November 2012 subject reports

MATHEMATICS HL

Overall grade boundaries

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 39	40 - 52	53 - 65	66 - 78	79 - 100

Series and differential equations

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 28	29 - 39	40 - 52	53 - 66	67 - 79	80 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 26	27 - 39	40 - 51	52 - 64	65 - 77	78 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 28	29 - 39	40 - 53	54 - 67	68 - 80	81 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 18	19 - 23	24 - 29	30 - 34	35 - 40

The range and suitability of the work submitted

There were a number of exceptionally well-produced portfolios this session, a tribute to the direction and guidance given to candidates by their teachers. Generally, the writing was clear, concise, and mathematically correct. Unfortunately, a few documents were extremely lengthy, containing an overwhelming excess of repetitive graphs using a cut-and-paste approach that did not enhance the work.

The assessment criteria were well understood by the teachers, who in many instances produced their own marking rubrics to assess their candidates' work consistently. Observations made by the moderating team are summarised below. Most of these observations and concerns are essentially the same as those made after the May 2012 session.

The tasks

The vast majority of the portfolios contained tasks that were taken from the older of the two publications in use, "Mathematics HL – The portfolio – Tasks for use in 2011 and 2012" with the most popular tasks still being "Patterns within Systems of Linear Equations" and "Modelling a Functional Building".

There were some tasks taken from the newer publication, "Tasks for use in 2012 and 2013" in the school samples, and a very small number of teacher-designed tasks submitted. One task, "Patterns from Complex Numbers", from the newer publication, proved to be quite problematic, with many students (and some teachers) misreading the instruction and producing a familiar and trivial consequence of De Moivre's theorem: that consecutive n-th roots of unity when connected form a regular polygon. However, the instruction in this task, stated for cube roots and later applied to n-th roots, specifically required the student to "Choose a root and draw line segments from this root to the other two roots", resulting in a tree by graph theory definition or, if viewed non-mathematically, a diagram resembling the tines of a rake.

The continued and repeated use of some teacher-designed tasks is of some concern. Some of these, involving "Lionel (the dog)" and "Pipelines" have been reused time and time again over the past decade, presenting more of a concern than tasks published by the IB that have expired. Schools that continue to use the same tasks each year, regardless of their presumed non-expiring status, risk the problem of plagiarism as solutions are easily available, not only online, but through their former students.

Candidate performance against each criterion

The candidates performed well against criterion A, although the occasional student used calculator notation or forgot the "dx" in an integral.

Communication skills have improved notably over the past few years, and there were many pieces of work which were exemplary. However, missing introductions and unlabelled graphs were still in evidence. Some student work, though correct, was far from concise, with a number of portfolios containing work in excess of thirty and forty pages! Moderators were overwhelmed and under-impressed with the repetitive and excessive number of pages of similar graphs. The emphasis needs to return to quality over quantity.

In terms of mathematical content, the candidates generally produced good work and the teachers have assessed the work well against criteria C and D.

However, in some type I work, the investigation was extremely limited, with little evidence of a pattern to warrant a conjecture, let alone a generalisation, particularly in the task, "Patterns within Systems of Linear Equations". Unfortunately, as it was in the May 2012 session, in the apparent haste to reach a generalisation, many students did not qualify their conclusion to consider limitations involving dependent systems of equations that produced coincident lines in this task.

In type II tasks, for example, in "Functional Building", the choice of an appropriate model should have been justified empirically by candidates through the use of a goodness of fit analysis, instead of merely assuming a quadratic model.

Some candidates did not pay enough attention in their type II task to the requirements in criterion D, and generated results to an inappropriate degree of accuracy, such as a millionth of a second or billionth of a metre. Also of concern was the failure to consider the reasonableness of parameter values, such as the choice of a “jogging speed” of 10 m/s in the task, “Running with Angie and Buddy”.

The use of technology varied considerably. Full marks were often given much too generously for the use of a large collection of similar graphs. Should a task such as “Functional Building” require the domain or range to be limited to positive values, the graphs should have been so adjusted.

Resourceful uses of technology were noted in the form of 3D graphs, graphs with sliders to adjust parameter values, and the use of conditional statements in dynamic spreadsheets.

