MATHS SL TIME ZONE 2

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0-16	17-31	32-45	46-57	58-69	70-82	83-100

This was the first session of the new course for mathematics SL. In general students seemed to be well prepared. However, as detailed below, there appeared to be some new areas of the syllabus with which candidates from some centres were unfamiliar. Details are also given below of ways in which the new requirements for Internal Assessment had not been fully implemented. Teachers should be sure they are working from the subject guide for mathematics SL, for first examinations in 2006, that was sent to schools in 2004.

All teachers are encouraged to complete G2 examination feedback forms. These are all read by the senior examining team at the Grade Award meeting and consideration is given to issues raised. G2 forms are available from your IB diploma co-ordinator or online on the OCC.

In response to some comments made on G2 forms in this session teachers are asked to note the following points:

- A standard level course can have a maximum of three hours external assessment thus increasing paper one to 1.5 hours means paper two now has to be 1.5 hours.
- The suggested teaching hours provided in the subject guide will not necessarily be reflected in the number of marks allocated to a particular topic in a particular session.
- Candidates should be familiar with the notation and the command terms detailed in the subject guide. These will be used in examinations without explanation.
- For paper one, the change in format from boxes with answer spaces to lines is intended to reflect the change in the assessment model. Correct answers with no working may not necessarily receive full marks. Therefore, the answer space has been removed to try to help candidates and to encourage them to show their working in a clear and organized way. Final answers should be written in the lined section and not against the question at the top.

Finally a couple of things that would make things easier for examiners.

- Students are required to write their answers in pen. If pencil is used it can be very difficult to read under artificial light.
- Please do not ask students to double over the green tags. It makes it extremely difficult to open out the papers for marking.

Standard level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	1-7	8-13	14-19	20-23	24-28	29-33	34-40

The implementation of the new syllabus has presented challenges to students, teachers and moderators. New tasks have had to be adopted or created with a new assessment rubric in mind. Teachers have been faced with learning the nuances of the assessment criteria, and transmitting them successfully to students. Moderators have had to deal with schools that have presented old material that is no longer appropriate, material assessed against the old criteria, and even forms that no longer apply. Despite all this, most schools have successfully made the transition to the new assessment scheme, and it is hoped that the feedback provided to all schools will ensure that the Internal Assessment in future sessions is more consistently and successfully implemented.

The range and suitability of the work submitted

Moderators have noted that most schools chose to offer tasks selected from the new teacher support material document (TSM). While this is certainly a wise choice at this point, it is hoped that teachers will feel more confident in setting tasks of their own design in future. Teachers who have bravely offered their own tasks will be able to use the information provided in feedback to the schools, and information contained here, to confirm or revise their tasks. Those teachers who design appropriate tasks or who modify TSM tasks are encouraged to share them through the Online Curriculum Centre (OCC), so that others can offer constructive feedback, or make use of them in their classes. This professional cooperation is much appreciated.

A concern that arose regarding the choice of tasks was that some tasks found in various resources, some old TSM tasks and some new ones specifically written with the IB Mathematics SL syllabus in mind, did not adequately meet the requirements of the tasks as described in the subject guide. Particularly, they did not offer students full opportunity to achieve well at every criterion. It is critical that teachers work through any task they intend to set and assess their work against all the criteria prior to assigning it to students, to ensure that their students can address each of the criteria levels. Otherwise students may be unintentionally penalized, as they might not be able attain the highest levels simply because the task does not provide for this.

Candidate performance against each criterion

As some of the old criteria (new criteria A, B, and E) have been maintained in the new rubric, teachers have been able to use their experience with these criteria to ably assess student work in the areas of Use of Notation/Terminology, Communication, and Use of Technology. Moderators have generally been able to confirm marks in these criteria wherever supporting comments have justified the assessment. Criterion E, Use of Technology, is now assessed for both tasks, and consequently takes on greater significance in the overall mark. Teachers are advised to plan for the appropriate accommodation of technology in the tasks they use. They should especially consider how students can give evidence of the technology used, and how resourceful the use has been to the development and enhancement of the work presented. The presence of printed output does not in itself constitute resourceful use.

