November 2014 subject reports

MATHEMATICS HL

Overall grade boundaries

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 25	26 - 37	38 - 50	51 - 62	63 - 73	74 - 100

Calculus

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 25	26 - 36	37 - 49	50 - 62	63 - 73	74 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 25	26 - 36	37 - 49	50 - 62	63 - 74	75 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 26	27 - 38	39 - 50	51 - 63	64 - 74	75 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 16	17 - 20

The range and suitability of the work submitted

There was a wide range of interesting topics and some schools are to be commended on the guidance given to students. Tasks included modelling real life situations and others provided a synopsis of mathematics that was found during the research process. Among the latter, common explorations included the Birthday Problem, the Buffon needle, game theory, solving the Rubik's cube and probabilities in a poker game. Although all these were suitable some topics did not allow students to show personal engagement apart from understanding the topic. Some explorations were also far too long, and sometimes too complex for peers to understand and also for the author to demonstrate good understanding of the mathematics involved.

It was also noted that some explorations were written in very small font to make them look concise, whereas others were very short with large diagrams, written in larger font and double-spaced.

Candidate performance against each criterion

Criterion A

In general students performed well in this criterion; however some explorations were far too long and therefore not concise. In some cases students had a rationale that was made up and not genuine. Some students did not write a clear aim and this often led to an incoherent piece of work that was not complete since they could not write a conclusion to put it all together.

Criterion B

Language, notation and terminology were generally correctly used. Students also used technology effectively and included diagrams, graphs and tables in the appropriate places, along the work. A single notational error can be condoned but repeated use throughout the exploration should be penalized. Marks were lost when students did a modelling exploration and did not define key terms.

Criterion C

More candidates are aware of the requirements for this criterion, and some will give contrived reasons for choosing the topic thinking that this might give an extra mark. In many cases this rationale was not supported in the rest of the exploration. A number of explorations were mere reproductions of papers found on the Internet or common advanced textbook problems. In this case students would receive some marks for mastering new techniques but it is difficult to justify top achievement levels. Exploration in a mathematical topic occurs when candidates use something that has already been explained or systematized as a tool to approaching a new question that they themselves have come up with. For example, candidates are allowed to recreate a Koch snowflake as long as they generate their own fractal. Some teachers are still under the impression that Personal Engagement is a measure of effort so that the choice of achievement levels becomes subjective.

Criterion D

A number of students did not offer any meaningful or critical reflection but wrote a conclusion summarizing results. Teachers often marked high on this criterion, which then led to achievement levels being marked down by the moderator. Students and teachers need to understand that for high achievement levels reflection needs to take place throughout the exploration, through candidates isolating themselves from the problem, to see it from another point of view and to analyse its limits, the connections it may have to other similar problems, other implications that might arise, aspects that could have been considered but were not, applications to other real world problems or contexts, discussing the techniques being used, the validity of the results obtained etc.… Once more, students who reproduced common textbook problems presented work that contained only superficial reflection.

Criterion E

Mathematical content was varied and while some explorations contained extensive mathematics others were very descriptive. Most explorations were also in the form of a mini research paper with mathematical content being reproduced, sometimes without demonstrating good understanding. In some cases students demonstrated an understanding of the technique being used but no significant understanding as to why the particular technique works. Some modelling explorations incorporated fitting data into a model, without any justification for choosing that model or development of the mathematical model used. Students need to be well guided when choosing their topics. It is sometimes easier to achieve good levels using simple HL mathematics content than going beyond the syllabus.

Recommendations and guidance for the teaching of future candidates

The exploration needs to be introduced early on and referred to frequently with references to appropriate topics as they arise during the learning experience. Teachers must dedicate the number of hours specified in the programme to guide the candidates throughout the process. It is important that teachers highlight the importance of proper referencing and citations.

Teachers are encouraged to read the new publications on the OCC; i.e., Academic Honesty in the IB educational context and Effective citing and referencing.

It is recommended that teachers provide candidates with mathematical stimuli in various forms, including but not limited to texts, web pages, specialized bibliographies, movies, video clips, photographs, paintings, graphic design, games of chance, board games, experiments, magic tricks, etc… The investigations that were part of the former internal assessment component may also serve as a source of ideas for candidates to develop their explorations creatively and constructively.

