November 2015 subject reports

Mathematics HL

Overall grade boundaries

Discrete

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 26	27 - 37	38 - 50	51 - 63	64 - 75	76 - 100

Calculus

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 27	28 - 37	38 - 50	51 - 62	63 - 75	76 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 37	38 - 51	52 - 64	65 - 77	78 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 39	40 - 52	53 - 65	66 - 78	79 - 100

Higher level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 16	17 - 20

The range and suitability of the work submitted

In general the explorations were based on topics chosen by the students. Some works had a very interesting stamp of creativity on the use of HL mathematics topics whereas others either had minimal mathematical content or were the reproduction of classical text book type problems. An interesting phenomenon which was also noticed in the May 15 session is the transcribing of mathematical videos found on “Numberphile” or “Khan Academy”. Whereas it is not unusual for such videos to act as stimuli for a topic, students should be reminded that it becomes very difficult to achieve top marks unless the teacher / moderator is able to find evidence of personal engagement and critical reflection in their written response. Many students chose modelling explorations. Unfortunately most of the candidates quoted a differential equation to model the phenomenon, defined the variables within the exploration context and integrated to obtain a relevant model without being able to interpret why or how the initial differential equation is valid. Some students chose topics that were well beyond the Mathematics HL course, for example the Gamma function, multi-variable integrals and vector calculus. The work produced was largely inaccessible to a peer group and they lacked adequate reasoning or evidence of understanding. In fact some explorations were so far removed from a teacher/moderator’s expected knowledge base that they were largely incomprehensible and very challenging to moderate. At the other end of the spectrum there were a number of superficial explorations that were not commensurate with the level of the course. Some of these were reports on the Fibonacci Numbers in nature.

A major concern remains the problem of citations. The importance of citations at every point of reference needs to be clearly made to students. It is recommended that teachers provide students with the document “Academic Honesty in the Diploma Programme” and discuss the possible consequences of malpractice.

Mostly students wrote explorations that were within the recommended number of pages, but some responses were too long.

Candidate performance against each criterion

Criterion A

In general candidates performed well against this criterion, with work being coherent and organized to different extents. It was noted that a number of students produced work that was excessively long and often appendices were included in an attempt to keep the length of the exploration within the specified limits.

Some teachers guided their students to produce a table of content and a word count and included this in their assessment rubric. There is no need for either of these in the Exploration. Some problems in this criterion were caused by students attempting to explain things that were beyond their own comprehension because the topic chosen was well beyond the level of the course. Some students also included interesting but unnecessary information which caused them to drift from the aim stated.

Criterion B

Most students performed well against this criterion. In some cases students produced irrelevant tables and repetitive graphs whereas in other cases the inclusion of diagrams and graphs would have added clarity to the mathematics explored.

Criterion C

Once more this criterion proved to be the more difficult for teachers to interpret although there was a general improvement over May/November 2014. It is very important that teachers and students alike understand the scope of this criterion. It was unfortunate to note that some teachers did not look for personal engagement in the students' work but assessed their students subjectively against this criterion. Many candidates did not know how to express their personal view on the topic being explored and ended up transcribing work that can easily be found in a textbook, on a website or a video clip. Teachers need to spend more time in discussion with students to establish ways in which personal engagement manifests itself in an exploration. On the other hand, some explorations were very original and well written so that one could hear the student voice coming through the written response.

Criterion D

This criterion was often not well understood by teachers and students. It should be noted that critical reflection has a metacognitive aspect to it that involves isolating a problem, looking at it from different perspectives and analysing the findings. Teachers often fail to recognize this and award achievement levels too liberally in this criterion. Teachers are advised to refer to the document "Additional notes and guidance on the Exploration" which can be found on the OCC. In general students who achieve high levels against this criterion also score highly against criterion C because as they attempt to overcome their perceived shortcomings they invariably manage to demonstrate personal engagement with their work.

Criterion E

The mathematical content continues to be very varied, ranging from very basic mathematics to extensions of the HL course well beyond the scope of the Exploration. Achieving a 6 still proved to be elusive on either count. Candidates who explored more abstract concepts beyond the HL curriculum were unable to demonstrate good understanding of the mathematics used. Some students produced nothing more than research papers that reproduced information gained from online sources. Some explorations based on modelling failed to go beyond the mechanical work of solving a differential equation and hence did not demonstrate thorough understanding.

Recommendations for the teaching of future candidates

There was evidence to suggest that some teachers were not dedicating the stipulated hours to the Exploration. It is imperative that 10 hours of teaching time are used to guide the students during the exploration process.

