November 2013 subject reports

MATHEMATICS HL

Overall grade boundaries

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 41	42 - 54	55 - 66	67 - 79	80 - 100

Series and differential equations

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 40	41 - 53	54 - 67	68 - 79	80 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 39	40 - 52	53 - 65	66 - 78	79 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 30	31 - 42	43 - 55	56 - 68	69 - 81	82 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 18	19 - 23	24 - 29	30 - 34	35 - 40

The range and suitability of the work submitted

The portfolios this session were generally well-written, starting with engaging introductions and concluding with thorough summaries that conveyed a good understanding of the tasks on the part of most candidates. On the other hand, the sheer length of some pieces of work, particularly those which extended well beyond forty pages, detracted from the clarity and conciseness expected in the work. It was also disappointing to note a number of portfolios that ended abruptly, showing incomplete work that only met some of the expectations given in the tasks.

The assessment criteria appear to have been well understood by teachers, although the awarding of full marks against Criterion F could not always be supported by the quality of the work shown.

The tasks:

As in the May 2013 session, nearly all of the portfolios this session contained tasks that were taken from the publication, “Tasks for use in 2012 and 2013”, with the two most popular being “Shadow Functions” and “Dice Games”. It is disturbing to note that the Subject Reports from the past three sessions have been ignored by some teachers, as the task, “Patterns from Complex Numbers”, was included in many samples with some students and teachers still misreading the instruction and producing a familiar and trivial consequence of De Moivre’s theorem: that consecutive n-th roots of unity, when connected, form a regular polygon. Significant deductions were made for only arriving at this conclusion as the intended generalisations were completely missed. Please note the advice given in each of the Subject Reports for May 2012, November 2012, and May 2013.

Candidate performance against each criterion

The candidates generally performed well against Criterion A, although the occasional student used calculator notation for multiplication or exponentiation, or did not use standard subscript notation as required.

As noted above, most students produced very well-written pieces of work with thorough explanations; however, untitled and unlabelled graphs were sometimes still in evidence. Moreover, moderators were not impressed with the excessively lengthy documents consisting merely of repetitive, “copied-and-pasted” pages of identical paragraphs with similar graphs and seemingly unending spreadsheet listings.

The candidates generally produced good work and the teachers have assessed the work well against Criteria C and D. However, it was disappointing to note that in some type I work, the “investigation” was only perfunctory, with little evidence of a search for patterns to warrant a conjecture, let alone a generalisation. It would appear that some students started with a prior notion of the end result without conducting an investigation.

The use of technology varied considerably. While some students merely illustrated their investigations with graphs, others used sliders in their graphs and dynamic spreadsheets with variable parameters. Full marks were often given much too generously for the use of a large collection of similar graphs. Also, in the type II tasks, graphs should have been adjusted to reflect the appropriate domain: positive integers only, all real numbers, etc.

There were many pieces of good and complete work; however, the awarding of full marks in Criterion F requires evidence of mathematical sophistication that extends beyond the mere completion of the task. It would also appear that some teachers incorrectly awarded full marks against this criterion for work that had been penalised against several other criteria.

Recommendations and guidance for the teaching of future candidates

The suggestions below have been offered to teachers in past Subject Reports, but may be worth noting again:

Teachers are expected to write directly on the work submitted, not only to provide feedback to their candidates, but information to the moderators as well. The use of Form B is suggested to allow the teacher to indicate more relevant and descriptive comments.

Include the background information to each portfolio task with each sample. A number of teachers have been very helpful in doing so, particularly with the use of Form A or through anecdotal comments, but many have not. Moderators find them very useful in determining the context in which each task was given when confirming the achievement levels awarded.

Include a solution key with each task submitted. Doing so will give the moderator some insight into the reasons for the teacher’s assessment of the work. Only a few teachers provided solution keys to the tasks with their samples. Some provided a marking schedule instead of a solution key.

When there is more than one HL teacher involved in marking the portfolio work, the use of a common marking scheme has been effective in ensuring consistency in marking.