Students need to understand the 5 criteria thoroughly before starting to develop their own exploration. They need to be guided to choose a topic wisely; one that incorporates Mathematics that allows them to demonstrate full understanding, and which is commensurate with the course. The topic should allow them to demonstrate personal engagement by possibly giving them the avenue to be original and creative. The topic should also be focused and the aim achievable within the set number of pages (6 to 12).

Teachers should refrain from making anecdotal comments about student commitment to the exploration process, as this has no bearing on criterion C. Personal engagement should be evident in the students own work. All student work should contain evidence of marking with errors being pointed out; this makes it easier for the moderator to confirm the teacher's marks.

Further comments

Although it is not stated that teachers need to supply information regarding material before the exploration process was started, it is highly recommended that this background information be given as it helps the moderator put the development of the exploration into perspective.

It might be useful for candidates to practice assessing explorations that are available in the TSM. This gives them an idea of what is expected of them in a project of this nature and it will ensure that candidates understand all 5 criteria. Class time might even be spent developing a group exploration with the help of the teacher as a means of understanding the methodology involved.

Having said all this the moderation of Internal Assessment with the wide variety of creative and interesting explorations continues to be an enjoyable task for moderators.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 28	29 - 42	43 - 58	59 - 75	76 - 91	92 - 120

General comments

The responses to this paper were very interesting, given that it is the first paper set on the new syllabus for the November cohort.

It is the case that, included in the N14 cohort, are many retake candidates (for various reasons) as well as the strong entry from some large centres. The performances on certain questions reflect that dichotomy.

The areas of the programme and examination which appeared difficult for the candidates

Working with inequalities. Combinatorics. Proofs and reasoning, using the conventions of mathematical arguments – in particular it is unwise to attempt to work backwards from a target result. Problem solving and the application of knowledge in unfamiliar situations.

The areas of the programme and examination in which candidates appeared well prepared

Routine applications of formulae, finding derivatives, basic integration, sketching graphs, simple probability.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This question was generally well done. A few candidates made a sign error for the horizontal translation. A few candidates expressed the required equations for the asymptotes as ‘inequalities’, which received no marks.

Question 2

(a) Most candidates obtained full marks.

(b) Many candidates obtained full marks, but some responses were inefficiently expressed. A very small minority attempted to use the exact roots, usually unsuccessfully.

Question 3

Generally well done. But there was a significant minority who didn't realise that they had to use calculus or completion of squares to minimise the length. Trying random values of s gained no marks. A number of candidates wasted time showing that their answer gave a minimum rather than a maximum value of the length.

Question 4

(a) This part was generally well done.

(b) Disappointingly, many candidates did not seem to understand the meaning of the word 'range' in this context.

Question 5

This question was generally well done. A significant number of candidates did not calculate the maximum value of C.

Question 6

Many candidates worked through this question successfully. A significant minority either made algebraic mistakes with the substitution or tried to work with an integral involving both x and u.

Question 7

This was a problem question for many candidates. Some quite strong candidates, on the evidence of their performance on other questions, did not realise that 'composite functions' and 'functions of a function' were the same thing, and therefore that the chain rule applied.

Question 8

An easy question, but many candidates exhibited discomfort and poor reasoning abilities. The difficulty for most was that the proposition was expressed in terms of an inequality. Hopefully, as most publishers of IB textbooks have realised, inequalities in such questions are within the syllabus.

Question 9

(a) The sketched graphs were mostly acceptable, but sometimes scrappy.

(b) Most candidates had some idea about the upper and lower quartiles, but some were rather vague about how to calculate them for this probability density function. Even those who integrated for the lower quartile often made algebraic mistakes in calculating its value.

Question 10

As the last question on section A, candidates had to think about the strategy for finding the answers to these two parts. Candidates often had a mark-worthy approach, in terms of considering separate cases, but couldn’t implement it correctly.

Question 11

(a)(b)(c) Generally very well done, even by candidates who had shown considerable weaknesses elsewhere on the paper.

(d) A productive question for many candidates, but some didn’t realise that a difference of areas/integrals was required.

(e) (i) Many candidates adopted a graphical approach, but sometimes with unconvincing reasoning.