There were many pieces of good and complete work; however, the awarding of full marks in criterion F requires evidence of mathematical sophistication that extends beyond the requirements of the task.

Recommendations and guidance for the teaching of future candidates

The portfolio tasks for this session may have been taken from the two documents mentioned above, but not taken from earlier publications. Please note that a significant penalty is applied in moderation for the use of “expired” tasks. Teachers may certainly choose to use a task proposed by a colleague or offered at a workshop, but be mindful that some such tasks may have been reused year after year and not always successfully by other teachers.

Teachers are expected to write directly on the work submitted, not only to provide feedback to their candidates, but information to the moderators as well. The use of Form B is suggested to allow the teacher to indicate more relevant and descriptive comments.

There has been a noticeable improvement in the provision of background information to each portfolio task that is required to accompany each sample, particularly with the use of Form A or through anecdotal comments. Moderators find them very useful in determining the context in which each task was given when confirming the achievement levels awarded.

A solution key for each task in the sample must accompany the portfolios in order that moderators can justify the accuracy of the work and understand the teacher’s expectations. Where there is more than one HL teacher involved in marking the portfolio work, the use of a common marking scheme has been effective.

Please be advised that for candidates completing the diploma in 2013, the tasks contained in the document, “Mathematics HL – The portfolio – Tasks for use in 2012 and 2013”, are the only published tasks eligible for use.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 30	31 - 44	45 - 60	61 - 77	78 - 93	94 - 120

The areas of the programme and examination which appeared difficult for the candidates

Candidates' work in mathematical induction was thorough, though not perhaps their underlying understanding, since the final reasoning mark in Question 12 was often not awarded.

Question 8b and Question 9 proved problematic.

Some very limited attempts at Question 10 showed a poor understanding (amongst some) of complex numbers in polar form.

The areas of the programme and examination in which candidates appeared well prepared

Binomial expansions, (implicit) differentiation and proof by induction (the proof for $n = k \Rightarrow n = k + 1$ true) were all tackled well, the latter impressing a number of examiners.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Many candidates scored full marks on this question, though their explanations for part a) often lacked clarity. Most preferred to use some kind of right-angled triangle rather than (perhaps in this case) the more sensible identity $\sin^2 \alpha + \cos^2 \alpha = 1$.

Question 2

This was generally very well answered. Those who failed to gain full marks often made minor sign slips. A surprising number obtained the correct simplified expression, but continued to rearrange their expressions, often doing so incorrectly. Fortunately, there were no penalties for doing so.

Question 3

This question was well answered in general. Part b(ii) was often the most problematic, usually because of candidates going to the trouble of finding an explicit and sometimes incorrect expression for $f\left(x-2\right)$

Question 4

This was answered very well. Candidates are very familiar with this type of question. Some lost a couple of marks by failing to find their final y coordinates, though only the weakest struggled with differentiation and so made little progress.

Question 5

A range of answers were seen to part a), though many more could have gained the mark had they taken time to understand the shape of the function. Part b) was done well, as was part c). In c), a number of candidates integrated by parts, but found the incorrect expression $-xe^{-x} + e^{-x}$.

Question 6

The best candidates used row reduction correctly in part a) and were hence able to deduce $b=1$ in part b) for an easy final 4 marks. The determinant method was often usefully employed in part a).

Question 7

Candidates using the sine rule here made little or no progress. With the cosine rule, the two values are obtained quite quickly, which was the case for a majority of candidates. A small number were able to write down the correct quadratic equation to be solved, but then made arithmetical errors en route to their final solution(s). Part b) was often left blank. The better candidates were able to deduce $k=3$, though the solution $k \geq 6$ was rarely, if at all, seen by examiners.

Question 8

Part a) proved an easy 6 marks for most candidates, while the majority failed to make any headway with part b), with some attempting to find the equation of their line in the form $y=mx+c$. Only the best candidates were able to see their way through to the given answer.

