MATHS HL

Overall grade boundaries

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 26	27 - 39	40 - 51	52 - 63	64 - 75	76 - 100

Series and differential equations

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 26	27 - 40	41 - 51	52 - 63	64 - 75	76 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 28	29 - 41	42 - 52	53 - 65	66 - 77	78 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 28	29 - 41	42 - 52	53 - 64	65 - 76	77 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 18	19 - 23	24 - 29	30 - 34	35 - 40

Many well-written portfolios were noted this session. Generally, both teachers and students appear to have understood the assessment expectations well; however, also in evidence were teachers who interpreted the criteria incorrectly. Observations made by the moderators are summarised below. Sadly, many are perennial comments that appear not to have been considered.

The tasks:

Most portfolio tasks were taken from the current Teacher Support Material (TSM) for Mathematics HL, with only a very small number designed by teachers. Teachers are encouraged to design their own tasks, keeping in mind the need to satisfy all criteria fully.

There were two concerns that arose with portfolio tasks:

1. Some teachers continue to use old tasks taken from the previous TSM. Those tasks do not fully satisfy the current assessment criteria; hence, a number of candidates lost a significant number of marks through no fault of their own. Unless appropriate modifications were made, these <u>older tasks should not have been used</u>.

2. The use of the new tasks for 2009 and 2010 is not only premature, but results in a 10-mark penalty for inclusion in this session. Though isolated, such instances were sad to note.

Candidates' performance

Most candidates performed well against criterion A. The use of computer notation seemed to be very limited; however, the inappropriate use of "^", "E09", and the like, continue to be missed by some teachers.

Many samples contained work that was well written. Where a student's work began with an introduction to the task, and comments, annotations, and conclusions accompanied the steps and results, the work was easy to read and follow, and earned high marks in criterion B. However, there were a few whose work seemed disjointed, providing nothing more than a question and answer format to the task. Unlabelled graphs and the relegation of tables to the appendix rate poorly in terms of an effective presentation.

Criteria C and D are meant to assess the mathematical content, and jointly comprise half of the total marks awarded to each piece of work. Generally, students have produced good work, and the assessments by their teachers have been appropriate. However, in some type I tasks, insufficient exploration and patterning rendered the quick formulation of a conjecture questionable. Where several intermediate general statements were derived, the proof of "the general statement", as opposed to "a general statement", needed to be evident to warrant full marks.

In type II tasks, variables should be explicitly defined. Some realisation of the significance of the results obtained in terms of the model when compared to the actual situation should have been provided, and students should have reflected on their findings. The analyses of data must be quantified, and if a regression analysis were appropriate, the student must have provided reasons for a particular choice. The use of software that automatically determines the "best" regression model often leaves little for the candidate to interpret by himself; consequently, little credit can be awarded.

The use of technology varied considerably. Full marks were given much too generously for an appropriate but not necessarily a resourceful use of technology, for example, in the inclusion of a scatter plot produced on a calculator. As one moderator remarked some time ago, technology must be used to do more than merely "decorate" the work. Students should be discouraged from including GDC key sequences – they are quite unnecessary.

There were many good pieces of work; however, the awarding of full marks in criterion F requires more than completion and correctness, but the evidence of mathematical sophistication.

Suggestions to teachers

Please be advised that tasks from the TSM must not be used as of the May 2009 examination session; consequently, they must not be used with candidates who started their diploma program after September 2007. (For November candidates, this requirement applies to all candidates sitting their examinations in November 2009 and 2010.) The use of any tasks from the current or older TSM will carry a 10-mark penalty as of the May 2009 session. Please refer to the document, "Mathematics HL – The portfolio – Tasks for use in 2009 and 2010" for suggested tasks.

Teachers should select tasks that provide students with a variety of mathematical activities suitable at higher level. Tasks taken directly from the Mathematics SL TSM do not meet HL requirements and will result in the candidates losing a significant number of marks.

Teachers are expected to write directly on their students' work, not only to provide feedback to students, but information to moderators as well. Some samples contained very few teacher comments. Moderation was extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks.