- Anecdotal comments about students' enthusiasm for the topic are not evidence of personal engagement.
- Teachers must indicate that they have marked the work with tick marks. On occasion errors were noted in student work that were not picked up by the teacher.
- Teachers should provide more feedback to students and information for the moderator on the body of the student's written response. Often these were missing and the comments on the form 5/EXCS were not relevant individual comments making it difficult for the moderator to justify the achievement levels being awarded.
- Teachers need to refer to the TSM as well as the document "Additional notes and guidance on the Exploration", both of which can be found on the OCC.
- Teachers should be strongly discouraged from mandating a particular type of exploration.
- Referencing continues to be a problem. Each reference, picture, graph must be cited at the point of reference. Failure to do this might result in the work being reviewed by the IB.
- One of the aspects of Approaches to Teaching and Learning in the DP is to encourage and stimulate students to develop research and writing skills in mathematics. This can be achieved by assigning mini-tasks, providing opportunities for reading and analyzing different forms of mathematical writing. Students should be encouraged to read mathematical articles about topics that are within their grasp.

Further comments

There seems to be a number of standard explorations. These include reproduction of video clips, SIR model for epidemics, RSA encryption and Ranking. This session also saw a larger number of explorations based on counting spirals on pine cones and pineapples to demonstrate the occurrence of Fibonacci numbers in nature.

Moderators continue to find the explorations much more interesting to moderate than the old tasks. However it seems that some students are choosing safe topics like statistics and projectile motion.

Teachers should discourage students from attempting to write an exploration on a topic which is largely inaccessible to them. Very often the students cannot demonstrate thorough understanding and can therefore not draw on any critical and meaningful reflection.

Higher level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 39	40 - 56	57 - 72	73 - 89	90 - 120

General comments

The comments of teachers support the examiners' view that the paper contained appropriately accessible questions for candidates throughout the expected HL range of competences.

However, examiners reported what seemed to be an increase in the level of careless work, including 'not reading the question'. There were some questions, where instructions to give answers in a particular form were ignored by a not insignificant number of candidates.

The poor level of algebraic manipulative skills and reasoning powers of the candidates in the middle range grades continues to be a concern.

A number of candidates are retaking the HL papers. It is to be hoped that such candidates receive some support from their school.

The areas of the programme and examination which appeared difficult for the candidates

Integration by substitution, particularly the incorporation of the changed limits. Interpretation of the idea of a vertical tangent to a curve in relation to the gradient function. Complete factorization of a trigonometric expression as a prerequisite to determining roots of an associated equation. The application of sums and products of roots to general polynomials. Working with abstract vectors. Finding the cube roots of a given complex number in an efficient and convincing way.

The areas of the programme and examination in which candidates appeared well prepared

Binomial expansions. Probability, particularly the use of tree diagrams. Calculus: Integration by parts; the chain rule for derivatives; implicit differentiation. Basic manipulation of complex numbers. Proof by induction.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Many candidates were unable to process the relations resulting from the application of the formulae for the arc length and area of a sector in the given context. Some who were more successful, still found answers in an unsatisfactory unsimplified form.

Question 2

This question was correctly answered by the majority of candidates. A few omitted the constant of integration.

Question 3

Well answered by the majority of candidates, with the occasional arithmetic slip in the simplification of the coefficients in the binomial expansion or the substitution of $x = 0.1$. Very few candidates multiplied out $(2+x)^4$ by brute force, and a few gave the expansion in descending order.

Question 4

Candidates showed good understanding of the chain rule and used a suitable method to calculate the normal line to a curve at a specified point. A significant number of candidates left the constant term of the answer as a fraction, contrary to the instructions given in the question.

Question 5

Most candidates made a good start, but a surprising number were then unable to use the limits correctly. Some candidates failed to use the fact that $\ln 1 = 0$.

Question 6

Generally good work on this question, often with the use of a tree diagram. However, some candidates were clearly unprepared for this sort of context.

Question 7

The majority of candidates realized that implicit differentiation was the appropriate tool, and successfully carried it out. Some candidates were convincing in ruling out a horizontal tangent, but others expected the examiner to read their minds. Finding points with a vertical tangent was more problematic for the majority. Some seemed to think that vertical implied an asymptote or, perversely, that the gradient was unity.

Question 8

Part (a) was almost universally answered competently, either using an application of the addition rule for the sine function, or by some graphical/transformational argument. The better candidates found part (b) to be a very easy induction question. Although most of the others were able to apply the logical machinery of the Principle of Mathematical Induction, they fell down in one of two ways: either not realizing that the $k+1$ st derivative of a function is the first derivative of the k th derivative; or not realizing that a complete proof required the use of the result from part (a).

Question 9

This was a question where very many middle range candidates had their lack of algebraic skills exposed. Most candidates recognized that double angle formulae had to be used, and were successful in that. Very few candidates were then able to reach the expected factorized form $(2\cos x-1)(\cos x-\sin x)=0$, from which the solutions fall out easily. More often than not, the working descended into a mess, or sometimes one factor was spotted and then lost on dividing out.