(ii) Poorly answered. Many candidates applied the suggested substitution only to one side of the inequality, and then had to fudge the answer.

Question 12

A fairly straightforward question for candidates confident in the use of and correct notation for relative position vectors. Sign errors were the most common, but the majority of candidates did not gain all the reasoning marks for part (c). In particular, it was necessary to observe that not only were two vectors parallel, but that they had a point in common.

Question 13

(a) Fairly successful.

(b) (i) Most candidates attempted to use the hint. Those who doubled the angle were usually successful – but many lost the final mark by not giving a convincing reason to reject the negative solution to the intermediate quadratic equation. Those who halved the angle got nowhere.

(ii) The majority of candidates obtained full marks.

(iii) This was poorly answered, few candidates realising that part of the integrand could be re-expressed using $\frac{1}{\cos^2 x} = \sec^2 x,$ which can be immediately integrated.

Recommendations and guidance for the teaching of future candidates

- Reinforce and provide examples of formal proofs, particularly in the context of the principle of mathematical induction and working with abstract vectors.
- Ensure that students show all calculations and reasoning steps in mathematical arguments – examiners cannot read the minds of candidates.

- Provide problem solving examples.
- Ensure that, as far as possible, retake candidates receive guidance and revision opportunities between examination sessions.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 46	47 - 60	61 - 73	74 - 87	88 - 120

General comments

The majority of candidates found this paper to be a demanding test of their mathematical knowledge and reasoning skills. In particular, most candidates experienced difficulties successfully answering the two most challenging questions on the paper, namely, Question 9 and Question 14. Despite this, it is pleasing to report that Questions 1, 3, 5 and 6 of Section A and Questions 10-13 of Section B were generally quite well done by the majority of candidates.

The areas of the programme and examination which appeared difficult for the candidates

- Normal distribution
- Kinematics
- Some aspects of trigonometry
- Questions requiring reasoning, formulation and interpretation
- Using a GDC table feature to solve an inequality

The areas of the programme and examination in which candidates appeared well prepared

- Vectors
- Probability
- Poisson distribution
- Calculus

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Reasonably well answered. A large number of candidates did not express their final answer correct to the nearest degree.

Question 2

In part (a), many candidates did not use the symmetry of the normal curve correctly. Many, for example, calculated the value of *x* for which $\mathrm{P}(X < x) = 0.99$ rather than $\mathrm{P}(X < x) = 0.01$.

In part (b), most candidates did not recognize that the required probability interval was $\mathrm{P}(60.15 \le X \le 60.25)$. A large number of candidates simply stated that $\mathrm{P}(X = 60.2) = 0.166$. Some candidates used $\mathrm{P}(60.1 \le X \le 60.3)$ while a number of candidates bizarrely used probability intervals not centred on 60.2, for example, $\mathrm{P}(60.15 \le X \le 60.24)$.

Question 3

Reasonably well done. Most candidates were able to obtain $x + y = 11$. Most manipulation errors occurred when candidates attempted to form the variance equation in terms of *x* and *y*. Some candidates did not apply the condition $x < y$ when determining their final answer.

Question 4

Reasonably well done. Most successful candidates determined that $s = 25t \Rightarrow \frac{\mathrm{d}s}{\mathrm{d}t} = 25$ from $x = 15t$ and $y = 20t$. A number of candidates did not use calculus while a few candidates correctly used implicit differentiation.

Question 5

Part (a) was reasonably well done. Some candidates made numerical errors when attempting to find a normal to π.

In part (b), a number of candidates were awarded follow through marks from numerical errors committed in part (a).

Question 6

A large number of candidates, either by graphical (mostly) or algebraic or via use of a GDC solver, were able to readily obtain $a = -1$. Most candidates who were awarded full marks however, made specific reference to an appropriate graph. Only a small percentage of candidates used the discriminant to justify that only one value of *a* satisfied the required condition. A number of candidates erroneously obtained $3a^3+a^2+5a-7=0$ or equivalent rather than $3a^3+a^2+5a+7=0$.

Question 7

Part (a) was reasonably well done. A number of candidates used $r=\frac{u_1}{u_2}=\frac{u_2}{u_3}$ rather than $r=\frac{u_2}{u_1}=\frac{u_3}{u_2}$. This invariably led to candidates obtaining $r=2$ in part (b).