Moderators find the background to each portfolio task very useful in determining the context in which the task was given when confirming the achievement levels awarded. This information must accompany each sample, either on Form A or through anecdotal comments.

If a teacher-designed task is submitted, a solution key must accompany the portfolios in order that moderators can justify the accuracy of the work, and appreciate the level of sophistication demonstrated in the work.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 33	34 - 49	50 - 63	64 - 77	78 - 91	92 - 120

The areas of the programme and examination that appeared difficult for candidates

On this paper candidates found difficulty with logarithms, some aspects of differentiation, discrete probability distributions, vectors, curve sketching and complex numbers. There are indications that a number of candidates were not prepared for questions on all aspects of the syllabus and that a number of candidates spent too much time on section A and hence ran out of time on section B.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been reasonably well prepared for questions on remainder and factor theorem, cumulative frequency curves, arithmetic sequences, implicit differentiation and proof by induction. There was no indication that candidates struggled with the arithmetic in any of the questions.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates made a meaningful attempt at this question. Weaker candidates often made arithmetic errors and a few candidates tried using long division, which also often resulted in arithmetic errors. Overall there were many fully correct solutions.

Question 2

There were fewer correct solutions to this question than might be expected. A significant number of students managed to combine the terms to form one logarithm, but rather than factorising, then expanded the brackets, which left them unable to gain an answer in its simplest form.

Question 3

There were many correct answers seen to this question. However a minority of students did not know what was meant by the inter-quartile range and in completing the frequency table in part (c) did not check that the frequencies added up to 100.

Question 4

There were many totally correct solutions to this question, but a number of candidates found two simultaneous equations and then spent a lot of time and working trying, often unsuccessfully, to solve these equations.

Question 5

Most candidates recognised that a method of integration by parts was appropriate for this question. However, although a good number of correct answers were seen, a number of candidates made algebraic errors in the process. A number of students were also unable to correctly substitute the limits.

Question 6

It was pleasing to see that a significant number of candidates understood that implicit differentiation was required and that they were able to make a reasonable attempt at this. A small number of candidates tried to make the equation explicit. This method will work, but most candidates who attempted this made either arithmetic or algebraic errors, which stopped them from gaining the correct answer.

Question 7

Part (a) was successfully completed by a significant majority of candidates. Part (b) was the first question that a significant majority of candidates struggled with. Only the better candidates understood that they needed to start with $y = \arctan x$ and then differentiate $x = \tan y$.

Question 8

Candidates found this question challenging with only better candidates gaining the correct answers. A number of students assumed incorrectly that the distribution was either Binomial or Geometric.

Question 9

There were a number of wholly correct answers seen and the best candidates tackled the question well. However, many candidates did not seem to understand what was expected in such a problem. It was disappointing that a significant number of candidates were unable to find the area of the hexagon.

Question 10

Only the better candidates were able to make significant progress with this question. Many candidates understood how to begin the question, but did not see how to progress to the last stage. On the whole the candidates' use of notation in this question was poor.

Question 11

Although the better candidates scored well on this question, it was disappointing to see that a number of candidates did not appear to be well prepared and made little progress. It was disappointing that a small minority of candidates were unable to sketch $y = \sin 2x$. Most candidates who completed part (a) attempted part (b), although not always successfully. In many cases the coordinates of the local maximum and minimum points and the equations of the asymptotes were not clearly stated. Part (c) was attempted by the vast majority of candidates. The responses to part (d) were disappointing with a significant number of candidates ignoring the hence and attempting differentiation which more often than not resulted in either arithmetic or algebraic errors. A reasonable number of candidates gained the correct answer to part (e), but a number tried to solve the equation is terms of $\sin x$ and $\cos x$ and made little progress.

Question 12

It was pleasing to see that the majority of candidates made a reasonable attempt at this question and that most had a reasonable idea of how a proof by induction works with matrices. However weaker candidates were often unable to show that if it was true for k, then it was also true for $k+1$. There were a reasonable number of correct answers to part (b) with many students recognising that they were being asked to work with the inverse.