Question 9

This was probably the least accessible question from section A. Most started by using the same value of *t* in attempting to find the common point, and so scored no marks. There were a number of very good candidates who set different parameters for t and correctly obtained $(3,1)$. There was slightly better understanding shown in part b), though some argued that the boats did not collide because their times were different, yet then provided incorrect times, or even no times at all.

Question 10

Many candidates were perhaps fortunate in this question due to there being several follow through marks available. Part a) was often done correctly. In part b), incorrect answers of 2 cis $\left(-\frac{\pi}{3}\right)$ were common, though many of these candidates often applied De Moivre's Theorem correctly to their answers. In c) the majority found $z^2=-3$ but could then get no further. The second part was often poorly done, with those rationalising the Cartesian form of z_1 having the most success. Part d) posed problems for a great many, and correct solutions were rarely seen. cis $\left(\frac{5n\pi}{6}\right)$ was often seen, but then finding $n=12$ proved to be a step too far for many. In general, the manipulation of complex numbers in polar form is not well understood.

Question 11

Virtually all candidates were able to score full marks in part a).

Part b) was again well answered, with errors coming in the form of careless slips when solving simultaneous equations. A surprising number of candidates were unable to write down the correct matrix for A^{-1}.

Part c) offered an easy source of marks for many. In the first part, straightforward attempts to solve equations simultaneously were the most successful, with matrix methods rarely seen. A number unfortunately lost some easy marks in the second part by using an incorrect gradient for l_2.

Question 12

Part a) proved to be an easy 3 marks for most candidates.

Part b) was often answered well, and candidates were well prepared in this session for this type of question. Candidates still need to take care when showing explicitly that P(1) is true, and some are still writing 'Let $n=k$' which gains no marks. The inductive step was often well argued, and given in clear detail, though the final inductive reasoning step was incorrect, or appeared rushed, even from the better candidates. 'True for $n=1$, $n=k$ and $n=k+1$' is still disappointingly seen, as were some even more unconvincing variations.

Part c) was again very well answered by the majority. A few weaker candidates attempted to find an inverse for the individual case $n=1$, but gained no credit for this.

Part d) was not at all well understood, with virtually no candidates able to tie together the hints given by connecting the different parts of the question. Rash, and often thoughtless attempts were made at each part, though by this stage some seemed to be struggling through lack of time. The inequality part of the question tended to be 'fudged', with arguments seen by examiners being largely unconvincing and lacking clarity. A tiny number of candidates provided the correct answer to the final part, though a surprising number persisted with what should have been recognised as fruitless working – usually in the form of long-winded integration attempts.

Recommendations and guidance for the teaching of future candidates

The underlying understanding shown of Proof by induction is often weak, and a fraction more time spent discussing the logic and reasoning behind this technique would be useful.
Manipulation of complex numbers is generally weak, including conversion of Cartesian to polar form (and vice versa), solving simple equations in $\mathbb{C}$ and (perhaps surprisingly) the use and understanding of De Moivre's Theorem.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 32	33 - 45	46 - 61	62 - 76	77 - 92	93 - 120

The areas of the programme and examination which appeared difficult for the candidates

- Algebraic manipulation, namely expanding perfect squares and simplifying products involving negative signs.
- Solving systems of equations involving parameters.
- Understanding of variance variation.
- Conditional probability and use of tree diagrams to solve problems.
- Area delimited by curves; forming definite integrals.
- Integrals by a given substitution.
- Implicit differentiation in the context of motion.
- Application of trigonometric identities.
- Complex conjugates and De Moivre's theorem.
- Use of a GDC to graphically obtain a maximum function value.

- Solving systems of equations involving parameters.
- Interpreting results and giving reasons.
- Rounding to an appropriate number of significant figures.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates seemed, in general, well prepared for this paper. Despite many students not being able to complete all the answers, the level shown by most of them was satisfactory. Many candidates, nevertheless, were able to complete all the examination with outstanding results. There was some good work on vectors, which ensured that many candidates did well on one of the section B questions. Candidates also performed well in sequences and series, determinant of a matrix, statistics, application of quotient rule to differentiate reciprocals of trigonometric functions and manipulation of expressions involving logarithms.