In part (b), most candidates were able to correctly find the first term and the common difference for the arithmetic sequence. However a number of candidates either obtained $r=2$ via means described in part (a) or confused the two sequences and used $u_1=\frac{3}{4}$ for the geometric sequence.

Question 8

Part (a) was not done as well as expected. A large number of candidates attempted to solve $5-(t-2)^2=0$ for *t*. Some candidates attempted to find when the particle's acceleration was zero.

Most candidates had difficulty with part (b) with a variety of errors committed. A significant proportion of candidates did not understand what was required. Many candidates worked with indefinite integrals rather than with definite integrals. Only a small percentage of candidates started by correctly finding the distance travelled by the particle before coming to rest. The occasional candidate made adroit use of a GDC and found the correct value of *t* by finding where the graph of $\int_0^4 5-(t-2)^2\,\mathrm{d}t+\int_4^x 3-\frac{t}{2}\,\mathrm{d}t$ crossed the horizontal axis.

Question 9

Most candidates found this a difficult question with a large number of candidates either not attempting it or making little to no progress. In part (a), a number of candidates attempted to show the desired result using specific regular polygons. Some candidates attempted to fudge the result.

In part (b), the overwhelming majority of candidates that obtained either $n=21$ or $n=26$ or both used either a GDC numerical solve feature or a graphical approach rather than a tabular approach which is more appropriate for a discrete variable such as the number of sides of a regular polygon. Some candidates wasted valuable time by showing that $C=\frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)}$ (a given result).

In part (c), the occasional candidate correctly commented that *C* was a good measure of compactness either because the value of *C* seemed to approach the limiting value of the circle as *n* increased or commented that *C* was not a good measure because of the disparity in *C*-values between even and odd values of *n*.

Question 10

This question was generally well done. Parts (a) and (b) were straightforward and well answered. Parts (c) (i) and (ii) were also well answered with most candidates correctly applying the second derivative test and displaying sound reasoning skills.

Part (c) (iii) required the use of the cosine rule and was reasonably well done. The most common error committed by candidates in attempting to find the value of QR was to use $\mathrm{PR} = \frac{14}{3}$ (cm) rather than $\mathrm{PR} = \left(\frac{14}{3}\right)^2$ (cm). The occasional candidate used $\cos 30° = \frac{1}{2}$.

Question 11

Parts (a), (b) and (d) were generally well done. In (a) (ii), some candidates calculated $1 - \mathrm{P}(X \le 3)$.

A number of candidates offered clear and well-reasoned solutions to part (c). The two most common successful approaches used to justify that the most likely number of complaints received is zero were either to calculate $\mathrm{P}(X = x)$ for $x = 0, 1, \ldots$ or find that $\mathrm{P}(X = 0) = 0.549 (> 0.5)$. A number of candidates stated that the most number of complaints received was the mean of the distribution $(\lambda = 0.6)$.

Question 12

Parts (a) and (b) were straightforward and were well done.

Parts (c) and (d) were also reasonably well done. A pleasingly large number of candidates recognized that an infinite geometric series was required in part (d).

Question 13

This question was done reasonably well by a large proportion of candidates. Many candidates however were unable to show the required result in part (a). A number of candidates seemingly did not realize how the container was formed while other candidates attempted to fudge the result.

Part (b) was quite well done. In part (b) (i), most candidates were able to correctly calculate $\frac{\mathrm{d}V}{\mathrm{d}h}$ and correctly apply a related rates expression to show the given result. Some candidates however made a sign error when stating $\frac{\mathrm{d}V}{\mathrm{d}t}$. A large number of candidates successfully answered part (b) (ii). In part (b) (iii), successful candidates either set up and calculated an appropriate definite integral or antidifferentiated and found that $t = C$ when $h = 0$.

In part (c), a pleasing number of candidates realized that the water depth stabilized when either $\frac{\mathrm{d}V}{\mathrm{d}t} = 0$ or $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$, sketched an appropriate graph and found the correct value of *h*. Some candidates misinterpreted the situation and attempted to find the coordinates of the local minimum of their graph.