Question 13

The response to Part A was disappointing. Many candidates did not know that they had to apply de Moivre's theorem and did not appreciate that they needed to find four roots. Part B started well for most candidates, but in part (b) many candidates did not appreciate the significance of *b* not lying on the real axis. A majority of candidates started (c) (i) and many fully correct answers were seen. Part (c) (ii) proved unsuccessful for all but the very best candidates.

Recommendation and guidance for the teaching of future candidates

- Students need to cover the entire syllabus.
- Students should be encouraged to pay attention to mathematical notation.
- Teachers should emphasise the importance of students setting out their procedures in a logical fashion.
- Most of the questions in this paper used common problem solving strategies and this should be a focus for candidates.
- Students need to practise papers of a similar style in order that they understand the need to balance their time.
- Students should be aware that they could be asked to sketch curves in this paper and hence should not be totally reliant on a calculator.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 30	31 - 47	48 - 61	62 - 76	77 - 90	91 - 120

The areas of the programme and examination that appeared difficult for candidates

It is disappointing to report that the majority of candidates were given an accuracy penalty, often on Question 1. Some candidates ignored the accuracy rules throughout the paper and it can only be assumed that they were completely unaware of the requirement to round answers to three significant figures.

Although candidates are generally well trained in the use of their GDCs, many candidates operated in the wrong angle mode, particularly in Question 6.

Some candidates have misconceptions concerning the conditions for a point to be a point of inflexion.

Some candidates showed a poor ability to undertake algebraic manipulation.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

Most candidates realised that the sine rule was the best option although some used the more difficult cosine rule which was an alternative method. Many candidates failed to realise that there were two solutions even though the question suggested this. Many candidates were given an arithmetic penalty for giving one of the possible of values $\hat{B}$ as $111.2°$ instead of $111°$.

Question 2

Many candidates misread the question and stopped at showing that the required term was the 115^{th}.

Question 3

Parts (a) and (b) were reasonably well done in general but (c) caused problems for many candidates where several misconceptions regarding the median were seen. The expectation was that candidates would use their GDCs to solve (a) and (b), and possibly even (c), although in the event most candidates did the integrations by hand. Those candidates using their GDCs made fewer mistakes in general than those doing the integrations analytically.

Question 4

Many candidates gained the first 4 marks by obtaining the equation, in unsimplified form, satisfied by m but then made mistakes in simplifying and solving this equation.

Question 5

Most candidates realised that some form of row operations was appropriate here but arithmetic errors were fairly common. Many candidates whose arithmetic was correct gave their answer as $k=3$ instead of $k \neq 3$. Very few candidates gave a correct answer to (b) with most failing to realise that stating that there was no common point was not enough to answer the question.

Question 6

Solutions to this question were extremely disappointing with many candidates doing the sketch in degree mode instead of radian mode. The two adjacent intercepts at 2.59 and 2.95 were often missed due to an unsatisfactory window. Some sketches were so small that a magnifying glass was required to read some of the numbers; candidates would be well advised to draw sketches large enough to be easily read.

Question 7

Part (a) was correctly solved by most candidates, either using the formula or directly from their GDC. Solutions to (b), however, were extremely disappointing with the majority of candidates giving $\sqrt{5}$, incorrectly, as their value of m. It was possible to apply follow through in (b) (ii) and (c) which were well done in general.

Question 8

Solutions were generally disappointing with many candidates being awarded the first 2 or 3 marks, but then going no further.

Question 9

Some candidates assumed that the decrease in population size was exponential / geometric and were therefore unable to gain the first 4 marks. Apart from this, reasonably good attempts were made by many candidates.

Question 10

Most candidates found this question to their liking and many correct solutions were seen. In (b), some candidates solved two equations for m and n but then failed to show that these values satisfied the third equation. In (e), some candidates used an incorrect formula to determine the coordinates of the mid-point of AB.

Question 11

Part A was well done by many candidates although an arithmetic penalty was often awarded in (b)(i) for giving the new value of the mean to too many significant figures. Candidates are known, however, to be generally uncomfortable with combinatorial mathematics and Part B caused problems for many candidates. Even some of those candidates who solved (a) and (b) correctly were then unable to deduce the answer to (c), sometimes going off on some long-winded solution which invariably gave the wrong answer. Very few correct solutions were seen to (d).