This examination session marks the last time when portfolios of student work are expected of candidates. In marking the students’ coursework in Explorations, please note that a different set of Assessment Criteria needs to be considered. Please review the Criteria and the Achievement Level descriptors carefully.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 33	34 - 46	47 - 62	63 - 79	80 - 95	96 - 120

The areas of the programme and examination which appeared difficult for the candidates

Transformations of $y = \ln x$. Domains and ranges of (composite) trigonometric functions. Candidates' underlying understanding of mathematical induction. Questions 8b, c, 11e, f and Question 12g proved problematic.

The areas of the programme and examination in which candidates appeared well prepared

Stationary points on curves, geometric series, applications of De Moivre's Theorem and proof by induction.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Many candidates scored full marks on what was thought to be an easy first question. However, a number of candidates wrote down two correct equations but proceeded to make algebraic errors and thus found incorrect values for p and q. A small number also attempted to answer this question using long division, but fully correct answers using this technique were rarely seen.

Question 2

This was very well answered and many fully correct solutions were seen. A small number of candidates made arithmetic mistakes in part a) and thus lost one or two accuracy marks. A few also seemed unaware of the formula $Var(X) = E(X^2) - E(X)^2$ and resorted to seeking an alternative, sometimes even attempting to apply a clearly incorrect $Var(X) = \sum (x_i - \mu)^2$.

Question 3

A number of candidates were able to answer a) and b) correctly but found part c) more challenging. Correct sketches for the inverse were seen, but with a few missing a horizontal asymptote. The range in part b) was usually seen correctly. In part c), only a small number of very good candidates were able to gain full marks. A large number used the point $(4,0)$ to form the equation $4a + b = 1$ but were unable (or did not recognise the need) to use the asymptote to form a second equation.

Question 4

The vast majority of candidates understood the meaning of a matrix being singular, and were able to calculate its determinant correctly. A number found $a + b = \frac{b-a}{a-b}$ but stopped at this point. Around two-thirds of candidates went on to find $a + b = -1$. Consistently seen amongst the very weakest candidates was an attempt to find the inverse of A^{-1}.

Question 5

The majority of candidates were able to apply implicit differentiation and the product rule correctly to obtain $3x^2\left(1+y^2\right)=0$. The better then recognised that $x=0$ was the only possible solution. Such candidates usually went on to obtain full marks. A number decided that $y=\pm 1$ though then made no further progress. The solution set $x=0$ and $y=\pm i$ was also occasionally seen. A small minority found the correct x and y values for the three co-ordinates but then surprisingly expressed them as $(0,0),(3,0)$ and $(-3,0)$.

Question 6

It was pleasing to see a great many clear and comprehensive answers for this relatively straightforward induction question. The inductive step only seemed to pose problems for the very weakest candidates. As in previous sessions, marks were mainly lost by candidates writing variations on 'Let $n=k$', rather than 'Assume true for $n=k$'. The final reasoning step still needs attention, with variations on '$n=k+1 \text{ true} \Rightarrow n=k$ true' evident, suggesting that mathematical induction as a technique is not clearly understood.

Question 7

This question was invariably answered very well. Candidates showed some skill in algebraic manipulation to derive the given answer in part a). Poor attempts at part b) were a rarity, though the final mark was sometimes lost after a correctly substituted equation was seen but with little follow-up work.

Question 8

Part a) often proved to be an easy 4 marks for candidates. A number were surprisingly content to gain the first 3 marks but were unable to make the final step by substituting $1-\sin^2 y$ for $\cos^2 y$.

Parts b) and c) were more often than not, problematic. Some puzzling 'working' was often seen, with candidates making little headway. Otherwise good candidates were able to answer part b), though correct solutions for c) were a rarity. The range $g\in[-4,\frac{4}{3}]$ was sometimes seen, but gained no marks.

Question 9

Part a) was answered well, and a very large proportion of candidates displayed familiarity and confidence with this type of change-of base equation.

In part b), good candidates were able to solve this proficiently. A number obtained only one solution, either through observation or mistakenly cancelling a $\ln x$ term. An incorrect solution $x=e^3$ was somewhat prevalent amongst the weaker candidates.

