November 2016 subject reports

Mathematics HL

Overall grade boundaries

Discrete

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 41	42 - 54	55 - 66	67 - 78	79 - 100

Calculus

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 30	31 - 42	43 - 54	55 - 67	68 - 79	80 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 41	42 - 54	55 - 67	68 - 78	79 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 41	42 - 54	55 - 67	68 - 79	80 - 100

Higher level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7

Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 16	17 - 20

The range and suitability of the work submitted

Candidates chose a good variety of interesting topics in this session. Problems occur when weaker candidates choose topics that are well beyond their ability to understand the mathematics involved. A number of explorations were purely descriptive research reports, which often dealt with topics beyond the syllabus that were poorly understood. It is important that candidates are told that they need to think independently and creatively and present mathematical ideas from their own perspective. Some topics that recur include the SIRS model and RSA encryption. Both these are at a level that is difficult for the average IB HL candidate to understand and although they are often applied to specific examples this is usually done in a superficial way. Some explorations still present statements, formulae, and images simply taken from online sources. The worst explorations came without citations at the point of reference. Teachers should inform students about the requirement for citations; otherwise, too many students might risk a malpractice decision.

With on screen marking some explorations were difficult to read. Teachers should annotate student work with a red pen, and the explorations should be scanned in colour for uploading. Students should also be aware that shading in pencil is often not picked up in a scan and this makes their work very difficult to moderate. Students and teachers could check scans before uploading to avoid this.

Candidate performance against each criterion

Criteria A

The majority of explorations seen were well organized with introduction, rationale, a body and a conclusion. Coherence was more variable. Some schools presented explorations with tables of content and a word count. Neither of these are appropriate for an Exploration. Some explorations often contained gaps in the explanations or were poorly expressed rendering the work incoherent. This could also indicate a lack of understanding (potentially penalized under E).

Criteria B

Most candidates were careful to define variables and key terms and most graphs were appropriately labelled

Criteria C

Personal engagement is frequently at a low level in the explorations that are mere “research reports”. There were many explorations where the topics (e.g. partial derivatives, multiplication of matrices, etc…) were introduced in precisely the manner they often are in textbooks. Such explanations did not show evidence of any personal engagement, merely transcribed presentations of other people's work.

Criteria D

Good reflection is usually seen throughout the exploration and drives its development. Much of the reflection seen was superficial and confined to a conclusion at the end of the work. Students should be encouraged to reflect on their work frequently and to report on their emergent thinking and how this thinking led them to the next part of their exploration.

Criteria E

The mathematical content varied greatly, from very basic to extensions well beyond the HL core. The choice of topic is key to performing well in this criterion. Teachers need to provide proper guidance to allow their students to choose topics that they can do justice to. Those who choose something too simplistic (e.g. substituting values for variables in a given equation) or too advanced will invariably fail to obtain a good score here as there is little or no evidence of understanding.

Recommendations for the teaching of future candidates

Teachers should ensure that the exploration is done after a reasonable amount of HL topics are covered. At the time of choosing topics, teachers need to be fully involved in guiding their students to choose topics that are appropriate for their exploration, allowing them to achieve appropriate levels in each of the criteria and avoiding pitfalls. Once the exploration is submitted the teacher must show evidence of marking with annotations and comments on the student work. Evidence that the mathematics has been checked for errors should also be present. This can take the form of tick marks next to the work.

Further comments

Marking explorations, appropriate annotations on students' written responses and processes to help make a good choice of topic need to remain an important part of workshops. It remains important that teachers are able to effectively communicate their reasoning behind their choice of levels of the criteria for moderators to be able to confirm their marking.

Higher level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 48	49 - 64	65 - 81	82 - 97	98 - 120

The areas of the programme and examination which appeared difficult for the candidates

Candidates showed difficulties in solving exponential and trigonometric equations, manipulating complex numbers, performing proofs, dealing with "show that" questions, performing algebraic manipulations, dealing with combined questions (eg exponential and quadratics), using of trigonometric identities beyond the most basic and manipulating complement and conditional probabilities.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates were well prepared to deal with basic probability questions and Venn diagrams, answer basic questions involving functions and related graph sketching, deal with implicit differentiation, find tangents to curves, get data from a graph, find stationary points and points of inflexion, perform one-step integration by parts, work with equations of lines and planes using vectors; use relationship between roots and coefficients of a quadratic. In most cases candidates were familiar with the structure of a proof by induction.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This question was very well answered with a few excellent candidates making calculation errors. Several methods were seen: substitution, elimination, row reduction and Cramer's rule – this last one more in Spanish scripts.