Question 12

It was disappointing to note that some candidates did not know the domain for $\arcsin$. Most candidates knew what to do in (b) but sometimes the wrong answer was obtained due to the calculator being in the wrong mode. In (c), the differentiation was often disappointing with $\arcsin\left(\frac{x}{3}\right)$ causing problems. In (f)(i), some candidates who failed to do (c) guessed the correct form of $f'(x)$ (presumably from (d)) and then went on to find $f''(x)$ correctly. In (f)(ii), the justification of a point of inflexion at $x = 0$ was sometimes incorrect – for example, some candidates showed simply that $f'(x)$ is positive on either side of the origin which is not a valid reason.

Recommendation and guidance for the teaching of future candidates

The majority of candidates failed to heed the following instruction on the front of the paper – 'Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures'. This resulted in an accuracy penalty being awarded to most candidates.

Candidates sometimes operate their GDC in the wrong mode when answering questions involving trigonometric functions. It has been suggested that candidates should normally set their GDC in radian mode when beginning to answer Maths HL questions since this is required more than degree mode. Teachers are recommended to give their candidates whatever advice they deem appropriate to solve this problem.

Paper three – Discrete mathematics

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 5	6 - 11	12 - 19	20 - 26	27 - 32	33 - 39	40 - 60

Paper three – Series and differential equations

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 12	13 - 20	21 - 26	27 - 31	32 - 37	38 - 60

Paper three – Sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 17	18 - 23	24 - 30	31 - 37	38 - 44	45 - 60

Paper three – Statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 18	19 - 23	24 - 29	30 - 35	36 - 41	42 - 60

General comments

Examiners felt that the four options papers provided scope for candidates to demonstrate their breadth of knowledge and mathematical competence. The small number of G2 comments indicated a satisfaction with the accessibility of the majority of questions asked. There was, however, evidence from the scripts that some candidates were unprepared to answer parts of questions on the Series and Differential Equations paper that went beyond core material. To a lesser extent that was true for the other options.

The areas of the programme and examination that appeared difficult for candidates

1. **Statistics and Probability**: Correctly stating hypotheses; applications of the Central Limit Theorem; conditional probability.
2. **Sets, relations and groups**: Providing convincing proofs that a set with binary operation has or has not the structure of a group; enumerating prime numbers.
3. **Series and differential equations**: Providing a rigorous approach to questions involving limits and the convergence/divergence of series; algebraic manipulation.
4. **Discrete:** Hexadecimal notation; Fermat's little theorem; Proofs involving f, e and v i.e. Euler's relation.

The levels of knowledge, understanding and skill demonstrated

A wide range of these elements was shown by candidates. Of course candidates were much more comfortable with the relatively routine parts of the options. At the top end, wide knowledge and manipulative skills were usually seen, but it was often not well expressed.

1. **Statistics and probability:** The competent use of a graphics calculator in hypothesis testing was usually apparent. Calculations involving specific distributions were also well carried out.
2. **Sets, relations and groups:** The following were handled competently: The construction of Cayley tables and their uses; the composition of functions; set operations.
3. **Series and differential equations:** Good skill and understanding was demonstrated in the two questions involving a differential equation. Many candidates were let down by poor manipulative skills in the follow-up parts of these and other questions.

4. **Discrete:** The niceties of the questions were often too much for the candidates and it was rare to see a good, complete solution that showed facility with the topic being examined. This was a little surprising since the paper was not overly difficult and candidates should have been able to do more than they did. It was the option with the fewest number of candidates.

The strengths and weaknesses of the candidates in the treatment of individual questions

1. Statistics and probability

Q1: It was disappointing that many candidates gave incorrect hypotheses. The binomial parameter had to be estimated from the data – the estimated value $p = 0.4$ should not appear in the hypotheses, and this also affects the number of degrees of freedom to be used in the x^2 test. Although marks were lost for the above conceptual error, the vast majority of candidates were able to demonstrate good GDC skills.

Q2: This question was generally well attempted as an example of the t-test. Very few used the Z statistic, and many found p-values.