Question 10

Only the very best candidates successfully answered this question. Few candidates seemed to be familiar with the formulae for the sum and product of roots of a general polynomial in terms of the coefficients.

Question 11

For part (a), most candidates were familiar with some form of De Moivre's theorem, but disappointingly few were able to apply it correctly and report cubic roots in the form asked for in the question. Parts (b) and (c) were often well done, although there were some errors in the simplification of final answers. Although there were some correct answers to part (d), most candidates who tackled this question reported the incorrect answer $p=6$.

Question 12

Most candidates made significant attempts to answer parts (a), (b), (c) and (d). The main source of error was in the simultaneous application of the product rule and the chain rule in finding the derivative of the given function. Coming to the graphical part (e), many candidates did not realize that, for example, the maximum of the graph is strictly to the right of the point with $x=0.5$, and therefore that the graph should not be overly symmetrical. Except for candidates who tried to integrate by parts, part (f) was often well done. Most candidates had some sort of idea about part (g), but some had difficulty expressing their thoughts in words.

Question 13

This was an accessible final question for those candidates who had experience of the use of abstract vectors in geometric questions. That enabled many candidates to gain marks in parts (a), (b)(i)(ii) and (c). Part (b)(iii) was more problematical for many because it involved equating two linear combinations of abstract vectors. Part (d) was rarely completely successfully answered, although many who tackled this part, did at least work out a correct vector product and found the coordinates of G. Some candidates adopted a nonsensical notation for linear combinations of vectors, for example: $-\mathbf{a}+\frac{1}{2}\mathbf{b}+\frac{1}{2}\mathbf{c}=\begin{pmatrix}-a\\ \frac{1}{2}b\\ \frac{1}{2}c\end{pmatrix}$.

Recommendations and guidance for the teaching of future candidates

Provide as much practice in algebraic manipulation as possible, and insist that students present their work in a convincingly logical fashion.

Emphasise the importance of reading questions thoroughly, and making sure that answers are given in the specific form required.

When past papers are used for practice or mock exam testing, make sure that students are ultimately shown the definitive mark schemes.

Ensure that students appreciate that some questions can bring together different parts of the syllabus.

Higher level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 46	47 - 62	63 - 78	79 - 94	95 - 120

General comments

A significant proportion of the cohort found this to be a rather accessible paper with a large number of candidates performing very well in both sections. However, two major concerns emanating from this paper and recent GDC-permitted HL papers were the unacceptably large number of candidates who had their GDC set in degrees mode and the large number of candidates who persisted with time-consuming and often erroneous analytic solution techniques rather than undertake routine judicious employment of a GDC.

It is incumbent upon teachers to engage students in sensible and open discourse to aid in determining sound principles for appropriate GDC use in Paper 2.

The areas of the programme and examination which appeared difficult for the candidates

- Not solving a system of equations with a GDC either by choice or through a lack of awareness
- Not calculating a standard deviation of a data set with a GDC either by choice or through a lack of awareness
- Determining the equation of a transformed sine graph
- Calculating the area of a region enclosed by a curve, the y-axis and a horizontal line
- Calculating a volume of revolution about the y-axis
- Applying conditional probability considerations in situations governed by specified probability distributions
- Using definite integration to find the distance travelled by a particle
- Using the vector equation of a line in two dimensions in a simple kinematics application
- Determining the domain and range of a composite function
- Transforming functions
- Stating the largest possible domain for which an inverse function exists
- Performing product rule differentiation
- Applying integral calculus techniques in context

The areas of the programme and examination in which candidates appeared well prepared

- Using the formula for $\mathrm{P}(A \cup B)$, using $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$ to show independence and using Bayes' theorem in a worded context
- Finding an unknown frequency in a frequency distribution table and calculating the mean of a data set
- Applying the area of a triangle formula $\frac{1}{2}bc\sin A$ and using the cosine rule
- Finding when a particle changes its direction of motion and finding a particle's maximum/minimum velocity
- Various calculations involving the normal, binomial and Poisson probability distributions in worded contexts
- Finding an expression for an inverse function
- Understanding the relationship between self-inverse functions

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Generally well done. A few candidates opted for a conditional probability justification for the independence of events A and B. Bizarrely, a number of candidates calculated $\mathrm{P}(A)\mathrm{P}(B) = 0.48 \times 0.65$ in part (a) and used the formula for $\mathrm{P}(A \cup B)$ in part (b).

Question 2

Candidates who used a GDC generally scored full marks whilst those candidates who did not use a GDC often wasted valuable time and frequently lost 3 of the 4 available marks. The occasional candidate did not give the coordinates of P to an acceptable accuracy.