Question 2

The majority of the candidates correctly answered this question. A few provided frequencies instead of probabilities or made careless arithmetic errors.

Question 3

This question was well answered in general. The most common errors were the use points estimated from the graph or exchange the values of a and c. Some weak candidates wasted time doing unnecessary calculations, showing difficulties in dealing with simple algebraic equations.

Question 4

Generally this question was well answered although some candidates gained the M but not the A mark in 4(a) due to calculation errors.

Question 5

This question was well answered in general. The most common error was considering $-2k$ as the sum of the roots; otherwise, if attempted, the majority answered this question correctly.

Question 6

This was the most challenging question for candidates. In part (c) in particular many candidates assumed what they wanted to prove: started assuming that the sequence given was arithmetic, and worked with AP formulae. However very strong candidates presented outstanding answers showing clear understanding of the topic. Definitely this question showed the difference between good and excellent candidates.

Question 7

There was a mixed response to this question. When attempted with the correct method, it was often answered correctly (with some students not rejecting the negative answer coming from the quadratic equation) but, surprisingly, in many cases candidates took logarithms on both sides, showing lack of understanding of this part of the syllabus.

Question 8

Generally this question was well answered, although many candidates had often issues in dealing with the algebra which lead to an incorrect answer in part (a), but most of these candidates still benefited from follow through full marks in part (b).

Question 9

(a) Most candidates achieved at least 4 of the 5 marks and very often full marks in this question. Implicit differentiation was used in most cases; occasionally the negative quotient of derivatives was used but with little success. The most common mistake in part (b) was forgetting the solution corresponding to the tangency point with the negative x-coordinate.

Question 10

(a) This question was usually well done, either using Venn diagrams or some variant of method 1. Some candidates attempted the "show that" in (a) and (b) assuming that the events were independent. In part (b)(i) many candidates got into a tangle with many failed attempts to arrive to the value given. Part (b)(ii) was better tackled than part (i) with many candidates arriving to the correct answer.

Question 11

(a), (b) and (d) were almost universally well done. In part (c) however very rarely both marks were awarded for this part. Most candidates either proved it was a stationary point or, assuming it to be a stationary point, showed it had to be a maximum. In part (e) many candidates had difficulties with the concavities or ignored the domain. A significant number of candidates only got the mark for the zeros. Part (f) was very often well done, although some candidates were awarded A0 for the final answer due to arithmetic issues. An alternative method using part (b) was seen in a few cases. In part (g) it was disappointing that many candidates left trigonometric functions unevaluated and/or disregarded the modulus sign. In part (h) the zero curvature was usually found but in most cases an erroneous reason was given or candidates just repeated the answer by saying zero curvature.

Question 12

Most candidates attempted parts (a) and (b) using De Moivre's theorem to solve the equation and giving its three roots, but in general these parts of the question were surprisingly poorly answered. Weaker candidates often could not even start off due to poor understanding of the basic complex number definitions required to be set out. Unfortunately often parts (c) and (d) were not attempted or included mistakes that showed lack of understanding of the algebra involved. However parts (c) and (d) were well handled by candidates with good algebraic skills.

Question 13

Parts (a) and (b) were well answered in general. Students showed awareness of the induction proof structure but very often not being able to prove the case $n = k + 1$ using the assumption that case $n = k$ true. Part (d) was difficult for many candidates. A common mistake, if attempted using a valid method, was not factoring but simplifying before giving the solutions. Often candidates ignored the domain given and gave answers containing extraneous solutions.