GDCs were properly used and answers were enhanced by this technology though many candidates could have got more marks if they had used this tool properly. Those who embraced the use of their GDC were in a much stronger position than those who felt the need to work through difficult problems by hand, either in part or completely.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates got full marks in this question. Some mistakes were detected when trying to find the number of terms of the arithmetic sequence, namely the use of the incorrect value $n = 132$; a few interpreted the question as the sum of multiples between the 100th and 500th terms. Occasional application of geometric series was attempted.

Question 2

Overall the question was pretty well answered but some candidates seemed to have mixed up the terms determinant with discriminant. In some cases a lack of quality mathematical reasoning and understanding of the discriminant was evident. Many worked with the quadratic formula rather than just the discriminant, conveying a lack of understanding of the strategy required. Errors in algebraic simplification (expanding terms involving negative signs) prevented many candidates from scoring well in this question. Many candidates were not able to give a clear reason why the quadratic has always two distinct real solutions; in some cases a vague explanation was given, often referring to a graph which was not sketched.

Question 3

This question was generally well done by most candidates. Those candidates who used GDC early in the answer managed to find the correct solution but often candidates used a lot of unnecessary algebraic simplification before resorting to the GDC. Some candidates did not use properties of logarithms correctly in order to secure an algebraic solution. A few misconceptions concerning properties of logarithms were noticed.

Question 4

Most candidates were successful in finding the correct value of the mean; however, the variance caused many difficulties. Many candidates affirmed that there were no differences in the variance as it remained constant; some others got wrong results due to premature rounding of figures. Many candidates lost the final mark because they rounded their answers prematurely, resulting in a very inaccurate answer to this question.

Question 5

This question was generally well done by most candidates. Some candidates recurred to a diagram to comprehend the nature of the problem but a few thought it was an arithmetic sequence.

A surprising number of candidates missed earning the final A1 mark because they did not read the question instructions fully and missed the accuracy instruction to give the answer correct to the nearest mm.

Question 6

Despite the fact that many candidates were able to calculate the speed of the particle, many of them failed to calculate the acceleration. Implicit differentiation turned out to be challenging in this exercise showing in many cases a lack of understanding of independent/dependent variables. Very often candidates did not use the chain rule or implicit differentiation when attempting to find the acceleration. It was not uncommon to see candidates trying to differentiate implicitly with respect to *t* rather than *s*, but getting the variables muddled.

Question 7

Part (a) was generally successful to most candidates; however the conditional probability was proved difficult to many candidates either because the unconditional probability of two correct games was found or the success in the second and third game was included. Many candidates used a clear tree diagram to calculate the corresponding probabilities. However other candidates frequently tried to do the problem without drawing a tree diagram and often had incorrect probabilities. It was sad to read many answers with probabilities greater than 1.

Question 8

Just a few candidates got full marks in this question. Substitution was usually incorrectly done and lead to wrong results. A cosine term in the denominator was a popular error. Candidates often chose unhelpful trigonometric identities and attempted integration by parts. Results such as $\int \sin^3 t \, \mathrm{d}t = \frac{\sin^4 t}{4} + C$ were often seen along with other misconceptions concerning the manipulation/simplification of integrals were also noticed. Some candidates unsatisfactorily attempted to use $\arcsin x$. However, there were some good solutions involving an expression for the cube of $\sin t$ in terms of $\sin t$ and $\sin 3t$ .Very few candidates re-expressed their final result in terms of x.

Question 9

This question proved challenging to most candidates. Just a few candidates were able to calculate the exact area between curves. Those candidates who tried to express the functions in terms of x instead of y showed better performances. Determining $\sqrt{x+3}$ only was a common error and forming appropriate definite integrals above and below the *x*-axis proved difficult. Although many candidates attempted to sketch the graphs, many found only one branch of the parabola and only one point of intersection; as the graph of the parabola was not complete, many candidates did not know which area they were trying to find. Not many split the integral correctly to find areas that would add up to the result. Premature rounding was usually seen and consequently final answers proved inaccurate.

Question 10

This was the most challenging question in part A with just a few candidates scoring full marks. This question showed that many candidates have difficulties with algebraic manipulations, application of De Moivre’s theorem and use of trigonometric identities. Although some candidates managed to calculate the square of a complex number, many failed to write down its conjugate or made algebraic errors which lead to wrong results in many cases. Just a few candidates were able to calculate the modulus and the argument of the complex number.