The greatest concerns arose in the assessment under criteria C and D. The new rubric assesses two major goals through these criteria; Processes and Results. However, the objectives of these goals differ according to the nature of the task, and thus criteria C and D have different assessment descriptors for investigative tasks (Type I) than for modelling tasks (Type II).

Type I tasks are intended to assess the students' abilities to work with mathematical patterns in numbers, expressions, shapes, etc. and to then generalize these patterns into a suitable mathematical statement. Aspects such as the validation of preliminary conjectures, exploration of scope and limitations of the variables, and some informal explanation as to why the statement is valid are also addressed.

Type II tasks are intended to assess the students' abilities to analyze raw data to develop a model function, consider how well the model fits the data and modify it as appropriate, show how it can be applied to other situations, and to critically interpret in context how reasonable the model is, what limitations apply, and what modifications might be necessary to improve the model. It is critically important that students explicitly identify the variables, parameters, and constraints used in the model. Students need to know such things before they work at the tasks, and therefore it is essential that teachers share and discuss the criteria with them. In particular it should be noted that an analytical approach that demonstrates the student's own knowledge of the mathematics involved must be used to develop the model prior to any use of regression features on a GDC or computer. Regression models are acceptable for comparison to the student-generated models, but are not mandatory. They do, however, provide a good opportunity for students to demonstrate their command of the appropriate technology.

A new criterion, F, offers the teacher an opportunity to assess holistically the quality of work presented. While there is no explicit link between performance on the other criteria and the mark awarded in criterion F, it is expected that only remarkable work, work that the teacher would stop and take admirable note of, should attain a mark of 2. On the other hand, it is expected that only a totally inadequate response would receive a mark of 0. It is expected that most assignments will achieve level 1.

Recommendations for the teaching of future candidates

Teachers are reminded that the subject guide and the TSM include specific instructions regarding the assessment of portfolios, including notes on the criteria to help explain their application. Note that, while teachers may offer advice to students as to whether they are on the right track with their work, they must not assess rough drafts and return them to students to modify prior to final submission.

Included below is a set of further notes that senior moderators have prepared to assist teachers in understanding the nuances of the criteria. A document containing fully commented criteria incorporating these further notes will be posted on the OCC. It is essential that teachers study the assessment of student work provided in the TSM. It is strongly recommended that teachers attend IBO teacher training workshops for further professional development.

Additional notes on applying the criteria

Criterion A: use of notation and terminology

Correct mathematical notation and symbols must be used eg π, rather than the word "pi". Calculator or computer notation should not be used. Notation such as ABS(x), 5.23E17, * etc, should not be used and such use will be penalised.

A single shortcoming would not preclude the awarding of level 2.

The terminology may depend on the task. In the case of Type I (Investigation) activities, terminology may include terms devised by the candidate (eg "slide", "shift", etc), provided that such terms reasonably reflect the appropriate mathematical concept.

Criterion B: communication

The "WHOA!" factor: If, in reading a candidate's work, the teacher has to pause to clarify where a result came from or how it was achieved ("WHOA! Where did that come from?!"), this generally indicates flawed communication.

Computer/calculator output may need clarification. Graphs generated by calculator or computer should present the variables and labels appropriate to the task. Hand-written labels may need to be added to screen dumps or printouts if the software doesn't provide for custom labels.

A single shortcoming would not preclude the awarding of level 3.

A "question and answer" format in the student's work does not represent the best form of mathematical communication, and the use of this format will likely preclude awarding a level 3.

Type I Criterion C: mathematical process

This criterion refers to the process of getting ready to produce the general statement. A student can achieve a 4 if everything is ready to produce the statement. (The statement does not need to be seen at this point.) The production of the statement and its correctness are assessed in D.

Testing further cases and commenting on the results is sufficient to award a level 5. "Tests the validity" does include commenting on the results of their testing. This applies to the general statement produced by the student, regardless of its correctness.

If a student gives a proof or justification of the correct statement, no further cases need be investigated to award a level 5.

Type I Criterion D: results

It is important to note the difference between "a (ie any) general statement" in level 2 and "the general statement" in level 3.

Type II Criterion C: mathematical process

Any form of definition, informal or implied, of variables, parameters, constraints, is acceptable eg labelling a graph or table, noting domain and range.

A qualitative analysis is sufficient to award a level 4.

