International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHS HL

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 26	27 - 40	41 - 52	53 - 64	65 - 75	76 - 100

Higher level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 18	19 - 23	24 - 29	30 - 34	35 - 40

The range and suitability of the work submitted

Most of the portfolio tasks were taken from the current Teacher Support Material (TSM) for Mathematics HL, but there were also several excellent new tasks submitted by a number of schools. Teachers are encouraged to design their own tasks, keeping in mind the need to satisfy all criteria fully. It should be noted that investigative tasks that preclude the use of technology, and modelling tasks in which the model is not created by the student, but given within the task, fall short of fulfilling the requirements of the portfolio.

Risking disastrous consequences for the candidates, some teachers appear not to be aware of the requirements under the new syllabus and continue to use old tasks taken from the previous TSM. Those tasks do not fully satisfy the current assessment criteria; hence, a number of candidates lost a significant number of marks through no fault of their own. Unless significant modifications are made, these older tasks should not be used. As announced in the *Coordinator Notes*, the use of any tasks from the current or older TSM will carry a 10-mark penalty as of the May 2009 session.

Tasks taken from the TSM for Mathematics SL are not at a suitable level for Mathematics HL and should not be used.

Candidate performance against each criterion

Most candidates performed well against criterion A. The use of computer notation seemed to be very limited. Correct terminology should include the use of correct mathematical vocabulary.

Many samples contained work that was well written. Where a student's work began with an introduction to the task, and comments, annotations, and conclusions accompanied the steps and results, the work was easy to read and follow, and earned high marks in criterion B. However, there were a few whose work seemed disjointed, providing nothing more than a question and answer format to the task. Unlabelled graphs and the relegation of tables to the appendix rate poorly in terms of an effective presentation.

Jointly, criteria C and D are meant to assess the mathematical content and comprise half the marks awarded to each piece of work. Generally, students have produced good work, and the assessments by their teachers have been appropriate. However, in some type I tasks, insufficient exploration and patterning rendered the quick formulation of a conjecture questionable. Where several intermediate general statements were derived, the proof of "*the* general statement", as opposed to "*a* general statement", needed to be evident to warrant full marks.

In type II tasks, variables should be explicitly defined. Some realisation of the significance of the results obtained in terms of the model when compared to the actual situation should have been provided, and students should have reflected on their findings. The analyses of data must be quantified, and if a regression analysis were appropriate, the student must have provided reasons for a particular choice. The use of software that automatically determines the "best" regression model often leaves little for the candidate to interpret on his own; consequently, little credit can be awarded to the candidate.

The use of technology varied considerably from the truly resourceful to the merely perfunctory. Full marks were given much too generously for an *appropriate* but not necessarily a *resourceful* use of technology, for example, in the inclusion of a single graph produced on a calculator. As one moderator remarked, technology must be used to do more than merely "decorate" the work. Students should be discouraged from including GDC key sequences – they are unnecessary and unwarranted.

There were many, many good pieces of work; however, the awarding of full marks in criterion F requires more than completion and correctness, but the evidence of mathematical sophistication.

Recommendations for the teaching of future candidates

Teachers should select tasks that provide students with a variety of mathematical activities suitable at higher level. Tasks taken directly from the Mathematics SL TSM do not meet HL requirements and are not considered acceptable.

The teacher who remains uninformed or chooses to ignore the changes to the portfolio assessment criteria is generally the reason for a significant loss of marks in moderation. This is completely unfair to the student and must be rectified.

Teachers are expected to write directly on their students' work, not only to provide feedback to students, but information to moderators as well. Some samples contained very few teacher comments. Moderation was extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks.

Moderators find the background to each portfolio task very useful in determining the context in which the task was given when confirming the achievement levels awarded. This information must accompany each sample, either on Form A or through anecdotal comments.

If a teacher-designed task is submitted, a solution key must accompany the portfolios in order that moderators can justify the accuracy of the work, and appreciate the level of sophistication demonstrated in the work.

Higher level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 31	32 - 50	51 - 63	64 - 76	77 - 89	90 - 120

The areas of the programme and examination that appeared difficult for the candidates

On this paper candidates found difficulty with statistics, justifying points of inflexion, transformation of curves, combinations, some aspects of calculus and sector areas. Many candidates suffered an accuracy penalty with some candidates incorrectly rounding throughout the paper.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been reasonably well prepared in that topics appeared to have been taught and that students had been exposed to the relevant concepts. A majority of candidates used the GDC reasonably well and showed the necessary working. On the whole candidates had success with the factor theorem, binomial theorem, geometric series and simple applications of calculus.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates successfully answered this question. One or two candidates tried to use long division, which often led them into algebraic difficulties.

Question 2

This was done well by most candidates. The most common error was to forget the negative sign.

Question 3

This response to this question was disappointing. Only part (b) was successfully answered by the majority of candidates. A significant number did not understand what was meant by the interquartile range or by the unbiased estimate of the population. A number of candidates tried to do part (c) without using the statistical functions on the calculator, which more often than not resulted in arithmetic errors.