Recommendations and guidance for the teaching of future candidates

- Emphasize the importance of reading the question carefully (e.g. notice plurals indicating two answers are needed, or read if a particular method is to be used) and not assuming what is not given in the question.
- Provide as much practice in algebraic manipulation as possible as part of the previous knowledge and during the course.
- Provide more practice in "show that" or "prove" questions.
- Remind students on the importance of understanding how the M, A and R marks work. Recommend them to show all their work and explain clearly in their response whatever required.
- Remind students that Section B should be answered on lined paper, not on graph paper and that all the work needs to be written in legible way. Insist on the importance of good presentation skills and organized work.
- Discourage students from accompanying their mathematical procedures with detailed (unnecessary) explanations of the steps carried out.

Higher level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 18	19 - 37	38 - 49	50 - 63	64 - 76	77 - 90	91 - 120

General comments

A good number of candidates found this paper reasonably accessible with a pleasing number submitting excellent scripts characterized by correct work, logical reasoning and argumentation and judicious use of a GDC. However, a recurring concern in HL Mathematics Paper 2 is the number of candidates who have their GDC set to degrees mode. This was obvious in questions involving trigonometric functions and applications of trigonometry. It was disappointing to note that a few candidates seemingly ran out of time early in Section B after completing an excellent Section A. It must be emphasized how important it is for teachers and students to have a very good appreciation of when to use a GDC and when to adopt a by-hand approach to a solution.

The areas of the programme and examination which appeared difficult for the candidates

- A lack of awareness that inbuilt GDC functionality can be used to perform routine calculations involving a discrete probability distribution.
- A lack of awareness of the need to have a GDC set in radians mode.
- Sketching a graph over a required domain.
- Solving an inequality involving a modulus function.
- Flexibility in the use of the sine and cosine rules.
- Interpreting and calculating a probability interval such as $\mathrm{P}\left(\left|X-\mu\right|<1.2\sigma\right)$.
- Use of a combination of sector and segment formulae to formulate an expression for the shaded area of two overlapping circles.
- Using alternative reasoning to justify that a function has an inverse.
- Finding a volume of revolution about the y-axis.
- Showing that a discrete probability distribution has two modes.
- Finding the minimum number of trials in a situation described by a binomial probability distribution.
- Calculations involving financial applications of geometric sequences and series.

The areas of the programme and examination in which candidates appeared well prepared

Finding the acute angle between two planes.

- Deriving a well-known result linking two consecutive Poisson probabilities and solving a related equation to find the value of μ.

- Finding a (constant) term in a binomial expansion.
- Finding the mean and standard deviation of a normal distribution.
- Finding a derivative using the quotient (product) rule.
- Finding an expression for the inverse of a function.
- Solving systems of two linear equations.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Generally well done although the overwhelming majority of candidates performed lengthy by-hand calculations rather than using inbuilt GDC functionality. In part (a), a number of candidates attempted to use $\sum x\left(\mathrm{P}\left(X = x\right)\right)^2$ rather than $\sum x^2\mathrm{P}\left(X = x\right)$ to calculate $\mathrm{E}\left(X^2\right)$. In part (b), a number of candidates stated $\mathrm{Var}\left(X\right) = \mathrm{E}\left(X^2\right) - \left(\mathrm{E}\left(X\right)\right)^2$ but used $\mathrm{Var}\left(X\right) = \mathrm{E}\left(X^2\right) - \mathrm{E}\left(X\right)$ instead. A few candidates determined that $\mathrm{Var}\left(X\right) < 0$.

Question 2

Generally well done. Some candidates made careless arithmetic mistakes when calculating the magnitude of a normal vector. A few candidates gave $15.0°$ (or equivalent) as their final answer. Some candidates successfully used the vector product approach.

Question 3

Despite some candidates having difficulty setting their work out in a clear and coherent manner, part (a) was generally well done. A large number of candidates were able to show the required result and then use it to find the value of μ in part (b). A few candidates did not use the hint from part (a) while a few others used an incorrect value of x.

Question 4

Generally well done. A number of valid methods were observed here including brute force expansion. The more frequently seen successful methods involved use of the general binomial expansion term ($T_{r+1} = \binom{12}{r}\left(4x^2\right)^{12-r}\left(-\frac{3}{2x}\right)^r$ and so $24 - 3r = 0 \Rightarrow r = 8$), noting the behavior of the powers of x or determining that $2n = 12 - n \Rightarrow n = 4$. A few candidates unfortunately neglected the binomial coefficient when attempting to form the required product. The occasional candidate gave $-3\ 247\ 695$ as their answer.