Question 3

Part (a) was generally well done. A number of candidates formed incorrect equations involving k and subsequently determined that k was not a positive integer.

Part (b)(i) was reasonably well done. In the expression for the correct mean number of goals scored that was derived in part (a), a number of candidates failed to recognize that they needed to add their k-value and 22 to the numerator and their k-value and 1 to the denominator.

Part (b)(ii) was not well done. A large number of candidates attempted unsuccessfully to calculate the standard deviation manually rather than directly with a GDC. Those that did use the GDC sometimes gave 4.77 (the incorrect standard deviation) as their answer.

Question 4

Part (a) was reasonably well done with most candidates able to determine that $C = 2$. A significant number of candidates found that $A = 3$ (rather than $A = -3$) and $B = 2$. A small number of candidates found the alternative correct answer of $A = 3$ and $B = -2$. This was a question where a candidate could use a GDC to quickly check their answer.

Part (b) was not well done. Again quite a few candidates attempted to solve their trigonometric equation manually. Other candidates did not pay heed to the equation's specified domain and gave answers that clearly did not match the given graph.

Question 5

In part (a), a significant number of candidates incorrectly expressed the area of the region R as an integral with respect to x rather than with respect to y. A number of candidates did not express their answer correct to four significant figures as specified. Some candidates calculated an area that was double the required area.

In part (b), a significant number of candidates attempted to calculate a volume of revolution about the x-axis rather than about the y-axis. A number of candidates gave 6.28 as their answer rather than the required 2π.

Question 6

This question was very well done with most candidates correctly interpreting what the question was asking. The use of Bayes' theorem rather than the use of a tree diagram was more common among the stronger candidates. Most candidates calculated that $\mathrm{P}(L') = 0.84$ but surprisingly, a number of candidates were unable to correctly calculate $\mathrm{P}(B \cap L')$ with 0.2 being a somewhat common numerator.

Question 7

This question was very well done. Some candidates did not recognize that they were dealing with the ambiguous case and found only one correct solution. The occasional candidate successfully used Heron's formula which is not prescribed in the syllabus.

Question 8

Unfortunately, a number of candidates attempted this question with their GDC set in degrees mode. Despite this, most candidates recognized the fundamental property of a continuous probability density function tested in part (a). Some candidates did not express their value of A correct to three decimal places as specified. A number of candidates attempted manual integration rather than use a GDC.

In part (b), it was apparent that many candidates did not understand what a mode of a continuous probability density function is. Many candidates who had their GDC set in degrees mode determined that the mode was 5. In addition, some candidates who had their GDC seemingly set in radians mode also determined that the mode was 5. It is highly likely that this latter group of candidates did not set an appropriate viewing window when graphing the function for $1 \le x \le 5$.

Part (c) was not well done with many candidates unable to determine that $\mathrm{P}\left(X \le 3 \mid X \ge 2\right) = \dfrac{\mathrm{P}\left(2 \le X \le 3\right)}{\mathrm{P}\left(X \ge 2\right)}$ and further, failing to recognize that $\mathrm{P}\left(2 \le X \le 3\right) = \int_2^3 f(x)\mathrm{d}x$. $\mathrm{P}\left(X \le 3 \mid X \ge 2\right) = \dfrac{\mathrm{P}\left(X \le 3\right)}{\mathrm{P}\left(X \ge 2\right)}$ was seen in quite a few instances as was a numerator containing a product of probabilities.

Question 9

Unfortunately, a number of candidates attempted this question with their GDC set in degrees mode. Despite this, part (a) was reasonably well done with most candidates attempting to solve $v(t) = 0$ for t either graphically or analytically.

Part (b) was reasonably well done with most candidates attempting to solve $v'(t) = 0$ for t either graphically or analytically. Some candidates unfortunately confused parts (a) and (b).

Part (c) was not well done with many candidates using limits determined in part (b) rather than those determined in part (a). A number of candidates attempted this question part with their GDC set in degrees mode.

Question 10

Despite being the most difficult Section A question, a pleasing number of candidates constructed elegant and well-reasoned solutions.

In part (a), the most common error was determining that $\boldsymbol{b} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$. A reasonable number of candidates who committed this error were awarded full follow-through marks in part (b).

In part (b), a good proportion of candidates recognized that Roderick must signal in a direction perpendicular to Ed's path. A number of correct solution methods were seen. The most popular solution methods included solving $\begin{pmatrix} 5 \\ 12 \end{pmatrix} \cdot \left(\begin{pmatrix} 11 \\ 9 \end{pmatrix} - \begin{pmatrix} -1+\frac{5}{3}t \\ 4+4t \end{pmatrix} \right) = 0$ (or equivalent) for t or solving $\frac{\mathrm{d}}{\mathrm{d}t}\left(\sqrt{\left(12-\frac{5t}{3}\right)^2 + (5-4t)^2} \right) = 0$ (or equivalent) for t.