Question 4

Most candidates had some success with this question. The major problem experienced by candidates was the fact that the common ration was negative, which then led to errors in the formula for the sum to n terms.

Question 5

Many candidates answered this question successfully. A small number tried to find the volume of the solid formed when A is rotated about the y axis and a small number misread the equation of the curve as $y =$ as opposed to $y^2 =$. In both cases these students were able to make little progress.

Question 6

The vast majority of candidates recognised the use of the chain rule in part a) and successfully found $\frac{dy}{dx}$ and most of these went on successfully to find $\frac{d^2y}{dx^2}$. In part (b) many candidates successfully found the x coordinates of the points of inflexion, but few candidates were able to give a correct justification.

Question 7

The vast majority of candidates were able to start the question, but a significant number of these did not gain the final correct answer. A number tried to find a unique solution to the question and a number justified that it had a non-unique solution without actually finding it.

Question 8

Part (a) was done successfully by the majority of students. However, part (b) caused many more problems. Relatively few recognised the significance of both the 3 and the negative sign.

Question 9

Many candidates successfully completed this question. Of those who started the question and did not gain full marks, the most common mistakes were to leave out the negative sign in the initial equations or to make arithmetic errors in solving the simultaneous equations.

Question 10

Most candidates seemed to have a reasonable idea of what to do here, and many correct answers were seen. Common errors were incorrect expansion of the determinant and an inability to solve the resulting cubic equation equal to zero.

Question 11

Again most candidates seemed to have a reasonable idea of what to do here and many correct answers were seen. For those that started the question and did not gain full marks, the most common error was in part (b) where a number of students did not realise the need to substitute the values of λ and μ back into the third equation to justify that it was a point of intersection.

Question 12

This question was not found easy by many candidates. A significant number of candidates clearly had difficulties in interpreting the mathematics from the question.

Question 13

Most candidates made a start on this question. However, relatively few gained full marks. A number tried a non-calculator approach to both parts, which nearly always led to the wrong answer. Those who tried a calculator approach often had the wrong window on the calculator, which led to partially correct answers.

Question 14

A significant number of candidates gained the correct answer to this question. Of those who started the question, the most common errors were to assume that either proving the line is parallel to the plane or that the line and the plane have a point in common was sufficient.

Question 15

A significant number of candidates gained full marks for this question. Problems resulted in part (a) when candidates did not realise the quadratic nature of the equation and in part (b) when they failed to complete the simplification and isolate x on only one side of the equation.

Question 16

This was one of the questions that candidates found the most difficult. Only a small number of candidates gained the correct answer and a significant number of the candidates who did not gain the correct final answer, did not explain what they were trying to do. Candidates should be aware that clear working in a question like this might allow marks to be awarded.

Question 17

There was a misprint in this question. The question stated $0 < a < \frac{\pi}{2}$ rather than $0 < a < 1$. Although this is regrettable, there was no evidence from the scripts that candidates had noted this error and those candidates that recognised integration by parts as the appropriate method were successful. Unfortunately only a minority of candidates recognised this.

Question 18

On the whole candidates found this question difficult, with relatively few gaining the correct answer. A significant number did not actually start the question and those that did often made assumptions about the triangles PRQ and PQS being right-angled.

Question 19

There were a number of correct answers seen to this question, which was pleasing. However, many candidates did not know how to start the question. A number also attempted a proof by induction, which led them into difficulties.

Question 20

The better candidates had some success with this question, being able to find y. However, only the best candidates succeeded in gaining fully correct solutions.

Recommendations and guidance for the teaching of future candidates

- Teachers should ensure that the statistics section of the syllabus is thoroughly taught.
- Students should be encouraged to pay attention to mathematical notation and accuracy.
- Teachers should emphasise the importance of students setting out their procedures in a logical fashion.
- When the word “exact” is given, students should realise that using a GDC approach is usually inappropriate.

Higher level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 30	31 - 46	47 - 61	62 - 75	76 - 90	91 - 120

The areas of the programme and examination that appeared difficult for the candidates

The topics found difficult by many candidates were the vector form of lines in 2-D and mathematical induction. Some candidates seem unable to understand terms such as ‘more than’ and ‘at least’ in questions on probability.

The areas of the programme and examination in which candidates appeared well prepared

The level of knowledge, understanding and skill demonstrated was reasonably good in general. Most candidates proved to be competent in the use of a GDC and used their calculator appropriately. Many candidates suffered an accuracy penalty somewhere in the paper. Some candidates need to be aware of the meaning of the word ‘hence’.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates solved (a) correctly, the main problem with the implicit differentiation being the $2xy$ term. In (b), however, some candidates appeared to be unfamiliar with the vector equation of a line in 2-D, so much so that they found it necessary to find the Cartesian equation first and then attempt, often incorrectly, to convert to vector form. Some candidates confused A with B and simply gave the equation of the normal at A. In this case, full follow through in (c) was not possible because their two lines were perpendicular.

Question 2 – Part A

Part(a) was well done by many candidates but in (b), some candidates, for no apparent reason, obtained the correct expression for $h(k(x))$ but then just stopped without attempting to apply the condition for equal roots.