Question 10

Part a) proved to be an easy start for the vast majority of candidates. Full marks for part b) were again likewise seen, though a small number shied away from considering the sign of their second derivative, despite the question asking them to do so.

Part c) again proved to be an easily earned 2 marks.

Many candidates lost their way in part d). A variety of possibilities for $g(x)$ were suggested, commonly $2xe^{-2x}$, $\frac{xe^{-x}}{2}$ or similar variations. Despite section ii) being worth only one mark, (and 'state' being present in the question), many laborious attempts at further differentiation were seen. Part diii was usually answered well by those who gave the correct function for $g(x)$.

Part e) was also answered well by those who had earned full marks up to that point.

While the integration by parts technique was clearly understood, it was somewhat surprising how many careless slips were seen in this part of the question. Only a minority gained full marks for part f).

Question 11

Part a) proved an easy start, though a few (weaker) candidates still believe $\overrightarrow{CA}$ to be $\overrightarrow{OC}-\overrightarrow{OA}$.

Part b) was an easy 3 marks and incorrect answers were rare.

Part c) was answered well, though reasoning sometimes seemed sparse, especially given that this was a 'show that' question.

Part d) proved more challenging, despite being a very standard question. Many candidates gained only 2 marks, either through correctly calculating the direction vector, or by successfully eliminating one of the variables. A number of clear fully correct solutions were seen, though the absence of '$\boldsymbol{r} =$' is still prevalent, and candidates might be reminded of the correct form for the vector equation of a line.

Part e) proved a puzzle for most, though an attempt to use row reduction on an augmented matrix seemed to be the choice way for most successful candidates.

Only the very best were able to demonstrate a complete understanding of intersecting planes and thus answer part f) correctly.

Question 12

Part a) has appeared several times before, though with it again being a 'show that' question, some candidates still need to be more aware of the need to show every step in their working, including the result that $\sin(-n\theta) = -\sin(n\theta)$.

Part b) was usually answered correctly.

Part c) was again often answered correctly, though some candidates often less successfully utilised a trig-only approach rather than taking note of part b).

Part d) was a good source of marks for those who kept with the spirit of using complex numbers for this type of question. Some limited attempts at trig-only solutions were seen, and correct solutions using this approach were extremely rare.

Part e) was well answered, though numerical slips were often common. A small number integrated $\sin n\theta$ as $n\cos n\theta$.

A large number of candidates did not realise the help that part e) inevitably provided for part f). Some correctly expressed the volume as $\pi\int\cos^4 x dx - \pi\int\cos^6 x dx$ and thus gained the first 2 marks but were able to progress no further. Only a small number of able candidates were able to obtain the correct answer of $\frac{\pi^2}{32}$

Part g) proved to be a challenge for the vast majority, though it was pleasing to see some of the highest scoring candidates gain all 3 marks.

Recommendations and guidance for the teaching of future candidates

Some otherwise standard techniques need particular care in their application. Intersections of planes, integration by parts, and even the remainder theorem were all questions where careless errors were seen. While there were some very high scoring and very well presented scripts, it should be impressed upon candidates that presentation is important, particularly for graph sketching questions, and those requiring careful algebraic working.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 48	49 - 64	65 - 79	80 - 95	96 - 120

The areas of the programme and examination which appeared difficult for the candidates

Stating answers to the required number of significant figures or decimal places, sketching clearly labelled and accurate graphs, calculating the modulus of a complex number z, using the scalar product to find a vector equation of a line perpendicular to a given line, formulating an equation from a diagram, using a non-routine trigonometric substitution to find an indefinite integral, finding the median of a continuous probability density function, recognising a binomially distributed random variable in a worded context, neglecting to evaluate a constant of integration, solving a differential equation that required separating variables, not recognising or being able to use alternative forms of acceleration, finding an inverse function including recognising conditions for which an inverse function does not exist and formulating a definite integral to calculate a volume of solid of revolution.