Question 5

In part (a), a good number of candidates produced a neat sketch over the correct domain and clearly indicated the location of the horizontal axis intercepts and the coordinates of the local maximum. However, a number of candidates attempted this question with their GDC set in degrees mode. A few candidates did not indicate the correct domain. Other errors included indicating that $x = -1$ was a vertical asymptote and sketching a graph that did not reach $(1,0)$

. In part (b), a number of candidates stated that the range was $y \le 1.68$ (or equivalent). Only a small number of candidates offered a fully correct solution to part (c). A number of candidates stated that $x < -0.189$ rather than $-1 \le x < -0.189$. A number of candidates attempted to solve $3x\arccos(x) > 1$ for x.

Question 6

This question challenged most of the cohort but was generally well attempted. A number of candidates determined that $\frac{dy}{dx} = \frac{1-145x}{143y}$ (or equivalent) rather than $\frac{dy}{dx} = -\frac{145x}{143y}$. Most errors were numerical, either in the determination of y or in the final calculation of $\frac{dy}{dt}$. Rather than give $\pm 2.13 \times 10^{-6}$ as their final answer, a substantial number of candidates gave either 2.13×10^{-6} or -2.13×10^{-6}.

Question 7

A number of candidates unfortunately attempted this question with their GDC set in degrees mode. Interestingly, a few candidates switched from radians mode to degrees mode when attempting part (b).

Part (a) was framed to be a simple application of the cosine rule and use of a GDC to solve $AC^2 - 8\cos\left(\frac{\pi}{9}\right)AC + 7 = 0$ (or equivalent). A number of candidates saw what was required and readily obtained 1.09 and 6.43. A large number of candidates ignored the instruction initially and applied the sine rule to find unknown angles. Some of these candidates eventually used the cosine rule to find the correct values for AC. Most who adopted this inefficient approach found only one correct value for AC.

Part (b) was reasonably well done. After making an error in part (a), a number of candidates were awarded full follow through marks. A few candidates found the area of each triangle but did not calculate the difference in areas.

Question 8

Reasonably well done. A number of successful candidates adopted inefficient solution approaches and did not use their GDC to its fullest capacity. A few candidates did not use $P(X > 42.52) = 1 - P(X < 42.52)$ and a number did not apply the inverse normal when setting up their linear equations in μ and σ.

Part (b) was not well done. The majority of candidates did not see that $P(|X - \mu| < 1.2\sigma) = P(\mu - 1.2\sigma < X < \mu + 1.2\sigma)$. A common error was to calculate $P(X < \mu + 1.2\sigma) = 0.885$. Only a very small number of candidates realized that

$\mathrm{P}(-1.2 < Z < 1.2) = 0.770$. Of course it was possible for candidates to use their values for μ and σ to obtain this answer.

Question 9

In part (a), a reasonable number of candidates were able to develop an expression for the area of the two overlapping circles. A large number of candidates did not attempt part (b) and only a small number of candidates were able to show that $\alpha = \arcsin\left(\frac{1}{4}\right)$. One successful approach involved obtaining $\sin\frac{\alpha}{4} = \frac{1}{4}$ by considering triangle ADM where M is the midpoint of BD. Another approach involved using the cosine rule in triangle ADB to obtain $1 - \cos\frac{\alpha}{2} = \frac{1}{8}$ and then obtaining $\sin\frac{\alpha}{4} = \frac{1}{4}$ from $\sin\frac{\alpha}{4} = \sqrt{\frac{1-\cos\frac{\alpha}{2}}{2}}$. In part (c), most candidates were unable to determine that $\theta = \pi - \frac{\alpha}{2}$. Hence only a small number were able to obtain $r = 1.69$.

Question 10

The most accessible parts of this question were parts (c) and (e). The large majority of candidates were able to successfully use the quotient (product) rule in part (c) and find a correct expression for $f^{-1}(x)$ in part (e).

