a natural flow between sections. Proper explanations should accompany any working, and all graphs and diagrams must be properly and consistently labelled, even if by hand.

Where students are investigating a mathematical behaviour, sufficient evidence should be presented before a conjecture is made. Once the conjecture is made, it should be verified by testing new values against both the general statement and the process that creates the pattern.

More effort needs to be made to explicitly identify variables, parameters and constraints at an early point in response to a modelling task. Analysis must involve mathematical skills covered in the syllabus. Students should show why and how they have arrived at the model and parameters used.

Graphs should show a plot of the original data (often the data is discrete) before a function is arrived at, followed then by various examples of functions that make for a better and better fit.

Students should appreciate that the ultimate goal is to model the real-life behaviour. Thus a thorough discussion of how reasonable and accurate the model is should be included. Reasonableness may include a discussion of the discrete nature of original data versus the continuous nature of the model function.

The use of technology must enhance the development of the task. Students should be able to offer more evidence in support of their work than if they did not have the technological tools available. They may require instruction as to how to best use the features of the calculator or software in order to achieve this.

The quality of work is often directly linked to the degree with which the student is engaged in the task. Teachers should try to help students see their work as a comprehensive and cohesive effort to address a mathematical situation with insight and understanding.

Teachers are reminded that they should establish an internal moderation process wherever more than one teacher is involved in the marking. A good idea is to cross-mark a number of the portfolios to ensure that the team is in agreement on key points of the assessment. The sample for moderation reflects the whole school so discrepancies in individual marking can make the process difficult.

Teachers should also compare their assessment with the moderated assessment by viewing the component scores available on IBIS. This can give a general idea of how generous or severe their marking might have been and may indicate that further reflection is necessary for certain criteria. Mark changes of 1 or 2 points are common and teachers should not consider this as a significant problem.

External assessment

Component grade boundaries

Paper 1

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 – 27	28 - 40	41 – 51	52 - 61	62 - 72	73 - 90

Paper 2

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 11	12 – 23	24 - 37	38 – 48	49 - 60	61 - 71	72 - 90

The areas of the programme and examination that appeared difficult for the candidates

Some centres appear not to cover the whole syllabus, which means that candidates from those centres were not able to make an attempt on several questions.

The topics with poor cover include :

- Binomial and Normal Distribution
- Probability
- Functions : their domains and ranges
- Differentiation and integration of more difficult functions
- Knowing which are the direction vectors in order to find the angle between two lines

Some candidates had difficulty in deciding the appropriate use of their GDC. See individual question information below.

Many candidates have difficulty in answering “show that” questions.

The skill of translating problem situations into an algebraic form proved challenging for many. Some adopted a trial and improvement approach but showed little coherent work to support this.

There were instances of accuracy errors due to rounding of intermediate answers.

The areas of the programme and examination in which candidates appeared well prepared

There were many fine papers and the work was generally set out in a clear, concise manner. In general, candidates demonstrated a broad range of knowledge. More candidates are also showing at least some working, which means they were at least getting some marks for their working.

See question analysis below for more details.

The strengths and weaknesses of the candidates in the treatment of individual questions

Paper 1

Question 1: (Geometric series)

Part (a) was well done. In part (b) the majority of errors occurred as a result of using the 29th power rather than the 30th power, or 1.13 rather than 1.013.

Question 2: (Binomial expansion)

This question was quite well done. Not squaring the "2" caused candidates to lose marks on a question that otherwise was well understood.

Question 3: (Composite and inverse functions)

Except for part (c), most candidates handled this question with ease. A few did not understand the notation for inverse, confusing it with reciprocal or derivative. Not understanding the domain of an inverse meant most candidates did not gain the final mark.

Question 4: (Probability)

Part (a) was handled well by most candidates. Parts (b) and (c) caused problems with many answering $\frac{68}{97}$ for part (c). They added the probabilities and did not subtract the intersection.

Question 5: (Integration)

The question was very well done with most students integrating correctly and finding the constant. Arithmetic mistakes were made in some cases.

Question 6: (Vectors)

On the whole, part (a) was done quite well. A few did not realize that the scalar product must be set equal to zero, while others made arithmetic mistakes. Part (b) was handled well by most candidates.

Question 7: (Matrices)

This question was well done by most candidates. The most common error occurred in part (b) where the value of x was not checked in both equations.

Question 8: (Normal distribution)

This question identified the students that have an understanding of the normal distribution and know how to handle it on their GDC. Those that did not, tried various other methods unsuccessfully or left it blank. Part (c) was done well by most all candidates. A few shaded a region centred on the mean.

Question 9: (Acceleration, velocity, and displacement)

In part (a) quite a few candidates simply evaluated the function at $t=1$ rather than differentiating. Those who were successful in part (b) found t by using logs or by solving on the GDC. In part (c) many candidates tried to find the distance using the formula $d=r\times t$ rather than integrating. Others knew they should integrate but did it incorrectly.