The areas of the programme and examination in which candidates appeared well prepared

Solving a straightforward matrix equation with a GDC, finding the first term and common difference of an arithmetic sequence, using a GDC to find the values of μ and of σ of a normal distribution, answering a routine conditional probability question, calculating a scalar product in terms of a trigonometric function, solving a trigonometric equation (a quadratic in sin θ), calculating the area of a segment from a sector and a triangle, applying the Poisson distribution in routine contexts, finding the mean of a continuous probability density function and determining an equation of a normal.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Generally well done by candidates who used a GDC. Candidates who used elimination techniques without a GDC often made algebraic errors. A few candidates attempted to calculate either $\boldsymbol{X} = \boldsymbol{BA}^{-1}$ or $\boldsymbol{X} = \boldsymbol{AB}$.

Question 2

Both parts were very well done. In part (a), a few candidates made a careless algebraic error when attempting to find the value of a or d.

In part (b), a few candidates attempted to find the value of *n* for which $u_n > 5000$. Some candidates used the incorrect formula $S_n = \frac{n}{2}[u_1 + (n-1)d]$. A number of candidates unnecessarily attempted to simplify S_n . Most successful candidates in part (b) adopted a graphical approach and communicated their solution effectively. A few candidates did not state their value of n as an integer.

Question 3

Part (a) was reasonably well done although more care was required when showing correct endpoint behaviour. A number of sketch graphs suggested the existence of either a vertical axis intercept or displayed an open circle on the vertical axis. A large number of candidates did not state the coordinates of the various key features correct to three significant figures. A large number of candidates did not locate the maximum near $x = 10$. Most candidates were able to locate the *x*-axis intercepts and the minimum. A few candidates unfortunately sketched a graph from a GDC set in degrees.

In part (b), a number of candidates identified the correct critical values but used incorrect inequality signs. Some candidates attempted to solve the inequality algebraically.

Question 4

Generally well done. Most candidates made correct use of the symmetry of the normal curve and the inverse normal to set up a correct pair of equations involving μ and σ. A few candidates expressed equations containing the GDC command term invNorm.

A few candidates did not express their answers correct to the nearest minute and a few candidates performed erroneous conversions from hours to minutes.

Question 5

Both parts were very well done. In part (a), most candidates successfully used a tree diagram.

In part (b), most candidates correctly used conditional probability considerations.

Question 6

Part (a) was reasonably well done. When multiplying and dividing by the conjugate of $a-\mathrm{i}$, some candidates incorrectly determined their denominator as a^2-1.

In part (b), a significant number of candidates were able to correctly expand and simplify $|z|$ although many candidates appeared to not understand the definition of $|z|$.

Question 7

Part (a) was very well done. Most candidates were able to use the scalar product and $\cos^2\theta = 1-\sin^2\theta$ to show the required result.

Part (b) was reasonably well done. A few candidates confused ‘smallest possible positive value’ with a minimum function value. Some candidates gave $\theta = 0.34$ as their final answer.

Question 8

Part (a) was very well done. Most candidates knew how to calculate the area of a segment. A few candidates used $r = 20$.

Part (b) challenged a large proportion of candidates. A common error was to equate the unshaded area and the shaded area. Some candidates expressed their final answer correct to three significant figures rather than to the four significant figures specified.

Question 9

Part (a) was not well done. Most candidates recognised the need to calculate a scalar product. Some candidates made careless sign or arithmetic errors when solving for λ. A few candidates neglected to express their final answer in the form ‘$\boldsymbol{r} =$’.

Candidates who were successful in answering part (a) generally answered part (b) correctly. The large majority of successful candidates calculated $|\overrightarrow{\mathrm{OP}}|$.

Question 10

Most candidates found this a challenging question. A large majority of candidates were able to change variable from x to u but were not able to make any further progress.

Question 11

Parts (a) and (b) were generally well done by a large proportion of candidates. In part (a) (ii), some candidates used an incorrect inequality (e.g. $\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X \leq 3)$) while in (a) (iii) some candidates did not use $\mu = 2.4$. In part (a) (iv), a number of candidates either did not realise that they needed to consider a binomial random variable or did so using incorrect parameters.