In parts (a) and (b), most candidates were able to determine the largest possible domain of f and state the equation of the vertical asymptote. A good number of candidates were able to determine the equations of the two horizontal asymptotes. In addition to one correct equation, some candidates gave $y = -1$ while others just found one correct horizontal asymptote (most often $y = -2$).

Part (d) proved to be one of the most challenging parts of the paper. Only a small number of candidates were able to justify that f has an inverse because $f'(x) < 0 \Rightarrow f$ is (strictly) decreasing (or alternatively $f'(x) \neq 0 \Rightarrow f$ has no turning points) and that one branch of the graph is above $y = -\frac{1}{2}$ and the other branch is below $y = -2$. Stating the domain of the inverse proved challenging for most. The most common error was to state $x \in \mathbb{R}, x \neq -2, -\frac{1}{2}$ (or equivalent).

Only a small number of candidates were able to calculate the correct volume of solid of revolution about the *y*-axis in part (f). A common error was to attempt to find a volume of solid of revolution about the *x*-axis. Some candidates stated integrals in the form $\pi\int_a^b \left(f(x)\right)^2 \mathrm{d}y$. A few candidates formulated the correct definite integral but made a syntactic error entering $\left(\ln\left(\frac{y+2}{2y+1}\right)\right)^2$ into their GDC.

Question 11

This question was accessible to all candidates with a few different solution pathways available. Parts (a) and (b) were generally well done although fudging was sometimes seen in part (a). The most common error in part (b) was to omit the 0.9 term from the equation $\frac{16+4a+b}{2000}\times 0.9 = 0.0027$.

Part (c) was reasonably well done with most candidates who found $a=-3$ and $b=2$ in part (b) able to show that the required result. Some fudging was seen in this question part.

In part (d), only a few candidates were able to use $\frac{\mathrm{P}(X=n)}{\mathrm{P}(X=n-1)}$ to justify the existence of two modes. The most popular approach was to attempt to justify the existence of two modes by using $\mathrm{P}(X=n)$. A number of candidates attempted to differentiate $\mathrm{P}(X=n)$, ignoring that $n \geq 3, n \in \square$.

In part (e), only a small number of candidates realized that they needed to solve either $\mathrm{P}(Y \geq 3) > 0.5$ where $Y \sim \mathrm{B}(x, 0.1)$ or $\sum_{n=0}^{x}\mathrm{P}(X=n) > 0.5$ for x. Quite a number attempted to solve the inequality $\mathrm{P}(Y \geq 1) > 0.5$.

Question 12

Parts (a) and (b) were reasonably well done although some ‘fudging’ was evident in parts (a)(ii) and (b)(ii). A lack of understanding of how A_2 is formed often led to incorrect expressions for A_3 and A_4 in part (b)(i).

In parts (c) and (d), a few candidates confused months and years while a larger number than anticipated used $n=180$ rather than $n=216$. For a question part this late in the paper, part (d) was reasonably well answered.

Only a few candidates were able to realize that $r=1.004^{12}$ and then go on to set up and solve the required equation in part (e). The majority of successful candidates adopted either a simple iterative approach or a trial and error approach.

Recommendations and guidance for the teaching of future candidates

- Ensure that students have their GDC set in radians mode.
- Use GDC-required worked examples and set GDC-required questions for students to

attempt in class and in tests and assignments.
- Gain more familiarity with the capabilities and functionality of a GDC and model efficient solution strategies when using a GDC.
- Give students plenty of opportunities to attempt questions that either require an explanation of a derived result or are framed as a ‘show that’ question requiring a clear and coherent argument. Discuss with students what constitutes a convincing mathematical argument.
- Model precise mathematical notation, particularly in the expression of domains and ranges and placing an emphasis on the difference between an open and a closed interval.
- Encourage students to draw a sketch diagram when solving normal distribution problems, particularly when absolute values are involved.

Higher level paper three discrete

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 18	19 - 27	28 - 33	34 - 39	40 - 45	46 - 60

General comments

There were candidates who were very well prepared for this paper who scored very highly. There were also a surprising number of candidates who had not really prepared for this Option. This was illustrated by, for example, no knowledge of the hand-shaking lemma, just writing down formula from the formula book that referred to other Options. The trial exam that they should have taken, should have exposed this lack of preparation.