Question 11

This question was generally well done by most candidates. It was evident that candidates had been well prepared in Poisson and normal distribution. In parts (a)-(d) candidates were usually successful and appropriate methods were shown although many candidates used labored algebraic approaches to solving simultaneous equations and wasted time answering part (d). Part (e) was very well answered by a smaller number of candidates but it was obviously more demanding in its level of abstraction.

Question 12

Part (a) was very well done by most candidates. Parts (b), (c) and (d) required a subtle balance between abstraction, differentiation skills and use of GDC.

In part (b), although candidates were asked to justify their reasoning, very few candidates offered an explanation for the maximum. Therefore most candidates did not earn the R1 mark in part (b). Also not as many candidates as anticipated used a graphical approach, preferring to use the calculus with varying degrees of success. In part (c), some candidates calculated the derivatives of inverse trigonometric functions. Some candidates had difficulty with parts (d) and (e). In part (d), some candidates erroneously used their alpha value from part (b). In part (d) many candidates used GDC to calculate decimal values for α and L. The premature rounding of decimals led sometimes to inaccurate results. Nevertheless many candidates got excellent results in this question.

Question 13

Although this was the last question in part B, it was answered surprisingly well by many candidates, except for part (e). Even those who had not done so well elsewhere often gained a number of marks in some parts of the question. Nevertheless the presence of parameters seemed to have blocked the abilities of weaker candidates to solve situations in which vectors were involved. Mathematical skills for this particular question were sometimes remarkable, however, calculations proved incomplete due to the way that planes were presented. Most candidates found a correct angle in part (a). Occasional arithmetic errors in calculating the magnitude of a vector and dot product occurred. In part (b) the vector product approach was popular. In some case candidates simply verified the result by substitution. There was a lot of simultaneous equation solving, much of which was not very pretty. In part (c), a number of candidates made errors when attempting to solve a system of equations involving parameters. Many of the results for the point were found in terms of k. It was notorious that candidates did not use their GDC to try to find the coordinates of the intersection point between lines. In part (d), a number of candidates used an incorrect point but this part was often done well.

Very few excellent answers to part (e) were seen using an efficient method. Most candidates attempted methods involving heavy algebraic manipulation and had little success in this part of the question.

Recommendations and guidance for the teaching of future candidates

- Remind students to read questions very carefully.
- Clarify the meaning of the various phrases used in probability questions, namely when conditional probability is involved and promote the use of diagrams to display information (eg tree diagrams).
- Candidates should be given strategies to integrate powers of trigonometric functions efficiently.
- Remind students that they should justify their results, namely in questions of the type ‘show that’, and encourage them to write more of their strategy for a problem down so that it can be well understood. They should also be encouraged to try different approaches to a problem rather than just one default method they always fall back on.

- If a graph obtained from the GDC is used, they should include a sketch with all relevant points labeled.

- Students need to be reminded about the 3 s.f. accuracy general rule and of the importance of using full accuracy of results as illustrated by Q4 on this paper, where premature rounding, even to 5 or 6 sf resulted in an inaccurate final answer. Most GDC allow copy & paste of values and help in avoiding errors due to early rounding.

- Students should be made aware of the advantage of using a GDC to solve equations in one, two or three variables and to find integrals as is their use in finding stationary points. There are still too many candidates who erroneously think they will gain more credit for laborious, error prone algebraic approaches. These candidates are penalized by wasting precious time with routine questions and often are unable to answer all the questions on the paper.

- Finally teachers should provide more practice of questions that require written mathematical reasoning, try to be more creative in proposing exercises to their students and provide a wide variety of scenarios that allow the development of problem solving skills in both real life and abstract contexts.

Paper three - Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 15	16 - 24	25 - 31	32 - 37	38 - 44	45 - 60

The areas of the programme and examination which appeared difficult for the candidates

The candidates were less happy when they had to think and to create proofs for themselves e.g. Q.5, than when they were doing known algorithms. It was clear that some (maybe re-sit candidates) were not really prepared for this option at all. The overall standard of the candidates was below what was expected.

