MATHS HL

Overall grade boundaries

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 25	26 - 37	38 - 50	51 - 63	64 - 76	77 - 100

Series and differential equations

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 24	25 - 37	38 - 49	50 - 62	63 - 74	75 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 25	26 - 37	38 - 49	50 - 62	63 - 74	75 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 25	26 - 37	38 - 50	51 - 63	64 - 76	77 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 18	19 - 23	24 - 29	30 - 34	35 - 40

Candidates produced excellent portfolios, and a few outstanding pieces of work were noted this session. Generally, the work was clearly marked, and the requisite forms were completed correctly. However, the teacher's solutions and the background to the tasks were often missing.

The tasks:

Virtually all portfolio tasks were taken from the current Teacher Support Material (TSM) for Mathematics HL, for use in 2009-2010. Popular choices were “Parabola Investigation”, “Investigating Ratios of Areas and Volumes”, “Modelling the Course of a Viral Illness and its Treatment” and “Designing a Freight Elevator”. There were very few teacher-designed tasks submitted. Teachers are encouraged to design their own tasks, keeping in mind the need to satisfy all criteria fully.

Candidates’ performance

Most candidates performed well against criterion A. The use of computer notation seemed to be very limited. However, the inappropriate use of “^”, “E09” and the like were still in evidence.

Good communication skills were amply demonstrated in some samples. Where a student’s work began with an introduction to the task, and comments, annotations and conclusions accompanied the steps and results, the work was easy to read and follow and earned high marks in criterion B. However, there were many students whose work did not stand on its own, particularly when there was no introduction to a task or when a “Question-and-Answer” format to a task was adopted. Unlabelled graphs and the relegation of tables to the appendix rate poorly in terms of an effective presentation and should have been penalised.

One significant concern this session was the excessive length of some student work. Using a “cut-and-paste” technique, many students produced work that was entirely too repetitive and long - exceeding 40 pages in some cases.

Generally, students have produced good work and the assessments by their teachers have been appropriate against criteria C and D. However, in some type I tasks, insufficient exploration and patterning rendered the quick formulation of a conjecture questionable. Further investigation was often not attempted once a conjecture was formed.

In type II tasks, variables should have been explicitly defined. Some realisation of the significance of the results obtained in terms of the model when compared to the actual situation should have been provided, and students should have reflected on their findings. The analyses of data must be quantified, and if a regression analysis were appropriate, the student must have provided reasons for choosing a particular model. The use of software that automatically determines the ‘best’ regression model leaves little room for interpretation by the candidate and should be avoided.

The use of technology was often superficial. Full marks were given much too generously for an appropriate, but not necessarily resourceful, use of technology; for example, in the mere inclusion of a graph of data. The awarding of full marks has little to do with the number of graphs, but with the extended use of technology that contributes significantly to the development of each task.

There were many good pieces of work. However, the awarding of full marks in criterion F requires more than completion and correctness; mathematical sophistication and insight are expected.

Suggestions to teachers

Teachers are expected to write directly on their students' work, not only to provide feedback to students, but to provide information to moderators as well. Some samples contained very few teacher comments. Moderation was extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks. The use of Form B, in lieu of the reverse of Form 5/PFCS, would permit space for specific and descriptive comments, as remarks that merely reflect the wording in the achievement levels are not particularly useful to the moderator.

Moderators find the background to each portfolio task very useful in determining the context in which the task was given when confirming the achievement levels awarded. This information must accompany each sample, either on Form A or through anecdotal remarks.

A solution key for tasks from the current publication, as well as for those designed by teachers, must accompany the portfolios in order that moderators can justify the accuracy of the work, and appreciate the level of sophistication expected of the students by the teacher.

The tasks contained in the current document have now been in use with students completing their diploma requirements in 2009. They can only be reused with students finishing their diploma program in 2010. Students starting their first year in 2009 should not be assigned these tasks.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 31	32 - 43	44 - 57	58 - 72	73 - 86	87 - 120

The areas of the programme and examination that appeared difficult for candidates

On this paper candidates found difficulty with functions, particularly finding the domain, complex numbers, matrices, proof by induction, integration of trigonometric functions and solutions to first order differential equations. Some candidates had difficulty with the level of arithmetic required.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been reasonably well prepared for questions on most aspects of differentiation, some aspects of integration, some aspects of series, and conditional probability. Notation and terminology were understood, and also written appropriately.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Many candidates answered part (a) successfully. For part (b), some candidates did not consider that the entire set of real numbers was asked for.