Question 11

This question was very well done with a number of candidates scoring full marks. Parts (a), (b) and (c) were well done. In part (a), a number of candidates who obtained full marks wasted some time by not using their GDC judiciously to solve the pair of equations directly. A few candidates incorrectly set their equations to 0.3 and 0.75 rather than $\Phi^{-1}(0.3)$ and $\Phi^{-1}(0.75)$ respectively.

In part (d), a significant number of candidates did not recognize this as a situation requiring conditional probability considerations.

Parts (e) and (f) were surprisingly well done. The most common error involved not specifying a correct inequality to represent the required probability. In part (e), some candidates incorrectly inferred from the phrase 'elevators arrive on average every 36 seconds' that $L \sim \mathrm{Po}(36)$. In part (f), it was pleasing to see the number of candidates that understood that 400 workers require at least 40 elevators.

Question 12

Part (a) was not well done. Some candidates identified correct interval endpoints but did not specify a closed interval. Other candidates applied the required compositions in an incorrect order. A few candidates attempted to find f.

Part (b) was reasonably well done. Most candidates explained why f does not have an inverse. A reasonable number of candidates accurately sketched the graph of the inverse. Some candidates attempted to sketch the graph of the reciprocal while other candidates sketched a restricted form of the original function.

In part (c), a large majority of candidates correctly calculated the inverse of a rational function with a significant proportion of these candidates expressing their final answer with correct function notation. A large number of candidates did not solve part (c) (ii) by inspection. Instead these candidates cross multiplied and embarked upon several lines of complicated and often incorrect algebra. In part (c) (iii), quite a few candidates were able to write down $\frac{2k(x)-5}{k(x)-2}=\frac{2x}{x+1}$ but were then unable to solve correctly for $k(x)$.

Question 13

In part (a), most candidates were able to apply the chain rule to successfully determine $f'(x)$. A surprising number of candidates were then not able to accurately apply the product rule with algebraic and sign errors prominent.

Parts (b) and (c), both 'show that' question parts, were not well done and sometimes proved challenging to mark. In part (b), a number of candidates used $f'(x)$ instead of $f''(x)$.

Only the very best candidates scored well in part (d). A number of candidates attempted to find the value of a which minimized P rather than I. A number of candidates incorrectly gave $P(a)=2a+60\mathrm{e}^{-\frac{a^2}{400}}$. In part (d) (iii), a few candidates gave the area under the roof as a definite integral with $\pm a$ as limits rather than ± 20. Candidates who attempted this part seemed confused with the various areas.

Recommendations and guidance for the teaching of future candidates

As mentioned previously, it is imperative that teachers and students undertake fruitful dialogue with regards to appropriate GDC use in Paper 2. In particular, discussions about appropriate mode settings, adjusting viewing windows so that key features of a graph can be readily identified and GDC inbuilt functionality need to be taught specifically if candidates are to demonstrate their true subject knowledge. Further, teachers must ensure that students are fully prepared for GDC-required assessment throughout the teaching of the course.

Conversely, teachers need students to understand that the command term 'show that' does not generally require the use of a GDC and often requires sound justification and reasoning.

Higher level paper three: discrete

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 15	16 - 23	24 - 28	29 - 34	35 - 39	40 - 60

General comments

There was quite a lot of standard techniques on this paper that should have enabled a well-prepared candidate to score well. The problem solving aspects of the paper in particular Q5 and parts of Q4 did cause some difficulties for all but the best candidates.

The areas of the programme and examination which appeared difficult for the candidates

Generally the Number Theory work was done less well than the Graph Theory. In particular the candidates seemed to be thrown by sigma notation and had trouble with finding the general solution to a Diophantine equation and dealing with congruences.

The areas of the programme and examination in which candidates appeared well prepared

Kruskal's algorithm, bipartite graphs, Hamiltonian and Eulerian circuits and cycles, and recurrence relations.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a) This was well done by candidates. They need to be aware that they must demonstrate the correct use of the algorithm in order to achieve full marks rather than just giving the solutions. Some candidates lost valuable time drawing out a graph of the towns, which was not required for this question.

(b) The key term in this question was 'justifying your answers'. In the case of a Hamiltonian path this meant listing the path, in the case of a cycle it meant explaining why one could not exist. For Eulerian trails and circuits a reference to the number of vertices of odd degree is all that was required.