In (b) (i), some candidates gave their value of *k* correct to three significant figures rather than correct to six decimal places. In parts (b) (i), (ii) and (iv), a large number of candidates unnecessarily used integration by parts. In part (b) (iii), a number of candidates thought the mode of X was $f(3)$ rather than $x = 3$. In part (b) (iv), a number of candidates did not consider the domain of *f* when attempting to find the median or checking their solution.

Question 12

In part (a) (i), a surprisingly large number of candidates were unable to correctly find *v* in terms of t. The most common error was to neglect the initial condition and write $\int 3\cos\frac{t}{4}\,\mathrm{d}t = 12\sin\frac{t}{4}$. Parts (ii) and (iii) were generally well done by those who answered part (a) (i) correctly. Due to errors made in part (a) (i), some velocity/time graphs displayed $v > 0$ for $0 \leq t \leq 8\pi$.

Parts (b) (i) and (iii) were generally well done. It was pleasing to see a large number of candidates able to solve a differential equation that required the separation of variables. Candidates that successfully answered part (a) (iii) generally used $s = \int_0^{\frac{\pi}{8}} 2\tan\left(\frac{\pi - 8t}{4}\right)\mathrm{d}t$ rather than starting with $\frac{\mathrm{d}v}{\mathrm{d}s} = -\frac{(v^2+4)}{v}$. In part (a) (ii), a significant proportion of candidates could not use either $\frac{\mathrm{d}v}{\mathrm{d}t} = v\frac{\mathrm{d}v}{\mathrm{d}s}$ or $\frac{\mathrm{d}v}{\mathrm{d}s} = \frac{\mathrm{d}v}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}s}$ to show the given result.

Question 13

In part (a) (i), successful candidates typically sketched the graph of $y = f(x)$, applied the horizontal line test to the graph and concluded that the function was not 1 – 1 (it did not obey the horizontal line test).

In part (a) (ii), a large number of candidates were able to show that the equation of the normal at point P was $9x + 12y - 9\ln 3 - 20 = 0$. A few candidates used the gradient of the tangent rather than using it to find the gradient of the normal.

Part (a) (iii) challenged most candidates. Most successful candidates graphed $y = f(x)$ and $y = x\,f'(x)$ on the same set of axes and found the *x*-coordinates of the intersection points.

Part (b) (i) challenged most candidates. While a large number of candidates seemed to understand how to find an inverse function, poor algebra skills (e.g. erroneously taking the natural logarithm of both sides) meant that very few candidates were able to form a quadratic in either e^x or e^y.

Only a few candidates successfully answered part (b) (ii).

Recommendations and guidance for the teaching of future candidates

- Raise awareness of the importance of basic graph sketching skills including careful consideration of domain, range, key features and endpoint behaviour.
- Raise awareness for the need to support an answer with an accurate sketch and/or indication of the method even when the answer can be obtained with GDC.
- Encourage candidates to use GDC to solve equations and integrate numerically; provide a wide range of problems that allow students to explore more advanced features of the GDC including a GDC's table, list or spreadsheet feature.
- Encourage candidates to store numerical answers obtained with GDC or show how to carry work through using enough significant figures so that the final answers are expressed correct to the appropriate degree of accuracy.
- Clarify the definition of a continuous probability density function.
- Encourage candidates to use mathematical notation, conventions and symbols rather than GDC command terms when answering probability distribution questions.
- Emphasize what it means to convincingly answer 'show that' examination questions and provide many examples of mathematical proofs.
- Clarify the meaning of each of the command terms in the Mathematics HL guide.
- Provide many examples of past examination questions and teach efficient ways of answering these examination questions; provide timed practice to improve candidates' efficiency in answering examination papers.

Paper three - Discrete mathematics

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 17	18 - 26	27 - 32	33 - 37	38 - 43	44 - 60

The areas of the programme and examination which appeared difficult for the candidates

Generally the number theory part of the programme was not so well prepared. There were few good attempts at either of the two questions on number theory.

The areas of the programme and examination in which candidates appeared well prepared

Graph theory seemed much better prepared, with many students at least attempting the questions in the correct vein.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Well answered by the majority of candidates.