The areas of the programme and examination which appeared difficult for the candidates

The candidates found it difficult with work on graphs and trees when they had to think rather than just apply an algorithm. Knowledge of what constituted a proper proof or “show that” was often lacking.

The areas of the programme and examination in which candidates appeared well prepared

Graph algorithms were known but were sometimes confused. Candidates could convert to a base 10 number. Candidates knew how to form the auxiliary equation for a recurrence relation and solve it.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a)(i) Converting the number to base 10 was done well but a large proportion of candidates did not read that x was the cube root of this number and thus made (ii) harder than it need be. Not all candidates realised that the cube root had to come out to be an integer which was a check on their working. (ii) Not as well done as (i) but reasonable attempts. The repeated division is an easier algorithm.

(b) Not well answered in many cases. The candidates did not naturally go to working modulo 8. There was a tendency to essentially say that 8 divided $9^i - 1$ without explaining why. There were also some very spurious arguments. The “if and only if” part of the question was often ignored.

(c) The instruction “using the method from part (b)” was often ignored.

Question 2

(a), (b) and (c) It was very surprising to discover how many students did not know what the “Handshaking Lemma” was in a Discrete paper. There were too many waffly non-proofs which essentially said “it does not exist because it does not exist”. These candidates sadly probably thought that they had gained some marks. The better candidates had the proper contradictions eg. $5 = 6$.

Question 3

(a) Tended to be all 3 marks gained or none of them (by candidates who did not think clearly about the information that had been given).

(b) It was really shocking to see that the majority of candidates thought that you could prove this general statement by just verifying it in a particular numerical case.

(c) There was good knowledge of how to form the auxiliary equation and solve the quadratic.

(d) Not all candidates followed the instruction to look at u_0. The value for B could have been tidied up better in several cases.

(e) Generally well answered.

(f) Not all candidates realised that use of Table on the GDC was a good way to solve this.

Question 4

(a) There were many good answers to all parts. Sometimes there was confusion with the twice a minimum spanning tree when the question specifically asked for the nearest neighbour algorithm.

(b) The candidate tended to gain full marks when they knew the method and none if they did not. Some candidates did not read carefully that it was vertex 5 that was to be deleted.

(c) The cycle was often found although not all candidates went back to vertex 1. There were some lists of the weights but not many candidates said that apart from the last one they formed an AP.

(d) Despite the hint given in this question the intuition required here was beyond most (but not all) candidates.

Recommendations and guidance for the teaching of future candidates

There were some candidates who did not put in any explanation or comments or reasoning and this lost them marks. It is always important that candidates read and re-read the question as carefully as possible. There are specific words used in the questions with specific meanings. The examiner is trying to guide them through the question. They should always try to see the point of the question and how one part can assist in a latter part. It is good to consider which part of the syllabus each question is testing. It is vital that all candidates do a trial exam that is marked correctively by their teacher and given back to them to study. Then they understand more exactly what they are expected to do. It is always beneficial to work on past IB papers and see how the marks are given to show them the standard that is required. As this is a calculator allowed paper, candidates should be taught to use their calculator efficiently and to save time e.g. use of Table. Candidates should be taught to realise that you cannot prove anything in Maths by starting with it and thus non-proofs that end in $0 = 0$ are always going to be treated with disfavour by examiners. It is good to teach students to check at the end of each part of a question is the answer that they have given, the type of entity that is required e.g. is it an integer, or a real to 3 significant figures, or a tree, or an expression etc. Candidates should know before the exam what to expect for the format of the paper. The instructions at the top of page 2 e.g. start each question on a new page, answers must be supported by working and/or explanations, are often being ignored.

Higher level paper three calculus

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 10	11 - 20	21 - 29	30 - 35	36 - 42	43 - 48	49 - 60

General comments

Generally the candidates found the paper to be accessible with many achieving very high marks. It was good to see that time was not a factor for the most of the candidates who were able to make a good attempt at all of the questions.