Candidates should be aware of some of the conditions for a Hamiltonian cycle not to exist, such as having a vertex with a degree of one. Dirac's theorem, quoted by several students, is not in the syllabus and is rarely useful for showing a graph is Hamiltonian, due to the conditions required for it to be applicable, and it cannot be used to prove a graph is not Hamiltonian.

Question 2

This was a standard question using techniques that the candidates should be very familiar with.

(b) There were several different techniques used successfully to solve this part. All are acceptable.

(c) The method for a repeated root in this part was not well known, with some candidates simply repeating the -1 in the standard formula for distinct roots.

Question 3

(a) This question fell into four parts with most candidates managing the first (to find the gcd). Many could then reverse the algorithm to find a single value for a and b, but surprisingly having reached this point many candidates could not write down the general formula. For those who could write down this expression the final part, which was the problem solving aspect to the question, was generally well done.

(b) Most candidates who completed this question successful took the ‘other’ route rather than the ‘hence’ one. They therefore broke down the exponent in their own way, usually using Fermat’s Little Theorem, sometimes without – though those who tried this latter route were often unsuccessful. It was again a fairly standard technique and so was surprising how many students were not able to approach it successfully.

Question 4

(a) and (b) Most candidates were able to successfully do these parts.

Parts (c), (d) and (e) were essentially a proof of the inequality $e \le 2v - 4$. The questions were set up to provide a structured route through the proof, though several candidates were unable to see the links between the parts.

(c)(i) The question mentioned that there were 4 faces and most students realized this included the outside face. Hence many gave the correct answer of degree 4 for each. The marks allocated to the question should have indicated to the candidates that the explanation in c (ii) was worth only one mark. Some unfortunately went to great lengths. The handshaking theorem was introduced on many occasions.

(d)(iii) Was well done, as most candidates knew that a bipartite graph could not contain any triangles. The necessity for a simple graph to have no loops or multiple edges was less well-known.

(e)(i) Candidates found this part difficult. The result is easily obtained from the answers to parts (c)(ii) and (d) but the link was not obvious to many. Part (ii) required the use of Euler’s relation, this was recognized by most candidates who attempted this part.

(f) This part could have been attempted without having completed the earlier parts. It is a proof that should have been learned by the candidates as it appears explicitly in the syllabus.

Question 5

(a)(i) This was well done by candidates.

(a)(ii) Candidates clearly found manipulation of sigma notation difficult. In fact, the expansion of the left hand side of the expression gives the right hand side very readily.

(b) Very few candidates managed this final part. It was a problem solving question requiring use of the two previous parts. Some gained some marks by using the result in (a)(i) to show the expression was divisible by 2.

Recommendations and guidance for the teaching of future candidates

Higher level paper three: calculus

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 17	18 - 22	23 - 28	29 - 33	34 - 39	40 - 60

General comments

Performance on the Calculus paper was generally disappointing. Whilst many students seemed to be prepared with superficial knowledge of the syllabus, it seemed as if sufficient practice with questions had been limited. There were some scripts appearing as if time had been a factor in poorer performance, although this was often as a result of applying lengthy methods where a simpler method had been expected.

The areas of the programme and examination which appeared difficult for the candidates

Students seemed to be more used to a rough conceptual idea of continuity and differentiability, rather than a more rigorous approach that is required. Many candidates did not understand what was meant by a graphical explanation. It was felt that candidates had difficulty making sense of the more complicated expressions of summations. Still the understanding and use of isoclines seemed to be very limited. Use of efficient calculator techniques to apply Euler's method.

The areas of the programme and examination in which candidates appeared well prepared

Candidates seemed well prepared with finding Maclauren series and also in general with the use of integrating factors to solve differential equations. Students were mostly able to use limits to show continuity. Whilst many candidates showed they had an understanding of Euler's method, there were too many errors in its application.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Many candidates did not use limits at all, and still more did not use limits in part (b). Some of those that did, lost marks for not applying first principles as stipulated in the subject guide.

Question 2

Generally quite well answered, although very few candidates used the identity in (a) to simplify (b). Most solutions were done by repeated differentiation resulting in loss of time and frequent errors along the way.

Question 3

Part (a) was generally well done, although there was a lack of rigour in many scripts.

Part (b) was poorly attempted. Many students attempted to apply a comparison test to $b_r = \frac{3^r}{r!}$ leading to no marks. The question clearly used the word "hence" and so candidates should be aware that the previous part must be used. Of those candidates that did compare to the correct series, very few took any notice of the fact that the series started from 7.

Question 4

There were a number of good responses to part (a) although also a surprising number of candidates who seemed to produce a lot of writing but nothing graphical. There were a lot of good answers for part (b) although there were more candidates than expected that could not find $\int_0^1 \frac{1}{1+x^2}dx$ in spite of it being in the formula book. Many who did find the integral correctly then proceeded to find $\arctan 1$ as either π or $\frac{\pi}{2}$. Such students did not seem to react when finding bizarre ranges for π. There were many good answers for (c) although some students did not achieve the marks through failing to show anything – merely stating the information given in the question. Part (d) was poorly attempted with very few correct answers. It seemed that most candidates were confused by the complexity of the statement.