Question 2

Part (a) was generally well answered. There were many examples of full marks in this part. Part (b) caused a few more difficulties, although there were many good solutions. Few candidates used the matrix to find the number of edges, preferring instead to draw the graph. A surprising number of students confused the ideas of having vertices of odd degree

Question 3

Surprisingly few good answers. Part (a) had a number of correct solutions, but there were also many that seemed to be a memorised solution, not properly expressed – and consequently wrong. In part (b) many failed to understand the question, not registering that x and y were digits rather than numbers. Part (c)(i) was generally well answered, although there were a number of longer methods applied, and few managed to do (c)(ii).

Question 4

Part (a) was generally well answered, with a variety of interpretations accepted. Part (b) also had a number of acceptable possibilities. In (c), although there were many good answers, a number of candidates failed to show what they were doing and so could not get full credit. Part (d) was generally well answered, but there were few good attempts at part (e)

Question 5

Many students were able to get partial credit for part (a), although few managed to gain full marks. There seemed to be very few good attempts at part (b), many failing at the outset to understand what was meant by 3^{3^m} .

Recommendations and guidance for the teaching of future candidates

It is clear that the number theory part of this option is the more difficult and so probably needs greater preparation.

Paper three - Series and differential equations

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 17	18 - 24	25 - 31	32 - 38	39 - 45	46 - 60

The areas of the programme and examination which appeared difficult for the candidates

On this paper, although candidates seemed reasonably well prepared for this option, they had difficulty with simplifying, manipulating, and integrating algebraic expressions and power series, and solving absolute value inequalities. Candidates found difficulty in working with the definition of limit of a sequence. There was some confusion in showing convergence of a sequence by attempting to apply a series convergence test.

They had some difficulty in approximating the sum of a Maclaurin series to the given degree of accuracy, and used the GDC rather than a reasoned argument.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates were able to successfully use definitions and theorems to justify results, as well as informally justify results using the GDC. They knew when to apply a particular convergence test for series, and the conditions necessary to apply the test. Most candidates were able to do the algebraic manipulation involved in partial fractions decomposition, and many could then work with the telescoping series to arrive at the correct sum of the series.

On the whole candidates also appeared to have been well prepared for calculating limits and dealing with indeterminate forms and in using Euler’s method to approximate the solution to a differential equation.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates were able to answer part (a) and many gained a fully correct answer. A number of candidates ignored the factor 2 in the numerator and this led to candidates being penalised. In some cases candidates were not able to identify an appropriate series to compare with. Most candidates used the Comparison test rather than the Limit comparison test.

Part (b) (i) was well answered but many candidates did not know what to do with the values found in part (i) to answer part (ii). Few candidates could not do the partial fraction decomposition, and some had arithmetic mistakes in arriving at the sum of the series.

Question 2

Most candidates were successful in answering part (a) using a variety of methods. The majority of candidates scored some marks, if not full marks. Surprisingly, some candidates did not have the correct graph for the function the sequence represents. They obviously did not enter it correctly into their GDCs. Others used one of the two definitions for showing that a sequence is increasing/decreasing, but made mistakes with the algebraic manipulation of the expression, thereby arriving at an incorrect answer. Part (b) was less well answered with many candidates ignoring the command terms 'show that' and 'find' and just writing down the value of the limit. Some candidates attempted to use convergence tests for series with this sequence. Part (c) of this question was found challenging by the majority of candidates due to difficulties in solving inequalities involving absolute value.

Question 3

Part (a) was well answered by most candidates. In a few cases calculation errors and early rounding errors prevented candidates from achieving full marks, but most candidates scored at least a few marks here. In part (b) some candidates failed to convincingly show the given result. Part (c) proved to be a hard question for many candidates and a significant number of candidates had difficulty manipulating the algebraic expression, and either had the incorrect expression to integrate, or incorrectly integrated the correct expression. Many candidates reached as far as separating the variables correctly but integrating proved to be too difficult for many of them although most realised that the expression on v had to be split into two separate integrals. Most candidates made good attempts to evaluate the arbitrary constant and arrived at a correct or almost correct expression (sign errors were a common error) which allowed follow through for part b (ii). In some cases however the expression obtained was too simple or was omitted and it was not possible to grant follow through marks.