The areas of the programme and examination which appeared difficult for the candidates

As always the more abstract areas of the programme, such as the mean value theorem, caused most problems. In addition finding the radius of convergence for a power series was also problematic for some candidates. Some of the differentiation was also poorly done. As always candidates need to be aware that in extended questions later parts are likely to require use of work done in earlier parts of the question.

The areas of the programme and examination in which candidates appeared well prepared

There were some excellent solutions from many candidates on the use of the integrating factor and finding a Maclaurin expansion. Most also knew how to apply L'Hôpital's rule, though the actual differentiation caused some difficulties.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a) This question was well done with many candidates comfortably managing to complete the correct integral

(b) Many candidates did this well. A few failed to multiply both sides of the expression by $1+x^2$, a greater number forgot to include, and so to find, the +*c* term.

Question 2

(a) This was a straightforward question which was well done by the candidates. Fortunately, given the clear instruction to find the terms using successive differentiation very few used standard results to generate the first four terms of the series.

(b) Candidates need to be aware that when the command term is 'deduce' it requires more than simply testing it in a few values. There also needs to be some justification of why this pattern should continue.

(c) Some candidates clearly knew this technique well and were able to produce some very successful answers. Too many candidates, perhaps through being distracted by the words 'ratio test', abandoned x altogether and so failed to reach an appropriate conclusion.

Question 3

(a) Despite a few who tried to use the quotient rule and so differentiate numerator and denominator together most candidates were clearly aware of how to apply L'Hôpital's rule.

(b) A straight-forward derivation of a formula which was well done by most candidates.

(c) On this occasion the candidates had to decide for themselves an appropriate test to use. It is possible to use the integral test but given the difficulties of the integration no one who attempted this approach scored more than 2 marks. Those who managed this part of the question successfully realized the link with part (a) and so used the limit comparison test. Looking for these kinds of links would be a useful approach for candidates to take during the five minute reading time.

When doing this test it is important that the candidate says explicitly what a finite limit implies ie. that the two functions converge or diverge together.

Many candidates were careless, as they have been on previous papers, in saying that $\frac{1}{\sqrt{x}}$ diverges by the p-series test, rather than its sum.

Question 4

(a) Unfortunately many candidates just gave the expression $f'(c) = \frac{f(b) - f(a)}{b - a}$ rather than stating the theorem and, in particular, giving the bounds for c.

(b) This question was generally well done. The most difficult part was (iv) and this caused many problems, particularly for those students who failed to realize that $f(h)$ is a constant. In part (vi), very few explained why $h = c$ could not be a solution.

(c) This was generally not well done, with many candidates not realizing they had to choose a specific function to replace $f(x)$. Further practice at these types of problem solving questions would be advisable.

Recommendations and guidance for the teaching of future candidates

As always with the calculus option those who display the necessary degree of rigour tend to do best. Examples of a lack of rigour were evident in questions 3(c) and 4(b), as mentioned above.

The importance of the five minutes reading time should be stressed, in particular for considering links between different parts of a question.

Higher level paper three sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 19	20 - 26	27 - 33	34 - 40	41 - 47	48 - 60

General comments

Many candidates showed good overall content knowledge and attempted to answer most of the questions. In general, although definitions were known and correctly quoted, difficulty was evident in applying these definitions to achieve or verify certain results.

The areas of the programme and examination which appeared difficult for the candidates

Not all candidates were familiar with the IB command terms, and the amount of working out necessary to achieve full marks. In particular, the working out shown for the command terms ‘sketch’ and ‘justify’ was at times insufficient. Although familiarity with definitions was evident, ability to apply definitions in particular cases was not always seen. An area of difficult for some candidates was the application of La Grange’s theorem and corollary, and cosets.

The areas of the programme and examination in which candidates appeared well prepared

Candidates were generally familiar with essential definitions and techniques used in this option, such as injective, surjective and bijective functions, equivalence relations, homomorphisms and isomorphisms, and properties of groups.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates answered parts (a)-(d) successfully. Some candidates did not know how to write permutations as disjoint cycles and others performed the order of composition incorrectly. Some candidates had difficulty with part (e), the order of a group of permutations. Quite a few had difficulty with part (f), i.e. finding the number of compositions of the cycles (12), (34) and (56).