Question 14

Most candidates found this a difficult question with a large number of candidates either not attempting it or making little to no progress. In part (a), most successful candidates squared both equations, added them together, used $\cos^2\theta + \sin^2\theta = 1$ and then simplified their result to show that $\sin(B + C) = \frac{1}{2}$. A number of candidates started with a correct alternative method (see the markscheme for alternative approaches) but were unable to follow them through fully.

In part (b), a small percentage of candidates were able to obtain $B + C = 30°\ (A = 150°)$ or $B + C = 150°\ (A = 30°)$ but were then unable to demonstrate or explain why $A = 30°$ is the only possible value for triangle ABC.

Recommendations and guidance for the teaching of future candidates

Teachers need to continue to highlight question wording that specifically asks for answers in a particular form.

Teachers need to spend more time discussing the symmetry of the normal probability density function curve. It is very important that students use appropriate sketches when answering questions involving the normal distribution. Such sketches can also serve to assist students when they appraise the reasonableness of final answers.

A large proportion of candidates encountered difficulties with questions that required formulation, interpretation and reasoning. Hence it is strongly recommended that teachers set activities and tasks that demand these forms of higher-order mathematical thinking.

In question 9, as *n* is a discrete quantity, the best approach to solving such problems with a GDC is by using a tabular approach rather than using a graphical approach. Hence it is important for teachers to ensure that students understand when it is appropriate to use a graphical solving approach and when it is appropriate to use a tabular solving approach.

Teachers need to discuss and highlight with students the potential limitations and anomalies associated with a GDC's numerical solve feature.

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Paper three - discrete mathematics

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 12	13 - 22	23 - 28	29 - 33	34 - 39	40 - 60

General comments

The general quality of work produced for this option paper was very poor. It was not that it was a difficult paper or that the candidates made mistakes. They just did not seem to know major parts of the syllabus and were easily confused.

The areas of the programme and examination which appeared difficult for the candidates

Candidate found it difficult when they had to prove statements e.g. Q5 (b). The general concept of proof was poor.

The areas of the programme and examination in which candidates appeared well prepared

The use of algorithms e.g. Euclidean, Dijkstra's was reasonable.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a) Well answered.

(b) Also well answered. A few candidates did not use the Euclidean algorithm to find the gcd as instructed.

(c) Many candidates essential said it was true because it was! There is only one mark which means one minute, so it must be a short answer which it is by using Fermat's Little Theorem.

(d) Some good answers but too many did not factorize as instructed, so that they could then spot the consecutive numbers.

(e) Only the better candidates realised that they had to find another factor of 2.

Question 2

Generally there were too many “waffly” words and not enough precise statements leading to conclusions.

(a) Misconceptions were: thinking that a few examples constituted a proof, thinking that the graph had to be connected, taking the edges as the pigeons not the degrees. The pigeon-hole principle was known but not always applied well.

(b).Similar problems as in (a).

(c) Many spurious reasons were given but good candidates went straight to the hand-shaking lemma.

Question 3

(a) Many candidates had the correct route and the cost. Not all showed sufficient working with their Dijkstra’s algorithm. See the mark-scheme for the neat way of laying out the working, including the back-tracking method. This tabular working is efficient, avoids mistakes and saves time.

(b) There was often confusion here between this problem and the travelling salesman. Good candidates started with the number of vertices of odd degree. Weaker candidates just tried to write the answer down without complete reasoning. All the 3 ways of joining the odd vertices had to be considered so that you knew you had the smallest. Sometimes a mark was lost by giving which routes (paths) had to be repeated rather than which roads (edges).

Question 4

(a) A variety of methods were used here. The Chinese Remainder Theorem method (Method 2 on the mark-scheme) is probably the most instructive. Candidates who tried to do it by formula often (as usual) made mistakes and got it wrong. Marks were lost by just saying 31 and not giving mod (55).

(b) Time was lost here by not using Fermat’s Little Theorem as a starting point, although the ad hoc methods will work.

(c) Although it said use parts (a) and (b) not enough candidates saw the connection.

Question 5

(a) Not all candidates wrote this answer down correctly although it was essentially told you in the question.