Question 5

Part (a) was predictably well done. There were many good solutions to part (b) although quite a few errors in the algebra along the way. Many students ignored the $+c$ and consequently were not able to find the correct particular solution. Whilst it was evident that most students had an awareness of Euler's method, it was surprising how many were not able to follow the method through accurately. There are good calculator techniques for this which have clearly not been developed. Very few students were able to get full marks for part (d) although there were a number obtaining partial marks. Part (e) caused more difficulties than it should. Candidates were unclear about what an isocline is or how it can be used. Of those that drew the isoclines, many only drew the top halves.

Recommendations and guidance for the teaching of future candidates

It is clear that teachers need to spend a little more time teaching the option. Some concepts are difficult and students need some time working on questions for the necessary depth of understanding. A more rigorous approach to continuity and differentiability is clearly required. Whilst the explanation “it has a sharp bit” might be helpful in developing the conceptual understanding of differentiability, it is not enough for questions in paper 3.

Teachers should expose students to examples of series that do not start at 1.

Reimann sums do need to be looked at in class and practice questions should be found.

Reliable and efficient calculator use is necessary for effectively using Euler’s method.

Higher level paper three: sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 18	19 - 24	25 - 32	33 - 39	40 - 47	48 - 60

General comments

The majority of the candidates were familiar with the content of the option and have attempted most of the questions. Candidates in general were able to state definitions but many had difficulties in applying them to prove the required results.

The areas of the programme and examination which appeared difficult for the candidates

Most candidates had difficulties in questions where they were required to show or prove results. In many cases, candidates made good attempts but used inappropriate notation and terminology. Mathematical communication was overall very poor.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates were familiar with the definition of injective and surjective function, definition of equivalence relation, definition of group and definition of kernel of homomorphism.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Almost all candidates attempted well this question. Most candidates were familiar with De Morgan law and were successful in applying it; distributive property was attempted often but with less success. A number of candidates attempted to establish the result using double inclusion method to establish equality of the sets. This latter group was less successful. A number of misconceptions were identified, namely $B \cap A' = B$. A number of candidates ignored the command term ‘prove’ and simply illustrated the result using Venn diagrams.

Question 2

Part (a) was well answered by most candidates. The most common situation where candidates were not awarded full marks were statement of definitions without any argument relating them to the question and an incorrect range in part (ii). Part (b) was less successful as many candidates just listed the properties without providing convincing evidence that these were true for the relation given. In a number of cases candidates were not clear about the names of the properties. The notation used was in many cases not appropriate either. Part (c) was less successful than expected with many candidates giving answers that did not correspond to a partition of the set □ but rather of the set □ .

Question 3

This question was poorly answered with very few candidates achieving full marks. Most candidates seemed unfamiliar with groups of permutations and the notation and terminology associated to these groups. In part (a) many candidates answered 24 instead of 12 without making any attempt to find the lowest common multiple of the lengths of the cycles. The same error was made in part (d) with many candidates stating 32 as the maximum possible order of an element of H. In part (b) many candidates showed lack of understanding of the question and answered 1. Part (c) was better answered but inappropriate notation was used in most cases.

Question 4

Most candidates were able to complete the table in part (a) but unfortunately many failed to obtain full marks in (b) either because they ignored the need for establishing all the properties required for $\{T, *\}$ to form a group or for attempting to justify associativity and commutativity based on examples. Part (c) was in general well answered although in some cases candidates provided unexpected answers like orders 4 and 5 showing little understanding of the meaning of order of an element and its relation with the order of the group itself. Part (d) was poorly attempted with many candidates showing unfamiliarity with cosets.

Question 5

Part (b) was correctly answered by many candidates but very few made good attempts to prove part (c). Part (a) was correctly answered by a few candidates but many others have attempted it. In many cases it was clear that candidates knew the definition of homomorphism of groups but were unable to apply it to prove that the given function was indeed a homomorphism between the groups given. A number of candidates attempted to justify the result using necessary but not sufficient conditions.

Recommendations and guidance for the teaching of future candidates

Candidates need to learn to communicate their ideas using correct mathematical notation.

Candidates need to practice formal proofs and learn when examples are sufficient and when more general arguments are required.

Candidates need to be exposed to different forms of representations of groups, namely groups of permutations and representation of elements as product of cycles.

Candidates need to be familiar of methods of proof of equality of sets, both using double inclusion method and application of properties of operations of sets, namely De Morgan Laws and distributive property.