Question 2

In both parts (a) and (b), many candidates gave poor sketches for the graphs asked for. In (a), the point (0,0) was often not excluded as part of the sketch, and the asymptotes were not clearly drawn in. For (b), the y-intercept (0,1) was often omitted. Some candidates mistakenly used the set D as the domain instead of the range. For both parts, many candidates simply quoted the definitions of injection, surjection and bijection, without applying those definitions to either their graphs or the functions used, as the command term 'justify' requires. Candidates generally answered the other parts of this question well.

Question 3

Part (a) of this question was answered fairly well by many candidates. Not all of candidates however successfully applied LaGrange's corollary to obtain all possible orders of the elements of {*G*,*}. Finding the elements with the orders that candidates did state was done successfully. Quite a few candidates had difficulty with part (b) and were not successful in finding the generator, the elements of set *H*, and the coset *aH*.

Question 4

Parts (a) and (b) were attempted by most candidates and well answered by many. Many candidates did not attempt part (c), and those who did, generally attempted the solution using proof by contradiction with some success. Some who attempted a direct proof were not successful.

Recommendations and guidance for the teaching of future candidates

- Candidates need to be mindful of all command terms in the HL syllabus in order to gain full marks on questions.
- Candidates need to take greater care in sketching graphs, being mindful to clearly identify all important points such as intercepts, and any asymptotes.
- Candidates need to know how to apply the definitions of surjective, injective and bijective functions to specific function examples.
- Candidates need to supply valid reasons to gain full marks when asked to justify their results.
- Candidates need to be familiar with methods of proof, for example, proof by contradiction, and should not use specific examples only to prove a statement.

Higher level paper three statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 18	19 - 25	26 - 33	34 - 41	42 - 49	50 - 60

General comments

The areas of the programme and examination which appeared difficult for the candidates

This examination did not cover any area of the programme that appeared to be generally difficult for the candidates. However, some candidates should be using their calculators more efficiently in questions like question 1, as explained more fully below.

The areas of the programme and examination in which candidates appeared well prepared

The majority of candidates were well prepared for this examination and performed well throughout the paper.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

As reported in the past, many candidates did not use their calculators efficiently in questions on this topic. In (a), it was not uncommon to see candidates calculating *t* using the formula and then finding the *p*-value using the cumulative *t* button. Then in (c) some candidates calculated the regression line using the formula and in (d) the *p*-value was again sometimes found via *t*. Remarkably, the arithmetic was usually carried out correctly but time was wasted. The intention of the question was to use the regression/correlation button in (a) and (c) and the *t*-test button in (d) which gave the answers immediately. In (c), some candidates gave the answer 27.9, failing to round to the nearest integer as required in the question. Many candidates failed to give the degrees of freedom in (b)(i) and (d)(i).

Question 2

Part (a) was well answered by the majority of candidates although some candidates seemed to confuse X and T . In (b), most candidates realised that it was the central limit theorem that allowed normality to be assumed. Part (c) was generally well answered although some candidates failed to use the mean and variance found in (a). Part (d) was the worst answered part question on the paper. Many candidates thought that Ray would be ringing, not John, and many failed to raise the initial probability found to the power 4.

Question 3

Part (a) was well answered by the majority of candidates. Part (b) was well answered by many candidates with very few arithmetic errors in the calculation of the final probability seen. Many candidates realised that there were several possibilities to consider.

Question 4

Part (a) was answered correctly by the majority of candidates with only a small minority adding the probability generating functions instead of multiplying. Although the question stated 'Write down', some candidates proved the result using the $\mathrm{E}\left(t^X\right)$ definition. Many candidates gave a correct solution to (b). This part required a certain amount of algebraic competence but most candidates were able to complete the proof correctly.

Recommendations and guidance for the teaching of future candidates

The main recommendation concerns the use of calculators in inferential questions. Usually, in this type of question, use of the appropriate calculator software will give the required result. The only danger in this approach is that if an error is made in inputting data, it is difficult to gain any M marks which may be available. So a necessary step is to check very carefully that the data have been inputted correctly.