(b) Very badly answered. Candidates seemed to think that they were being told this relationship (so used it to find u(2)) rather than attempting to prove it.

(c) This distinguished the better candidates. Some candidates though that they could use the method for homogeneous recurrence relations of second order and hence started solving a quadratic. Only the better candidates saw that it was a combined AP/GP.

(d) The best candidates saw this but most had not done enough earlier to get to do this.

Recommendations and guidance for the teaching of future candidates

Although this option involves graphs and trees there was no need for candidates to use graph paper for some of their answers! It made it more difficult to read the answers of candidates that did this, with the papers being scanned. Candidates lost marks by not reading carefully enough what the question actually said and using the hints in the wording of the questions. If a candidate introduces a variable that is not given in the question then they need to say what it stands for so that the examiner can follow their working. They have to remember that they are trying to communicate to the examiner so careful use of words and diagrams can only assist them. Candidates need to be prepared for proofs as well as algorithms and know that "waffly" words rarely gain many marks. Looking at the structure of proofs on the mark-schemes of previous exams will help. For example, you cannot start with what you are trying to prove and examples are not proofs. I cannot really emphasis those last two points enough and we should all be getting this message across. With many of the points mentioned above, careful corrective marking of a trial exam should have assisted the candidates, if they were prepared to learn. It is important that the whole syllabus is covered in the teaching.

Paper three - calculus

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 12	13 - 19	20 - 25	26 - 32	33 - 38	39 - 60

General comments

Generally candidates did well on the early parts of the paper but found the problem solving aspects of the later questions more difficult. One area of concern was the seeming lack of knowledge with regard to isoclines. These are new in the syllabus and this is the first time they had appeared in an exam paper. Some otherwise very successful candidates failed to pick up marks on this question.

The areas of the programme and examination which appeared difficult for the candidates

The following topics caused difficulties for the students; isoclines, differentiation from first principles the mean value theorem.

The areas of the programme and examination in which candidates appeared well prepared

The candidates were well prepared for the questions on integral test, finding a radius of convergence, integrating factor and Euler's method.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a) This was well-done by the candidates. Most knew the required test and could perform it accurately. It should be noted that if a question tells you to use a particular test you do not need to verify the necessary conditions for the test to hold.

(b) (i) This was well done by the majority of candidates.

(ii) Most candidates were aware that they needed to test the end-points of the interval. Most also spotted the link when $x = 1$ with part (a), though stating it was a p-series with an exponent <1 was also acceptable. For the alternating series formed when $x = 3$, the candidates were expected to state that the individual terms were a decreasing sequence, tending to zero.

Question 2

(a) This was generally well done. Some candidates did not realize $e^{-\ln t}$ could be simplified to $\frac{1}{t}$

(b) &(c) These parts were well done by the majority of candidates

(d) Some candidates ignored the instruction to prove from first principles and instead used standard differentiation. Some candidates also only found a derivative from one side.

Question 3

This was a fairly straightforward question on isoclines. As said above, many candidates did not seem to realize what was meant by this term, though many still managed to obtain marks for part (a).

Some candidates did not show numbers on the axes.

Parts (b) and (c) were attempted by very few candidates. Few recognized that the gradient of the curve had to equal the value of *k* on the isocline.

(d) Those candidates who knew the method managed to score well on this part. On most calculators a short program can be written in the exam to make Euler's method very quick. Quite a few candidates were losing time by calculating and writing out many intermediate values, rather than just the *x* and *y* values.

Question 4

(a) Most candidates picked up this mark for realizing the common ratio was $-x^2$

(b) Quite a few candidates did not recognize the importance of 'hence' in this question, losing a lot of time by trying to work out the terms from first principles.

Of those who integrated the formula from part (a) only a handful remembered to include the '+*c*' term, and to verify that this must be equal to zero.

(c) Most candidates were able to achieve some marks on this question. The most commonly lost mark was through not stating that the inequality was unchanged when multiplying by $y-x$ as $y>x$.

(d) The first part of this question proved to be very straightforward for the majority of candidates.

In (ii) very few realized that they had to replace the lower variable in the formula from part (c) by zero.

(e) Candidates found this part difficult, failing to spot which function was required.

(f) Many candidates, even those who did not successfully complete (d) (ii) or (e), realized that these parts gave them the necessary inequality.