The areas of the programme and examination in which candidates appeared well prepared

The candidates generally knew how to apply algorithms.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a) This was quite well answered. Some candidates did not make their method clear and others showed no method at all. Some clearly had a correct method but did not make it clear what their final answers were. It is recommended that teachers look at the tabular method with its backtracking system as shown in the mark scheme as an efficient way of tackling this type of problem.

(b) Fairly good knowledge shown here but not by all.

(c) Some good answers but too much confusion with methods they partly remembered about the travelling salesman problem. Candidates should be aware of helpful connections between parts of a question.

Question 2

(a) and (b) This should have been a fairly straight forward question. It just required methodical thinking. As expected there were mistakes of missing solutions and also giving duplicate ones. Some candidates were very poor and did not even really know what a tree was. There was a hint in the question with the number of marks given, that some candidates seemed to have picked up on.

Question 3

(a) The Euclidean algorithm was well applied. If it is done in the format shown in the mark scheme then the keeping track method of the linear combinations of the 2 original numbers makes part (b) easier.

(b) Again well answered but not quite as good as (a).

(c) Surprisingly, since it is basic bookwork, this part was answered very badly indeed. Most candidates did not realise that -10 was the number to multiply by. Sadly, of the candidates that did do it, some did not read the question carefully enough to see that a positive integer answer was required.

(d) Again this is standard bookwork. It was answered better than part (c). There were the usual mistakes in the final answer e.g. not having the two numbers, with the parameter, co-prime.

Question 4

(a) Fermat's little theorem was reasonably well known. Some candidates forgot to mention that p was a prime. Not all candidates took the hint to use this in the next part.

(b) This was not answered well. Candidates did not use the information given to consider the variable modulo 5, 13 and 11 and then apply the Chinese Remainder Theorem. Many candidates just considered it mod 11 and thus often did not consider all possibilities.

Question 5

Only the top candidates were able to produce logically, well thought-out proofs. Too many candidates started with what they were trying to prove i.e. that G and its complement were isomorphic. Other candidates thought that one example constituted a proof without systematically considering all possibilities.

Recommendations and guidance for the teaching of future candidates

Not all candidates were starting a fresh piece of paper for each of the questions. Although this option involves graphs and trees there was no need for candidates to use graph paper for some of their answers! It made it more difficult to read the answers of candidates that did this with the papers being scanned. If a candidate introduces a variable that is not given in the question then they need to say what it stands for so that the examiner can follow their working. Candidates need to be prepared for proofs as well as algorithms and know that "waffly" words rarely gain many marks. Looking at the structure of proofs on the mark-schemes of previous exams will help. With many of the points mentioned above, careful corrective marking of a trial exam should have assisted the candidates, if they were prepared to learn.

Paper three - Series and differential equations

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 17	18 - 23	24 - 31	32 - 39	40 - 47	48 - 60

The areas of the programme and examination which appeared difficult for the candidates

The following topics seemed to cause problems for the candidates. Use of integrals to estimate sums of series, telescoping series and, in general, more rigour was required in tests on series than was provided by some candidates.

The areas of the programme and examination in which candidates appeared well prepared

Differential equations, Eulers Theorem and Partial Fractions

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This question allowed candidates to demonstrate a range of skills in solving differential equations. Generally this was well done with candidates making mistakes in algebra rather than the techniques themselves. For example a common error in part (a) was to go from $\ln y = \ln x + c$ to $y = x + c$

Question 2

Part (a) was done well. We would recommend that candidates write down the equation they are using, in this case, $y_{n+1} = y_n + 0.1\sqrt{x_n + y_n}$, to ensure they get all the method marks. Beyond this the answer is all that is needed (or if a student wishes to show working, simply each of the values of x_n and y_n) . Many candidates wasted a lot of time by writing out values of each part of the function, perhaps indicating they did not how to do it more quickly using their calculators.

Part (b) Surprisingly when drawing the graph a lot of candidates had (0.01, 2.8099) closer to 2.80 than 2.81

Most realised that the best possible estimate was given by the *y*-intercept of the line they had drawn.