In the development of the model, it was intended that students initially use an analytic approach, and use the regression tool (and possibly their knowledge of regression) to support their findings.

Type II Criterion D: results

"Appropriate degree of accuracy" means appropriate in the context of the task. It may be interpreted in terms of the level of reasonableness expected to earn a level 3, 4 or 5. A minor error in accuracy (eg using 10 sf instead of 2 or 3) might not prevent a student progressing from level 3 to level 4, but could preclude them from progressing from level 4 to level 5.

Criterion E: use of technology

While printed output is not required, some statement confirming appropriate use of technology (from the teacher or student) is necessary to achieve level 3.

Note that using a computer and/or a GDC to generate graphs or tables may not significantly contribute to the development of the task, and so may not merit awarding a level 3.

The emphasis in this criterion is on the contribution of the technology to the mathematical development of the task rather than to the presentation/communication.

Criterion F: quality of work

Award level 2 only if the work presented is beyond ordinary expectations. The teacher will take pause to admire the quality of such work ("Wow! Now, that's impressive!").

Only a totally inadequate response would receive 0.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-15	16-30	31-40	41-51	52-62	63-73	74-90

General comments

G2 summaries

Comparison with last year's paper

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
1	9	37	30	5

- **Suitability of question paper:**

	Too easy	Appropriate	Too Difficult
Level of difficulty	1	130	15
	Poor	**Satisfactory**	**Good**
Syllabus coverage	6	57	83
Clarity of wording	4	57	93
Presentation of paper	3	58	107

The areas of the programme and examination that appeared difficult for the candidates

- There are still problems with the understanding of the concepts of logarithms.
- Many candidates had difficulty interpreting the sign of the first and second derivatives from the graph of a function.
- Probability still causes lots of difficulties with many candidates.
- Many do not know how to "justify their answer". Candidates are inclined to make claims without seeing the necessity to back the claims up with numerical facts.
- Dealing with trig functions and their derivatives is a problem for many.

The areas of the programme and examination in which candidates appeared well prepared

- There were a few problems with the accuracy issue.
- Many candidates now have a good understanding of the potential for their graphic display calculator (GDC), and used them appropriately, although there can be a tendency to over rely on the GDC.
- There is an improvement in the knowledge of vectors (although the question is an easy one).

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1 (Geometric series)

This question was very well done. A few candidates found the sum of the first fifteen terms rather than the value of the fifteenth term in part (b).

Question 2 (Vectors)

This question was quite well done with only a few candidates considering using a Cartesian method and failing to give the answer in vector form.

Question 3 (Mode, Median and Mean)

This question tested the basic understanding of this topic and was quite well done.

Question 4 (Logarithms)

Part (a) was not well done, while part (b), which required the use of logs to solve the equation was well done. It suggests that while many candidates are quite happy to use logarithms in the solution of problems they don’t necessarily have a clear understanding of the underlying concept.

Question 5 (Discrete random variables)

While most candidates could do well in part (a) there were many who could not do part (b). Too many wanted to divide $\mathrm{E}(X)$ by 5 (or 15), thus showing a lack of understanding of the concept of expected value.

Question 6 ($f'(x)$ and $f''(x)$ from a graph)

Many candidates could identify the sign of the first derivative but could not do so for the second. Understanding of the link between the behaviour of the graph and the sign of the second derivative is weak.

Question 7 (The quadratic function)

This question was quite well done. Many candidates realised they had to complete the square and made attempts (not always successful). Many got the sign in part (b)(ii) wrong.

Question 8 (Sectors and Segments of circles)

This question was quite well done with most candidates at least writing down the two equations for the first two A marks. Some chose a difficult way to eliminate one of the variables and got “bogged down” in algebra. A few made a simple mistake early on.

Question 9 (Volumes of solids of revolution)

This is a standard question on this new topic. Most candidates were able to write down the expression so gained three marks. . Not many used their GDCs for the final part. They were then defeated by the algebra and there were only a few fully correct solutions

Question 10 (3x3 Matrices)

This question was well done with most candidates being able to use their GDCs to find the inverse of a 3x3 matrix. Many candidates wrote the matrices round the wrong way in part (b) but went on to get part (c) correct, indicating that the noncommutative property of matrix multiplication is not widely understood.