Recommendations and guidance for the teaching of future candidates

It is very important that teachers are aware of the recent changes in the syllabus. In their revision of past papers candidates will not come across questions of isoclines, or mean value theorem, so it is important that these (and other new topics) are revised separately.

Students should learn how to do Euler's method quickly on a calculator.

Exam technique needs to be emphasized, particularly in extended questions. Candidates should be very aware when the question says 'hence' and should look where they can get back into a question even if they cannot do earlier parts (for example 3d or 4f).

Paper three - sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 12	13 - 18	19 - 25	26 - 33	34 - 40	41 - 60

The areas of the programme and examination which appeared difficult for the candidates

Candidates found it difficult to formulate coherent proofs and frequently missed crucial steps in reasoning. Understanding the difference between the rigour of a proof and a simple conceptual explanation seemed to be a major problem. Many students did not understand permutations, or at least did not understand the cyclic notation, and many did not know what a homomorphism is. There were surprisingly many conceptual errors in finding subsets.

The areas of the programme and examination in which candidates appeared well prepared

Candidates seemed to know how to complete a Cayley table with modular arithmetic, and understood the concepts of the order of elements. Most students were aware of the conditions for an equivalence relation, knew what injections and surjections are, and knew what Lagranges theorem is.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

The first two parts of this question were generally well done. It was surprising to see how many difficulties there were with parts (c) and (d) with many answers given as {4}, {11} and {14} for example.

Question 2

Most students indicated an understanding of the concepts of Injection and Surjection, but many did not give rigorous proofs. Even where graphs were used, it was very common for a sketch to be so imprecise with no asymptotes marked that it was difficult to award even partial credit. Some candidates mistakenly stated that the function was not surjective because 0.5 was not in the domain.

Question 3

Many students were unable to start the question, seemingly as they did not understand the cyclic notation. Many of those that did understand found it quite straightforward to obtain good marks on this question.

Question 4

Part (a) was well answered by those who understood what a homomorphism is. However many candidates simply did not have this knowledge and consequently could not get into the question. Part (b) was well answered, even by those who could not do (a). However, there were many who having not understood what a homomorphism is, made no attempt on this easy question part. Understandably many lost a mark through not simplifying p^{-2} to p^{5}. Those who knew what a homomorphism is generally obtained good marks in part (c).

Question 5

Many students obtained just half marks in (a) for not stating the requirement of the order to be finite. Part (b) should have been more straightforward than many found. In part (c) it was evident that most candidates knew what to do, but being a more difficult question fell down on a lack of rigour. Nonetheless, many candidates obtained full or partial marks on this question part. Part (d) enabled many candidates to obtain, at least partial marks, but there were few students with the insight to be able to answer part (e) satisfactorily.

Recommendations and guidance for the teaching of future candidates

An emphasis on formal proof and rigour is necessary for this option and teachers should be aware of that, both when choosing the option, and whilst preparing the students for the exam. Students need to develop an attention to detail and sufficient example questions and subsequent feedback need to be given if they are to be ready for the exam. Clearly all areas of the syllabus needs to be covered as missing something simple like the cycle form of permutations can render an easy question impossible for the candidates. There are a number of proofs in the syllabus that simply have to be learned. It is important that candidates are made aware of this.

Paper three - statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 15	16 - 23	24 - 29	30 - 36	37 - 42	43 - 60

The areas of the programme and examination which appeared difficult for the candidates

Most candidates seem to know the definition of the probability generating function but many are uncomfortable using these functions.

Rather surprisingly, many candidates were unable to deal with the question involving the standard error of the mean. Also, many candidates seem to be unaware of the distinction between critical value and critical region.

Most candidates know that to find the cumulative distribution function given the probability density function requires integration but some candidates are unable to carry out this process.

The areas of the programme and examination in which candidates appeared well prepared

Candidates are generally well prepared in the area of correlation and regression.