Question 2

Some candidates did not consider changing the number to modulus-argument form. Among those that did this successfully, many considered individual values of n, or only positive values. Very few candidates considered negative multiples of 3.

Question 3

Several candidates, including those who answered parts (a) and (b) correctly, unsuccessfully attempted to solve part (c) by constructing a system of equations and solving simultaneously.

Question 4

Very few candidates attempted part (a), and of those that did, few were successful. Part (b) was answered fairly well by most candidates.

Question 5

Many candidates answered both parts successfully. Some candidates made unsuccessful attempts at solving by expanding and synthetic division.

Question 6

Most candidates answered this question successfully. Some made arithmetic errors.

Question 7

Quite a variety of methods were successfully employed to solve part (a). Many candidates did not attempt part (b).

Question 8

Part (a) was done successfully by many candidates. However, very few attempted part (b).

Question 9

A reasonable number of candidates answered this correctly, although some omitted the 2nd point of inflection.

Question 10

Only the best candidates were able to make significant progress with this question. Quite a few did not consider rotation about the y-axis. Others wrote the correct expression, but seemed daunted by needing to integrate by parts twice.

Question 11

Parts (a), (b) and (c) were answered successfully by a large number of candidates. Some, however, had difficulty with the arithmetic. In part (d) many candidates showed little understanding of sigma notation and proof by induction. There were cases of circular reasoning and using n, k and r randomly. A concluding sentence almost always appeared, even if the proof was done incorrectly, or not done at all.

Question 12

This was the least accessible question in the entire paper, with very few candidates achieving high marks. Sketches were generally done poorly, and candidates failed to label the point of intersection. A ‘dummy’ variable was seldom used in part (a), hence in most cases it was not possible to get more than 3 marks. There was a lot of good guesswork as to the coordinates of the point of intersection, but no reasoning showed. Many candidates started with the conclusion in part (c). In part (d) most candidates did not distinguish between the inequality and strict inequality.

Question 13

A large number of candidates did not attempt part (a), or did so unsuccessfully. It was obvious that many candidates had been trained to answer questions of the type in part (b), and hence of those who attempted it, many did so successfully. Quite a few however failed to find all solutions.

Recommendation and guidance for the teaching of future candidates

- Students should be exposed to a wide range of contexts when exploring individual topics. Links between topics should be highlighted
- Students need to refresh basic arithmetical operations, such as work with fractions and decimals
- The organization of a ‘show that’ or proof needs emphasising, i.e. a logical progression from hypotheses to conclusion
- Basic functions work and graph sketching need to be emphasized, including the aspects that must be shown, e.g. roots, asymptotes, etc

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 10	11 - 21	22 - 40	41 - 56	57 - 72	73 - 88	89 - 120

The areas of the programme and examination that appeared difficult for candidates

Many candidates were inclined to produce untidy and disordered solutions which sometimes failed to achieve the marks. There were questions in the paper for which it would be more appropriate to use a calculator, but many students appeared unable to distinguish when a calculator solution would be the easiest and most appropriate.

The areas of the programme and examination in which candidates appeared well prepared

There were many students who appeared well prepared for the whole syllabus, although others simply seemed unprepared in all but small areas of the syllabus. Generally students were well prepared for the calculus questions.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

Responses to this question were surprisingly poor. Few candidates recognised that the easier way to answer the question was to use a graph on the GDC. Many candidates embarked on fruitless algebraic manipulation which led nowhere.

Question 2

There were many successful answers to this question, as would be expected. There seemed to be some students, however, that had not been taught the vector geometry section

Question 3

Generally a well answered question.

Question 4

Most candidates were able to answer this question well.

Question 5

Many students used incorrect limits to the integral, although many did correctly let the integral equal to 0.5.

Question 6

Part (a) was well answered, whilst few candidates managed to correctly use conditional probability for part (b).