Question 10: (Box and whisker plot)

Many candidates answered this question correctly. Some had only the endpoints correct. It was quite obvious which candidates were not familiar with the definition of the interquartile range.

Question 11: (Area of a sector)

Overall, this question was quite well done. Candidates seem to connect the perimeter with $r+r+\text{arc length}=20$ and therefore could show the given statement. Substituting in the expression and solving for r was handled quite nicely by most students.

Question 12: (Quadratic function)

Surprisingly, few students used the discriminant to find the possible values of q. Some did successfully factorize. In part (b) some candidates did not use the greater value of q, which caused them to get a negative answer. Many candidates were successful with part (c) regardless of parts (a) and (b).

Question 13: (Natural log function)

Many candidates correctly found the answers to parts (a), (bi), and (c). The difficulty arose with part (bii) in finding the range of f. In most cases the GDC was used successfully in this problem, particularly in part (c).

Question 14: (Transformations)

Most candidates handled this question with ease. The errors occurred when the endpoints of $g(x)$ went beyond the domain. Parts (b) and (c) were well done.

Question 15: (Trigonometric differentiation)

The need to use trigonometric identities was realized by most candidates. Many wrote equivalent statements but then did not differentiate. Marks were lost due to the failure to show all steps in the "show that" part of the question. Part (b) was answered by only a few candidates.

Paper 2

Question 1: Triangle trigonometry

This question was generally very well answered. A few candidates would have been better served spending an extra minute or two looking for the most efficient solution method, as some of the more convoluted techniques ended up costing valuable time later in the examination. More candidates used the cosine rule than Pythagoras' Theorem to find AC even though they recognised angle ABC was $90°$.

In part (c) very few candidates realised that there was a second possible value for angle DBC, nearly all of these immediately rejected the obtuse angle. Candidates were able to gain full marks whichever angle they worked with (or even for working through with both as was strictly correct).

Question 2: Mean of grouped data, problem solving

In part (a) some candidates did not show enough work. In a "show that" question, more working must be shown than might be in a regular question. Good progress was often made on parts (b) and (c). Candidates who could not set up equations sometimes resorted to trial and improvement. Their solution techniques were usually poorly documented.

Question 3: Vectors

More candidates made good progress on this 3-D vector question than in the past. In part (b), some candidates tried to use the equation that they were trying to prove; in a "show that" question, working backwards in this way is not acceptable. A number of candidates left their answers to part (c)(i) as $t = 8$, instead of giving the time when the airplanes meet.

In part (d) about half the candidates chose the correct direction vectors with some working with $8\boldsymbol{d}_1$ and $8\boldsymbol{d}_2$. Many candidates who chose incorrect vectors did not identify them clearly. The responses illustrate that many candidates do not know which part of the vector equation represents the direction and that many do not know the difference between a position vector and a direction vector.

Question 4: Probability

More candidates seemed confident attempting this probability question than in the past. Not many used a Venn diagram, which could have helped to clarify their thinking. Many more candidates than last year used the definition of independence somewhere in their working. Many candidates were unable to process the information in part (d) into an equation.

Question 5: Functions and calculus

Many candidates made good progress on this question. Some tried to use an antiderivative to find the value of the definite integral in part (b) which led to a lot of working with no marks. A GDC evaluation was expected. A number of candidates felt comfortable skipping some parts of the question while still working on later parts. In particular, a good number of candidates correctly set up an integral equation in part (d).

It is surprising that many who correctly used the Fundamental Theorem of Calculus and worked through part (d) to give a quadratic equation then made errors in solving it.

Recommendations and guidance for the teaching of future candidates

- Teachers should check the information given in the subject guide and make sure candidates are aware of the meaning of the command terms and notation that may be used in questions.
- Centres need to be sure they cover the whole syllabus.
- Give candidates practice in knowing when it is appropriate to use the GDC and when analytic approaches are called for. In particular, A GDC approach may be the best or even the only way of evaluating some definite integrals and finding solutions of some equations. If a graph is used, a sketch of the graph must be included as is clearly stated in the exam booklet itself.
- When working with the calculator, candidates should be careful to carry through more than three significant figures in their working and only round required answers.
- Calculator syntax and notation should not be used. Candidates must show their set-up in mathematical notation.
- Candidates need more practice in integration and differentiation of more difficult functions and their use in problem solving.

- Much more work needs to be done on probability, especially in recognizing and analyzing normal probability situations.
- Give students practice in **showing that** certain results are true. Each step of working/reasoning must be clearly shown. Generally, these types of questions are not to be done with the graphing calculator. It is also important that candidates do not work in reverse and simply verify that the answer is correct.
- Give students practice in giving explanations for results and be tough in the marking of such explanations, demanding accuracy and clarity.