Question 4

Part (a) of this question was accessible to the vast majority of candidates, who recognised that L‟Hôpital‟s rule could be used. Most candidates were successful in finding the limit, with some making calculation errors. Candidates that attempted to use $\lim_{x\to 0}\frac{\sin x}{x}=1$ or a combination of this result and L‟Hôpital‟s rule were less successful. In part (b) most candidates showed to be familiar with the substitution given and were successful in showing the result. Very few candidates were able to do part (c) successfully. Most used trial and error to arrive at the answer.

Question 5

Overall candidates made good attempts to parts (a) and most candidates realized that the graph contained the origin; however many candidates had difficulty rendering the correct shape of the graph of f'. Part b(i) was also well answered although some candidates where not very clear and digressed a lot. Part (ii) was less successful with most candidates scoring just part of the marks. A small number of candidates answered part (c) correctly with a valid reason.

Recommendations and guidance for the teaching of future candidates

Candidates seemed to be reasonably well prepared with the concepts, definitions and theorems of this option. Most of their difficulty came with the core skills needed in this option, i.e., ability to successfully manipulate algebraic expressions, solving inequalities, and integration. More practice is needed in these basic core areas. When teaching future candidates bear in mind that:

- Students need to have a good understanding of the core calculus and master differentiation and integration techniques to be successful with this option.
- Students need to be made aware of the importance of showing working out whenever the command terms indicate that some work is required.
- Students need many opportunities to explain in writing their results and justify their answers, as many seem to struggle in providing clear valid reasons.
- Students need to be made aware of the risks of early rounding that may lead to inaccurate or incorrect answers and respect the instructions about level of accuracy of answers required in general and for particular questions.

Paper three - Sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 5	6 - 10	11 - 19	20 - 27	28 - 34	35 - 42	43 - 60

The areas of the programme and examination which appeared difficult for the candidates

- Candidates were particularly weak in providing a logical sequence of steps to justify their results.
- Definitions and theorems needed to justify results were often correctly quoted, but not convincingly applied in solving the given problem.
- Candidates had difficulty proving that two infinite groups are isomorphic, proving that a group is cyclic, and proving a statement is not true by finding a counter-example.
- Most candidates had difficulty with finding equivalence classes.
- Many candidates had difficulty with algebraic manipulations.

The areas of the programme and examination in which candidates appeared well prepared

- Candidates seemed to know fairly well the statements of definitions and theorems in this option.
- Candidates were able to successfully state Group axioms, and make reasonable progress in showing that a given set under a binary operation forms a group.
- Candidates were fairly successful with matrix operations, sets axioms, and equivalence relations.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

For part (a), given the command term ‘show that’ and the number of marks for this part, the best approach is a graphical one, i.e., an informal approach. Many candidates chose an algebraic approach and generally made correct statements for injective and surjective. However, they often did not follow through with the necessary algebraic manipulation to make a valid conclusion. In part (b), many candidates were not able to provide valid counter-examples. In part (c) It was obvious that quite a few candidates had not seen this type of function before. Those that were able to find the inverse generally did not justify their result, and hence could not earn the final R mark.

Question 2

In part (a), many candidates could not provide a logical sequence of steps to show that G is cyclic. In particular, although they correctly quoted Lagrange’s theorem, they did not always consider all the orders of a, i.e., all the factors of 12, omitting in particular 1 as a factor. Some candidates did not state the second generator, in particular a^{-1}. Very few candidates were successful in finding the required subgroup, although they were obviously familiar with setting up a Cayley table. For part (c), no candidate was able to provide a counter-example.

Question 3

In part (a), many candidates attempted to find the inverse of a 3x3 matrix using the known method for a 2x2. Very few candidates actually thought of multiplying the given matrix with its given inverse in order to arrive at the identity matrix. Although most candidates could successfully list the group axioms, very few could verify them, in particular closure. In part (b), most candidates could not identify the isomorphism.

Question 4

Part (a) was fairly well answered by many candidates. They knew how to apply the equivalence relations axioms in this particular example. Part (b) however proved to be very challenging and hardly any correct answers were seen.