Question 7

Most students were able to state the conjugate root, but many were unable to take the question further. Of those that then recognised the method, the question was well answered.

Question 8

Implicit differentiation is usually found to be difficult, but on this occasion there were many correct solutions. There were also a number of errors in the differentiation of e^{xy}, and although these often led to the correct final answer, marks could not be awarded.

Question 9

Very few completely correct answers were given to this question. Many students found *a* to be 0 and many failed to provide adequate sketches. There were very few correct answers to part (c) although many students were able to obtain partial marks.

Question 10

This was generally a well answered question.

Question 11

There were a lot of arithmetic errors in the treatment of this question, even though it was apparent that many students did understand the methods involved. In (a) many students failed to realise that $\overrightarrow{AB}$ should be a multiple of the cross product of the two direction vectors, rather than the cross product itself, and many students failed to give the final answer as coordinates.

Question 12

Many students were unable to get started with this question, and those that did were generally very poor at defining their variables at the start.

Question 13

There were some good attempts at this question, but there were also many candidates that were unable to maintain a clearly presented solution and consequently were unable to obtain marks that they should have been able to secure. Those that attempted part (b) usually made a good attempt.

Recommendation and guidance for the teaching of future candidates

- As always it needs to be mentioned that students need to cover the entire syllabus if they are to be properly prepared to take this exam
- Students need to pay attention to producing clear, orderly solutions if their work is to obtain full credit.
- Many students need to have greater practice at using the GDC and recognising where it may be the most appropriate method in a question.

Paper three – Discrete mathematics

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 16	17 - 22	23 - 30	31 - 38	39 - 46	47 - 60

Paper three – Series and differential equations

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 15	16 - 21	22 - 28	29 - 34	35 - 41	42 - 60

Paper three – Sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 16	17 - 22	23 - 28	29 - 34	35 - 40	41 - 60

Paper three – Statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 18	19 - 23	24 - 31	32 - 38	39 - 46	47 - 60

The areas of the programme and examination that appeared difficult for candidates

1. **Statistics and probability**: The calculation of Type II error probabilities; problems involving the mode of a discrete distribution

2. **Sets, relations and groups**: Determining whether or not a function is a bijection; isomorphism of groups; theoretical questions involving groups

3. **Series and differential equations**: The formal ε, N definition of convergence; applying appropriate tests of convergence

4. **Discrete:** The use of Fermat's little theorem; the use of powers of the adjacency matrix to determine the number of walks between vertices

The areas of the programme and examination in which candidates appeared well prepared

1. **Statistics and probability:** The use of the GDC in carrying out statistical tests

2. **Sets, relations and groups:** The construction of Cayley tables for specific groups; equivalence relations

3. **Series and differential equations:** The solution of homogeneous differential equations; partial fractions

4. **Discrete:** The solution of simultaneous congruences using a variety of methods; the use of algorithms in graphs

The strengths and weaknesses of the candidates in the treatment of individual questions

1. Statistics and probability

Q1: In (a), some candidates incorrectly gave the hypotheses in terms of $\bar{x}$ instead of μ.. In (b), many candidates found the correct critical values but then some gave the critical region as $2.45 < \bar{x} < 2.55$ instead of $\bar{x} < 2.45 \cup \bar{x} > 2.55$. Many candidates gave the critical values correct to four significant figures and therefore were given an arithmetic penalty. In (c), many candidates correctly defined a Type II error but were unable to calculate the corresponding probability.

Q2: In (a), it was disappointing to note that many candidates failed to realise that the question was concerned with the mean lengths of the jumps and worked instead with the sums of the lengths. Most candidates obtained correct estimates in (b)(i), usually directly from the GDC. In (b)(ii), however, some candidates found a *z*-interval instead of a *t*-interval.

Q3: Many candidates found the expected frequencies correctly although some calculated an expected frequency for ‘8’ instead of ‘8 or more’. The calculation of the chi-squared statistic was generally well done, either directly from the GDC or using the formula.

Q4: Many candidates made a reasonable attempt at (a)(i) and (ii) but few were able to show that the mode is the integer part of $n+1\ p$. Part (b) also proved difficult for most candidates with few correct solutions seen.

2. Sets, relations and groups

Q1: This question was generally well answered.