Question 3

Most candidates correctly obtained the result in part (a). Many then failed to realise that having obtained this result once it could then simply be stated when doing parts (b) and (d)

In part (b) the calculation of the integral as equal to 1 only scored 2 of the 3 marks. The final mark was for stating that 'because the value of the integral is finite (or 'the limit exists' or an equivalent statement) then the series converges. Quite a few candidates left out this phrase.

Candidates found part (c) difficult. Very few drew the correct series of rectangles and some clearly had no idea of what was expected of them.

Though part (e) could be done without doing any of the previous parts of the question many students were probably put off by the notation because only a minority attempted it.

Question 4

Candidates and teachers need to be aware that the Limit comparison test is distinct from the comparison test. Quite a number of candidates lost most of the marks for this part by doing the wrong test.

Some candidates failed to state that because the result was finite and not equal to zero then the two series converge or diverge together. Others forgot to state, with a reason, that $\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges.

In part (b) finding the partial fractions was well done. The second part involving the use of telescoping series was less well done, and students were clearly not as familiar with this technique as with some others.

Part (c) was the least well done of all the questions. It was expected that students would use explicitly the result from the first part of 4(b) or show it once again in order to give a complete answer to this question, rather than just assuming that a pattern spotted in the first few terms would continue.

Candidates need to be informed that unless specifically told otherwise they may use without proof any of the Maclaurin expansions given in the Information Booklet. There were many candidates who lost time and gained no marks by trying to derive the expansion for ln $(1+x)$

In part (d) the link with 4(b) was spotted by many candidates. As always candidates need to be aware that when a question says ‘hence’ alternative methods eg L’Hopital’s Rule will not score any marks.

In part (e) some candidates used L’Hopital’s Rule and were awarded the mark.

Many candidates saw the link in part (f) with part (e) and gave a good solution to this question.

Recommendations and guidance for the teaching of future candidates

- Be aware of calculator shortcuts in iterative processes.
- The test for the convergence or otherwise of a series must include a full conclusion.
- Be aware of which Maclaurin series are given in the Information Booklet and emphasise to students that these do not need to be derived unless explicitly told to do so.

Paper three - Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 22	23 - 29	30 - 35	36 - 42	43 - 60

The areas of the programme and examination which appeared difficult for the candidates

Candidates seemed to have difficulties displaying the correct level of rigour for this level. Their reasoning was frequently unclear or nor present at all, therefore not obtaining the marks. It was clear that although students had some idea of the concepts involved in most questions, they had difficulties in applying that to different situations.

The areas of the programme and examination in which candidates appeared well prepared

Generally it was clear that candidates had a knowledge of all areas of the curriculum and so were able to attempt all questions. A good understanding of relations was evident as was a sound understanding of finite groups.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Generally this question was well answered, with students showing a sound knowledge of relations. There were a few candidates who mixed reflexive and symmetric qualities and marks were also lost because reasoning was either unclear or absent. Most students were able to offer counterexamples for transitivity in parts (a) and (b) but a number lost marks in failing to give adequate working to show transitivity in parts (c) and (d). That said, there were a pleasing number of good solutions here showing all the required rigour. Whilst most students were able to identify part (c) as an equivalence relation, surprisingly few gave the correct equivalence classes.

Question 2

Solutions to this part were surprisingly disappointing. Most students were able to get the mark for part (a) but many candidates failed to go successfully beyond that point. In part (b) many candidates knew to show that $f\left(\begin{pmatrix} a \\ b \end{pmatrix}\right) = f\left(\begin{pmatrix} c \\ d \end{pmatrix}\right) \Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}$ but simply deduced $a + 2b = c + 2d \Rightarrow a = c \text{ and } b = d$. In a similar way, those that used matrix methods simply removed the matrix without determining existence. Similar problems were found in following parts, although of the later parts, several candidates did manage to obtain the marks for part (d).