Question 11 (Functions and their tangents)

Most got part (a) correct but were confused in part (b) as to how to proceed. Many candidates thought that the derivative of $f(x)$ equalled the equation of the tangent. However they often then went on to gain follow through marks for correct working in part (c).

Question 12 (Probability)

This question was poorly done. A solution to the question could be simplified by drawing a Venn diagram but even candidates who drew one were not able to use it to solve the question. Very few got part (c) correct . Not many candidates saw the need to back up their arguments in part (c) with appropriate calculations.

Question 13 (The product rule)

For a later question this wasn't too badly done. Many got part (a) correct but many did not give exact answers.

Question 14 (Trig functions and their derivatives)

Hardly anyone got part (a) correct, in spite of the answer being immediately available from the GDC. Some successfully used the chain rule to differentiate the function for part (b) but there were a few who didn't realise that the second derivative of s is the acceleration. Very few realised the GDC or the derivative of the acceleration was needed for part (c).

Question 15 (Interpretation of statistical diagrams)

This question was well done. Candidates showed a good understanding of the concepts. A number of weaker candidates seemed to perform quite well with this question.

Recommendations and guidance for the teaching of future candidates

- Obviously, addressing those areas identified as weaknesses is one recommendation.
- Probability as always, seems an area requiring special focus.
- The fact that some candidates could not successfully "complete the square" and interpret the resulting expression with the transformation of the basic quadratic function suggests the need for more emphasis on non-GDC solutions to problems.
- Teaching recognition of when the GDC approach is best also needs attention.
- The significance of the term "**exact**" needs to be emphasised and the concept of the difference between "exact" and "approximate" must be taught. Many candidates appear to think that exact is the answer they get on their calculator display!
- Make the candidates aware of the need to show working and method, as a correct answer without working no longer automatically gains full credit.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-13	14-26	27-41	42-52	53-63	64-74	75-90

General comments

G2 summaries

Comparison with last year's paper

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
2	20	44	14	2

Suitability of question paper:

	Too easy	Appropriate	Too Difficult
Level of difficulty	4	131	4
	Poor	Satisfactory	Good
Syllabus coverage	4	61	76
Clarity of wording	0	50	91
Presentation of paper	1	38	103

The areas of the programme and examination that appeared difficult for the candidates

The biggest area of difficulty for most candidates was probability and statistics, and more specifically, conditional probability and normal distribution calculations. A more general area of weakness was **showing** a result to be valid. The **meaning** of the scalar product of two vectors also challenged many candidates.

The areas of the programme and examination in which candidates appeared well prepared

Candidates handled 2 x 2 matrix algebra and arithmetic series calculations very well and seemed to have a good understanding of inverse and composite functions, and of basic differentiation principles. Apart from the scalar product difficulties, many candidates showed a fair level of competence with 3-dimensional vectors.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1(Matrices and Arithmetic Series)

In general, this question was done very well. The most common error was failing to **show** why the product of two matrices had a particular value. Doing the calculation by GDC is not sufficient. The way matrix multiplication works needs to be demonstrated. A different kind of problem for some candidates arose from not reading the question sufficiently carefully, resulting in calculation of the n^{th} term instead of the sum of the first n terms. Many candidates correctly spotted the patterns being generated and were able to write down the final answers without difficulty.

Question 2 (Inverse and Composite Functions, Graphing and Integration of Rational Functions)

The majority of candidates displayed a good understanding of inverse and composite functions and scored well in the early part of this question. A few candidates either did not know what composition meant, or mistook the inverse for the derivative. Some excellent graphs of the rational function were produced, though some clearly put in more time and effort than a simple **sketch** required. Common failings were lack of asymptotes, or branches which curved away from the asymptotes. It should be stressed that most GDCs, in their standard graphing mode, will show vertical asymptotes as part of the graph. Though this was not a major problem in this question, it is important to understand that the vertical asymptote is not part of the graph. It is also important to stress that the **equation** of an asymptote should be an equation, not $x \neq 2$ or simply 2 as was often seen. Many candidates could not integrate $\frac{1}{x-2}$, and those candidates who could did not always understand the request for an **exact** answer. Though a strictly correct answer for the antiderivative would involve absolute value signs, normal parentheses are acceptable for mathematics SL. Most candidates could shade the area represented by the integral, but a significant number just shaded the area between the curve and the horizontal asymptote.