Q2: In many cases the attempts at showing that *f* is a bijection were unconvincing. The candidates were guided towards showing that *f* is an injection by noting that $f'(x) > 0$ for all *x,* but some candidates attempted to show that $f(x) = f(y) \Rightarrow x = y$ which is much more difficult. Solutions to (c) were often disappointing, with the algebra defeating many candidates.

Q3: Most candidates were able to show, in (a), that *R* is an equivalence relation although few were able to identify the required equivalence class. In (b), the explanation that *S* is not transitive was often unconvincing.

Q4: Part (a) was well answered by many candidates. Few candidates, however, were able to make even a reasonable attempt at (b). Many candidates seem to know that isomorphism is something to do with having the same underlying structure but have no idea of the formal definition involving a bijection from one to the other.

Q5: Solutions to this question were extremely disappointing. This property of subgroups is mentioned specifically in the Guide and yet most candidates were unable to make much progress in (b) and even solutions to (a) were often unconvincing.

3. Series and differential equations

Q1: Many candidates were able to make a reasonable attempt at this question with many perfect solutions seen.

Q2: Many candidates obtained the required series by finding the values of successive derivatives at $x = 0$, failing to realise that the intention was to start with the exponential series and replace *x* by the series for $\mathrm{e}^x - 1$. Candidates who did this were given partial credit for using this method. Part (b) was reasonably well answered using a variety of methods

Q3: Most candidates were able to find the limit of the sequence. Very few correct solutions were seen to (b), indicating perhaps that the ε, N definition of convergence, although mentioned specifically in the Guide, is not generally taught.

Q4: Parts (a) and (b)(i) were generally well done. Although many candidates realised that the series in (b)(ii) was telescoping, some made arithmetic errors in summing the series to infinity.

Q5: Solutions to this question were generally disappointing. In (a), many candidates were unable even to find an expression for the *n*th term so that they could not apply the ratio test. In (b), few candidates were able to rewrite the *n*th term in the form $\sum(-1)^n \sin\left(\frac{1}{n}\right)$ so that most candidates failed to realise that the series was alternating.

4. Discrete

Q1: Solutions to this question were extremely variable with some candidates taking several pages to give a correct solution and others taking several pages and getting nowhere. Some elegant solutions were seen including the fact that the members of the two sets can be represented as $2 \bmod 4$ and $2 \bmod 5$ respectively so that common members are $2 \bmod 20$.

Q2: Part (a) was well answered in general. In (b), however, only a handful of candidates realised that the solution involved raising the adjacency matrix to the power 4. Many candidates tried to count the number of walks by enumerating them but never got beyond 10. Part (c) was well answered.

Q3: Part (a) was well answered by many candidates. The isomorphism in (b) was better handled than the isomorphism in the Sets paper but that is of course because it is easier in this context.

Q4: Questions similar to (a) have been asked in the past so it was surprising to see that solutions this time were generally disappointing. In (b), most candidates changed the base 7 number 126 to the base 10 number 69. After that the expectation was that Fermat's little theorem would be used to complete the solution but few candidates actually did that. Many were unable to proceed any further and others used a variety of methods, for example working modulo 7,

$$5^{69} = (5^2)^{34}.5 = 4^{34}.5 = (4^2)^{17}.5 = 2^{17}.5 \text{ etc}$$

This is of course a valid method, but somewhat laborious.

Q5: Many candidates made a reasonable attempt at showing that bipartite implies cycles of even length but few candidates even attempted the converse.

Recommendation and guidance for the teaching of future candidates

Statistics and Probability

Ensure that the concept of critical values is well understood.

Ensure that candidates understand when to use a z-test and when to use a t-test.

Teach the use of the simpler formula for calculating the chi-squared statistic for candidates who prefer a manual calculation.

Sets, relations and groups

Ensure that the isomorphism of groups is better understood.

Ensure that theoretical questions on groups are better handled.

Series and differential equations

Ensure that candidates are familiar with the formal ε, N definition of convergence.

Ensure that candidates understand when to use the standard tests for convergence of series.

Discrete

Ensure that problems involving the use of Fermat's little theorem are better handled.

Ensure that candidates are familiar with using powers of matrices for counting walks.