Higher level paper three: statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 18	19 - 28	29 - 35	36 - 42	43 - 49	50 - 60

The areas of the programme and examination which appeared difficult for the candidates

Based on the solutions to Question 4(f) it is clear that many candidates believe that in order to show that two random variables X and Y are identically distributed, it is sufficient to show that $E(X) = E(Y)$. It would be useful to show them a counter example, eg X is $B(10, 0.3)$ and Y is $Po(3)$.

Candidates appear to find it difficult to ensure that they use the correct regression line in problems where the value of one variable is used to predict the value of another variable.

Candidates need to be quite clear that in order to obtain a cumulative distribution function, summation should be used for discrete random variables and integration for continuous random variables.

The areas of the programme and examination in which candidates appeared well prepared

Candidates are generally well prepared in the use of the graphical display calculator although there is a small minority who fail to exploit the full capability.

Candidates appear in general to be giving better solutions to problems involving estimation and probability generating functions.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Part (a) was correctly answered by most candidates. Those candidates who gave only an incorrect answer were not given the M mark. Candidates should perhaps be advised to indicate a method in this type of problem, for example here candidates who indicated that the confidence limits were $\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$ gained the M mark even if they then gave an incorrect answer.

Part (b) was also well answered, although a couple of errors were occasionally seen. Some candidates used 1.6449 instead of 1.96 as their z-value, this was immediately identified by the incorrect answer 271. Other candidates used only half the interval in their inequality, this was immediately identified by the incorrect answer 97.

Question 2

Almost every candidate gave the correct value of r in (a), usually using the statistics menu on the calculator although a minority of candidates used the calculator to give sums and sums of squares and then used the appropriate formula to calculate r. This method is not recommended since it gives a significant opportunity for arithmetic error.

Part (b) was well answered by most candidates with the following errors occasionally seen. Some candidates gave incorrect hypotheses, using r or μ as the parameter, others gave a contradictory conclusion, for example, 'Reject H_0, moisture content and strength are independent'.

Many candidates gave an incorrect solution to (c), usually because of incorrectly using the regression line of moisture content on strength. This was possibly just carelessness in noting which set of data is in which numbered memory. These incorrect results emphasise that care should be taken when dealing with regression lines to ensure that the correct one is used.

Question 3

In this question, candidates had to realise that there were three possibilities of selection, a boy and a girl, two boys or two girls. In the event, the majority of candidates considered only the first possibility so that 0.885 was by far the most common incorrect answer.

Question 4

Solutions to (a) were often disappointing. Most candidates began correctly by stating that the probability function was $\frac{1}{4}\left(\frac{3}{4}\right)^{u-1}$ but many candidates then used integration instead of summation to find the cumulative distribution function.

Part (b) was correctly answered by most candidates, using the software on the calculator.

Part (c) was well answered by most candidates. It seemed in many cases that candidates were familiar with the derivation of the probability generating function for the general case and they were able to translate this into the special case in the question.

Part (d) was well answered by most candidates. Some candidates wasted time by evaluating the mean and variance using the probability generating function instead of using the formulae in the formula booklet.

Part (e) was well answered with most candidates differentiating $G_w(t)$ correctly and then putting $t = 1$. Some candidates used the calculator to carry out the differentiation numerically and this was accepted.

Part (f) was answered correctly by only a small minority of the candidates. Most candidates who attempted (f) showed only that $E(V) = E(W + 3)$ and concluded, incorrectly, that $V = W + 3$. Most of the few candidates who used probability generating functions correctly failed then to state that equal probability generating functions implies equal distributions.

Question 5

In (a), some candidates misunderstood the question, thinking that every possible value of X had probability p. This had the effect of making much of the question inaccessible. Those candidates who read the question correctly usually obtained the correct answer for $E(X_1)$ and then correctly deduced the unbiased estimator for p. A small minority gave the unbiased estimator as $E\left(\frac{X_1 - 3}{3}\right)$. Although close to the correct solution, it was not accepted since it is not an estimator.

Part (b)(i) was well answered by most candidates. In (b)(ii), however, a fairly common error was to write $\mathrm{Var}\left(\frac{1}{3}X_2 - kX_2\right) = \left(\frac{1}{9} + k^2\right)\mathrm{Var}(X_2)$ instead of $\mathrm{Var}\left(\frac{1}{3} - k\right)^2 \mathrm{Var}(X_2)$.

Some candidates wasted time evaluating $Var\left(X\right)$ which was not needed.

Recommendations and guidance for the teaching of future candidates

Some candidates need to be more careful in extracting information from the calculator.

Candidates should be aware that there is a uniqueness theorem for probability generating functions which needs to be used to show that random variables are identically distributed.