Question 3 (Maxima, Trigonometric Identities, Properties of Triangles)

This question directed candidates through a proof that the triangle of maximum area with one side and the perimeter given was isosceles. Most candidates were successful by one means or another in finding the maximum value of the quadratic expression and the value of x at which that maximum value occurred. Candidates who used their GDC to find these values often earned only 2 of the 3 marks by failing to sketch the graph on which their conclusions were based. Most candidates could apply the cosine rule to the circumstances of the question, though a number had difficulty expressing z in terms of x. Greater difficulty was encountered in showing how the expression for $\cos Z$ could be obtained, but many were successful in this regard, and most candidates were able to use the area formula involving $\sin Z$ to show how the expression for the square of the area was obtained. The part causing the greatest difficulty involved using the identity $\sin^2 x + \cos^2 x = 1$ to obtain a quadratic expression for the square of the area. Few candidates made the connection between the final expression for the square of the area, and the work done in the first part of the question.

Question 4 (Normal Probability, Conditional Probability)

This question was poorly done by most candidates. The work on normal probability seemed unknown to many. The newness of this topic to the syllabus was no doubt a major contributing factor to this situation. Parts (a) and (b) were very basic applications of normal probability and a reasonable number of correct solutions were seen, GDC based calculations being very common. Part (c) was also a basic normal probability calculation, but somewhat trickier to set up, owing to the necessity to recognize that the middle 90% involves finding z values for 5% and 95%. Many found the values for 10% and 90%. Parts (d) and (e) represented a classic conditional probability question, with one input based on the normal probability calculation in part (a). It would appear that this connection intimidated some candidates who probably were quite familiar with conditional probability. For those candidates attempting parts (d) and (e), a common error was to simply add the probability of a girl being taller than 170 cm and a boy being taller than 170 cm without taking into account the probability of 0.6 of being a girl and 0.4 of being a boy. Unfortunately, many of the candidates who successfully answered part (d) were not able to use the results of those calculations to answer part (e). In marking this question considerable tolerance was allowed for the range of answers resulting from premature or incorrect rounding of preliminary answers. If the final answer is to be accurate to 3 significant figures, then the values of any preliminary results when used in further calculations should be accurate to at least 4 significant figures. Skilled calculator use should not require any preliminary rounding at all.

Question 5 (Three dimensional vectors)

Candidates did surprisingly well on this question, given past difficulty with two dimensional vectors and the newness of three dimensional vectors to the syllabus. Most candidates could find the vector joining two points and correctly calculate its magnitude. Slightly fewer candidates could calculate the various scalar products, the biggest problem for many of them being writing the scalar product itself as a vector. This error not only prevented candidates from showing the scalar product is zero in each case, but also showed a very serious misunderstanding of what the term “scalar” in scalar product means. Those candidates who calculated the scalar products correctly had no difficulty in concluding the angles between adjacent edges were all 90º. Many candidates obtained the volume correctly, even if they missed the scalar product work. Parts (d) and (e) were answered correctly by significantly fewer candidates, but many still did quite well with finding the coordinates of H, and a considerable number of candidates were successful in choosing appropriate vectors for the diagonals and using the scalar product to find the angle between these vectors.

Recommendations and guidance for the teaching of future candidates

The most important thing teachers can do is to make sure the entire syllabus has been covered. It seemed apparent that at a number of schools, insufficient time had been devoted to some of the topics new to this syllabus, most specifically probability and statistics. Probability and statistics represent 30 of the 140 hours or over 21% of the syllabus, a proportion exceeded only by calculus at 26%. The challenges that teachers face in giving this much attention to this topic are recognized, but these challenges must be met if students are to do well in the course. A more general area of weakness which could benefit from greater emphasis is the “show that” type of question. Providing practice with this type of problem on tests and assignments is strongly recommended. A further point that should be emphasized to students is the necessity to support work done with graphic display calculators, particularly when graphs are involved, by sketching those graphs as part of their solutions. A simple sketch without use of graph paper is all that is required, as per instructions at the beginning of the question paper.