Question 3

A surprising number of candidates were unable to answer part (a) and consequently were unable to access much of the rest of the question. Most candidates however, were successful in parts (a), (b) and (c), and it was pleasing to see the preparedness of candidates in these parts. There were also many good answers for parts (d) and (e) although the third part of (e) caused the most problems with candidates failing to provide sufficient reasoning. Few candidates managed good responses to parts (f) and (g).

Question 4

Most candidates were able to answer part (a) indicating preparation in such questions. Many students failed to identify the command term “state” in parts (b) and (c) and spent a lot of time – usually unsuccessfully - with algebraic methods. Most students were able to offer satisfactory solutions to part (d) and although most showed that they knew what to do in part (e), few were able to complete the proof of associativity. Surprisingly few managed to answer parts (f) and (g) although many who continued to this stage, were able to pick up at least one of the marks for part (h), regardless of what they had done before. Many candidates interpreted the question as asking to prove that the group was Abelian, rather than proving that it was an Abelian group. Few were able to fully appreciate the significance in part (i) although there were a number of reasonable solutions.

Recommendations and guidance for the teaching of future candidates

- Candidates should be aware of command terms used in IB exams and be alert to the consequences.
- Candidates need to practice applying ideas learnt in different situations.
- Candidates need to practice constructing mathematical arguments that clearly communicate their ideas and arguments.
- Candidates should practice thinking about the question as a whole, and consequences of their answer on their previous solutions. For example in question 2, many students should have realised with their solution to part (d) that their solutions to (b) and (c) were incomplete and could have gone back to gain the missed marks.

Paper three - Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 17	18 - 24	25 - 33	34 - 42	43 - 51	52 - 60

The areas of the programme and examination which appeared difficult for the candidates

There were no particular areas that the candidates seemed weak on. There were many very good candidates that dealt with all questions competently. Conversely, weak candidates had difficulties in all areas. Scripts tended to be good or poor with little middle ground.

The areas of the programme and examination in which candidates appeared well prepared

The general standard shown by the students was very pleasing and above expectations.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This was generally well answered. Some students did not read the question carefully enough and see the comparisons made between the Hypergeometric distribution and the Binomial distribution, with 5 trials (some candidates went to 10 trials) in each case. Part (h) caused the most problems and it was very rare to see a script that gained the reasoning mark for saying that *A* and *B* were independent events. This question was a good indicator of the standard of the rest of the paper.

Question 2

This was very well answered indeed with very many candidates gaining full marks including, pleasingly, part (b). Candidates who could not do question 2, struggled on the whole paper.

Question 3

(a) There were many reasonable answers. In (i) not all candidates explained their method so that they could gain good partial marks even if they had the wrong final answer. A common mistake was to give an answer above 3. It was pleasing that almost all candidates had (ii) and (iii) correct, as this had caused problems in the past. In (iv) it was amusing to see a few candidates work out 5% using conditional probability rather than just write down the answer as asked.

(b) It was pleasing that almost all candidates realised that it was a *t*-test rather than a *z*-test.

There was good understanding on how to use the calculator in parts (ii) and (iii). The correct confidence interval to the desired accuracy was not always given.

The most common mistake in question 3 was forgetting to take into account the variance of the sample mean.

Question 4

This was well answered as the last question should be the most difficult. It seemed accessible to many candidates, if they realised what the distributions were. The goodness of fit test was well used in (c) with hardly any candidates mistakenly combining cells. Part (e) was made more complicated than it needed to be with calculator solutions when a bit of pure maths would have sufficed. Part (f) caused some problems but good candidates did not have too much difficulty.

Recommendations and guidance for the teaching of future candidates

- Not all candidates were starting a fresh piece of paper for the questions.

- If a candidate introduces a variable that is not given in the question then they need to say what it stands for so that the examiner can follow their working. Candidates were losing marks by not giving their answers to the desired degree of accuracy, either that stated in the question or as given in the general rubric. In Statistics the calculator can be used most effectively and efficiently. However it is always wise to explain the method being used as the question paper states that “Full marks are not necessarily awarded for a correct answer with no working”. Conversely if the wrong answer is given and no working is shown it can only gain zero marks whereas if explanation had been given it could have gained almost full marks. With many of the points mentioned above careful corrective marking of a trial exam should have assisted the candidates, if they were prepared to learn.