Most candidates are confident in the use of the negative binomial distribution.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Part (a) was correctly answered by most candidates. Some graphs were difficult to mark because candidates drew their lines on top of the ruled lines in the answer book. Candidates should be advised not to do this. Candidates should also be aware that the command term ‘sketch’ requires relevant values to be indicated. In (b), most candidates realised that the cumulative distribution function had to be found by integration but the limits were sometimes incorrect. In (c), candidates who found the upper and lower quartiles correctly sometimes gave the interquartile range as [0.5,2]. It is important for candidates to realise that that the word range has a different meaning in statistics compared with other branches of mathematics.

Question 2

Part (a) was well answered, using the negative binomial distribution $NB(3, 0.2)$, by many candidates. Some candidates began by using the binomial distribution $B(5, 0.2)$ which is a valid method as long as it is followed by multiplying by 0.2 but this final step was not always carried out successfully. Part (b) was well answered by the majority of candidates. In (c), candidates who used the binomial distribution $B(8, 0.2)$ were generally successful. Candidates who used the negative binomial distribution

$Y \approx NB(3, 0.2)$ to evaluate $P(Y > 8)$ were usually unsuccessful because of the large amount of computation involved.

Question 3

Solutions to (a) were often disappointing with few candidates gaining full marks, a common error being failure to state that

$E(XY) = E(X)E(Y)$ or $\mathrm{E}\big((X-\mu_x)(Y-\mu_y)\big) = \mathrm{E}(X-\mu_x)\mathrm{E}(Y-\mu_y)$ in the case of independence.

In (b), the hypotheses were sometimes given incorrectly. Some candidates gave H_1 as $\rho \neq 0$, not seeing that a one-tailed test was required. A more serious error was giving the hypotheses as $H_0 : r = 0, H_1 : r < 0$ which shows a complete misunderstanding of the situation. Subsequent parts of the question were well answered in general.

Question 4

Solutions to the different parts of this question proved to be extremely variable in quality with some parts well answered by the majority of the candidates and other parts accessible to only a few candidates. Part (a) was well answered in general although the presentation was sometimes poor with some candidates doing the differentiation of $G(t)$ and the substitution of $t = 1$ simultaneously. Part (b) was well answered in general, the most common error being to state that $Var(2X - Y) = Var(2X) - Var(Y)$.

Parts (c) and (d) were well answered by the majority of candidates. Solutions to (e), however, were extremely disappointing with few candidates giving correct solutions. A common incorrect solution was the following:

$$G_{X+Y}(t) = G_X(t)G_Y(t)$$

Differentiating,

$$G'_{X+Y}(t) = G'_X(t)G_Y(t) + G_X(t)G'_Y(t)$$

$$\mathrm{E}(X+Y) = G'_{X+Y}(1) = \mathrm{E}(X)\times 1 + \mathrm{E}(Y)\times 1 = 2\lambda$$

This is correct mathematics but it does not show that $X+Y$ is Poisson and it was given no credit. Even the majority of candidates who showed that $G_{X+Y}(t)=\mathrm{e}^{2\lambda(t-1)}$ failed to state that this result proved that $X+Y$ is Poisson and they usually differentiated this function to show that $E\left(X+Y\right) = 2\lambda$.

In (f), most candidates stated that $G_{X+Y}(1)=1$ even if they were unable to determine $G_{X+Y}(t)$ but many candidates were unable to evaluate $G_{X+Y}(-1)$. Very few correct solutions were seen to (g) even if the candidates correctly evaluated $G_{X+Y}(1)$ and $G_{X+Y}(-1)$.

Question 5

Solutions to this question were generally disappointing.

In (a), the standard error of the mean was often taken to be σ (1.2) instead of $\frac{\sigma}{\sqrt{n}}$ (0.3) and the solution sometimes ended with the critical value without the critical region being given.

In (b), the standard error was again often incorrect. In (c), the question was often misunderstood with candidates finding the weighted mean of the two means,

ie $0.9 \times 5.2 + 0.1 \times 4.6 = 5.14$ instead of the weighted mean of two probabilities. Without having the solution to (c), part (d) was inaccessible to most of the candidates so that very few correct solutions were seen.

Recommendations and guidance for the teaching of future candidates

Candidates should be familiar with the definitions and applications of probability generating functions.

Candidates should clearly understand that the standard deviation (standard error) of $\bar{X}$ is $\frac{\sigma}{\sqrt{n}}$ and not σ. It is recommended that this result should be derived in class.