Question 5

For part (a), candidates who chose to prove the given statement using the properties of Sets were often successful with the proof. Some candidates chose to use the definition of equality of sets, but made little to no progress. In a few cases candidates attempted to use Venn diagrams as a proof. Part (b) was challenging for most candidates, and few correct answers were seen.

Recommendations and guidance for the teaching of future candidates

- Candidates need to know and practice the difference between a formal proof and an informal justification.
- Candidates need to be given many opportunities to apply the concepts, definitions and theorems using a variety of examples and contexts.

- Candidates need to be aware of, and make use of, counter-examples
- Candidates should be made aware of well-known groups, such as the Klein V-group, S3 groups, etc., that would be useful for both examples and counter-examples.
- This option could be introduced early in the program so as to provide ample opportunities for explicit links to core topics and structures (for example, groups of functions under operations such as differentiation, composition, etc.)
- Candidates would benefit from doing many past papers in order to better gauge how much working out is required to justify results.

Paper three - Statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 10	11 - 20	21 - 29	30 - 36	37 - 42	43 - 49	50 - 60

The areas of the programme and examination which appeared difficult for the candidates

Some candidates appeared to be unaware of what has to be done in a ‘show that’ question. In this situation, no credit is given for simply writing down the answer, the justification must be given.

It was noted that, in several instances, incorrect hypotheses were given. A fairly common error was to give the hypotheses the wrong way around.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates are confident in using their graphical display calculator.

Most candidates are able to solve problems involving linear combinations of normal random variables.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

In (a)(i), the candidates were required to show that the estimate of the mean is 113.21 so that those who stated simply ‘Using my GDC, mean = 113.21’ were given no credit. Candidates were expected to indicate that the interval midpoints were used and to show the appropriate formula. In (a)(ii), division by either 999 or 1000 was accepted, partly because of the large sample size and partly because the question did not ask for an unbiased estimate of variance. Solutions to (c) were often badly written, often quite difficult to understand exactly what was being stated.

Question 2

This question was well answered in general. The most common error was to calculate the variance of the total weight in (b) as $4^2 \times 14^2 + 6^2 \times 12^2$ instead of $4 \times 14^2 + 6 \times 12^2$.

Question 3

It was disappointing to see that some candidates wrote incorrect hypotheses, eg 'H_0 : Data are binomial ; H_1 : Data are not binomial' without specifying any parameters. In (a), the degrees of freedom were sometimes given incorrectly as 6. Part (b) caused unexpected problems for many candidates who misunderstood the question and gave 'increase the number of trials' as their answer.

Question 4

It was again disappointing to see many candidates giving incorrect hypotheses. A common error was to give the hypotheses the wrong way around. Candidates should be aware that in this type of problem the null hypothesis always represents the status quo. Also, some candidates defined 'd = time before – time after' and then gave the hypotheses incorrectly as H_0 : $d = 0$ or $\bar{d} = 0$; H_1 : $d > 0$ or $\bar{d} > 0$. It is important to note that the parameter being tested here is $E(d)$ or μ_d although μ was accepted.

Question 5

Parts (a) and (b) were well answered by most candidates. The most common error in (a) was to calculate $E(2X+3Y)$ correctly as 12 and then state that, because the sum is Poisson, the variance is also 12. Many of these candidates then stated in (b) that the sum is Poisson because the mean and variance are equal, without apparently realising the circularity of their argument. Although (c) was intended as a possible hint for solving (d) and (e), many candidates simply noted that $X+Y$ is $P_o(5)$ which led immediately to the correct answer. Some candidates tended to merge (d) and (e), often unsuccessfully, while very few candidates completed (e) correctly where the need to insert $t!$ in the numerator and denominator was not usually spotted.

Recommendations and guidance for the teaching of future candidates

Candidates need to be more careful in stating the hypotheses in questions on inference. These are easy marks for careful candidates but, unfortunately, many candidates state incorrect hypotheses.

Candidates should be encouraged to write their solutions in a more legible fashion. It is sometimes extremely difficult to read the candidate's solution and no marks can be awarded if the examiner is unable to read the solution.