Q3: Most candidates understood the context of this question, and the negative binomial distribution was usually applied, albeit occasionally with incorrect parameters. Good solutions were seen to part(b), using lists in their GDC or trial and error.

Q4: This was the only question on the paper with a conceptually 'hard' final part. Part(a) was generally well done, either by integration or by use of the standard formulae for a uniform distribution. Many candidates were not able to provide convincing reasoning in parts (b) and (c)(iii). Part(c)(ii), the application of the Central Limit Theorem was only very rarely tackled competently.

Q5: It was pleasing to see the many correct solutions to parts (a) and (b). Part(c) was simply an application of conditional probability, but this was rarely properly carried through.

2. Sets, relations and groups

Q1: It was surprising and disappointing that many candidates regarded 1 as a prime number. One of the consequences of this error was that it simplified some of the set-theoretic calculations in part(b), with a loss of follow-through marks. Generally speaking, it was clear that the majority of candidates were familiar with the set operations in part(b).

Q2: This question was generally well done, with the exception of part(b)(iii), showing that the operation is non-associative.

Q3: This question was generally well done. In part(a), the quickest answer involved showing that squaring the function gave the identity. Some candidates went through the more elaborate method of finding the inverse function in each case.

Q4: There was a mixed response to this question. Some candidates were completely out of their depth. Stronger candidates provided satisfactory answers to parts (a) and (c). For the other parts there was a general lack of appreciation that, for example, closure and the existence of inverses, requires that products and inverses have to be shown to be members of the set.

Q5: The majority of candidates were able to compute the composite functions involved in parts (a) and (b). Part(c) was satisfactorily tackled by a minority of candidates. There were more GDC solutions than the more obvious approach of factorizing a difference of squares. Some candidates seemed to forget that m and n belonged to the set of integers.

3. Series and differential equations

Q1: Part(a) was well done by many candidates. In part(b)(i), however, it was disappointing to see so many candidates unable to differentiate $x(\ln x - 1)$ correctly. Again, too many candidates were able to quote the general form of a Taylor series expansion, but not how to apply it to the given function.

Q2: Part(a)(i) caused problems for some candidates who failed to realize that the integral can only be tackled by the use of partial fractions. Even then, the improper integral only exists as a limit – too many candidates ignored or skated over this important point. Candidates must realize that in this type of question, rigour is important, and full marks will only be awarded for a full and clearly explained argument. This applies as well to part(b), where it was also noted that some candidates were confusing the convergence of the terms of a series to zero with convergence of the series itself.

Q3: Most candidates failed to realize that the first step was to write $f(x)$ as $(1+ax)(1+bx)^{-1}$. Given the displayed answer to part(a), many candidates successfully tackled part(b). Few understood the meaning of the 'hence' in part(c).

Q4: Many candidates successfully obtained the displayed solution of the differential equation in part(a). Few complete solutions to part(b) were seen which used the result in part(a). The problem can, however, be solved by direct differentiation although this is algebraically more complicated. Some successful solutions using this method were seen.

4. Discrete

Q1: (a) Many did not seem familiar with hexadecimal notation and often left their answer as 12101514 instead of CAFE.

(b) The Euclidean algorithm was generally found to be easy to deal with but getting a general solution in part (iii) eluded many candidates.

(c) Rewriting the congruence in the form $9x = 3 + 18\lambda$ for example was not often seen but should have been the first thing thought of.

Q2: Setting out clearly the steps of the algorithms is still a problem for many although getting the correct spanning tree and its length were not.

Q3: (a) Some candidates were obviously not sure what was meant by 'product of primes' which surprised the examiner. Few good solutions to part (b) were written. There were some reasonable attempts at part (c) using powers rather than Fermat's little theorem.

Q4: Many candidates seem only to have a weak understanding of the requirements for the proof of a mathematical statement.

Recommendation and guidance for the teaching of future candidates

The examiners share a concern that too few students seem able to demonstrate that they have explored the topics in the program to the depth expected. Many students are able to perform routine operations successfully but then flounder once slightly more difficult and applied situations are encountered. We get the impression that perhaps teaching pressures have limited the time available for students to experience the wide range of more challenging questions within their chosen option.