Question 2 – Part B

Part (a) was well done by most candidates. In (b), however, the reason given for there being only eight distinct values was sometimes unconvincing. In spite of the word '**hence**', some candidates failed to see the link between (c) and the earlier work and attempted, often unsuccessfully, to solve the equation from scratch.

Question 3 – Part A

In (a), candidates often misinterpreted the terms 'at least' and 'more than'. These and other similar expressions often occur in questions on probability and candidates need to be aware of their exact meaning. In (a), although full marks were given for writing down only the answer, it would have been advisable to explain, as far as possible, the method being used so that some marks could be awarded even if the final answers were incorrect. Part (b) was reasonably well done although the presentation of the solution was often poor with little explanation of what was being done. Most candidates solved the inequality $e^{-2x} < 0.01$ by taking logs although a variety of different correct methods using a GDC were seen. A trial and error method was accepted although this approach is not recommended because it might be time-consuming in a different situation.

Question 3 – Part B

Most candidates knew what had to be done in (a) but not all were able to do the necessary integration. Some candidates, for some unknown reason, took the lower limit to be $-2\sqrt{3}$ which led to only partial follow through in (b) and (c) because the question was simplified by this error. Some candidates thought that the mode was f(0) rather than 0. Candidates who used their GDC to find the mode, giving answers such as 6×10^{-10}, were given no credit. Some candidates ignored the word 'exact' in (a) and (c) for which only limited credit was given.

Question 4

Part (a) was well done by many candidates, although parts of the graph were sometimes omitted due to inappropriate choice of window. The asymptote $x = 0$ was sometimes missed and inaccurate values were given for the intercepts and coordinates of the maximum and minimum points. In (b) and (c), many candidates simply substituted their answers from (a) into the equations and obtained a small number which they then stated was 0. This was not accepted as a correct method. Solutions to (d) were often disappointing with some candidates being unfamiliar with the properties of the relevant trigonometric functions.

Question 5 – Part A

This was well done by many candidates although it was disappointing to see some candidates writing $(1 + t^2)$ instead of $(1 + t)^2$, either through carelessness or thinking that they were the same thing. Some candidates omitted the arbitrary constant in (a).

Question 5 – Part B

The layout of proofs by induction continues to be poor in general. In many cases, it is impossible to see what is being assumed and sometimes the result for $n = k + 1$ is simply taken from the result to be proved rather than going back to the algebraic process being investigated.

Recommendations and guidance for the teaching of future candidates

- Ensure that candidates are aware of the accuracy rules. Many candidates are losing a mark for incorrect rounding.
- Concentrate on proof by induction, a topic which continues to cause problems for many candidates.
- Remind candidates of the need for an arbitrary constant when integrating.

Higher level paper three

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 12	13 - 21	22 - 28	29 - 34	35 - 41	42 - 60

General comments

Options A and C remain the most popular options. The paper seemed to be of the required standard and there was a full range of scripts, even one that received full marks.

Not all candidates cooperated with the first request on page 2 to begin each question on a new page which makes the marking a little more difficult.

The requirement 'all numerical answers must be given exactly or to three significant figures unless stated otherwise in the question' is also too often overlooked.

The areas of the programme and examination that appeared difficult for the candidates

Section A: this option was covered quite well. Appropriate use of the GDC could have been better. Understanding of the null and alternate hypotheses was not as good as it should have been. The Poisson distribution presented more difficulty than expected.. Understanding of type I and type II errors.

Section B: Clarity in defining sets, proving set identities, equivalence classes all appeared to present difficulties for the candidates.

Section C: Successive differentiation to get the Maclaurin series, the binomial series, successive use of differentiation in l'Hopital's rule and changing the variable in the differential equation proved difficult.

Section D: Generally well done but clear proofs were not seen very often.

The areas of the programme and examination in which candidates appeared well prepared

Section A: Use of the normal distribution. Application of the χ^2 test. Use of the calculator and *p*-values.

Section B: Equivalence relations. Group properties and proof. Composition of permutations and generators

Section C: Limits, Maclaurin series, differential equations.

Section D: Base/modular arithmetic, gcd, algorithms for spanning trees, Euler's relation, and divisibilty.

The strengths and weaknesses of the candidates in the treatment of individual questions

Section A:

1. Parts (a) and (b) were well done although some students quoted a normal curve equation without any apparent reason. The hypotheses were muddled in part(c) although many got the correct *p*-value. The simplicity of this last part using the GDC escaped some students.

2. (a) was quite well done although too many candidates did not combine columns with low expected frequencies. Here again the question is greatly facilitated if the candidate knows how to use the GDC efficiently.

Part (b) proved beyond most students.

There was some confusion about which groups should be combined in part (c) and again the hypotheses were sometimes not correct. The arithmetical calculations were easily done with the GDC.

3. In part (a) too many candidates thought it necessary to prove that the distribution mean is $\frac{1}{\lambda}$ rather than just quoting it from the formula book. This resulted in a loss of time. Part (b)(i) was found to be easy but the other parts of this question were not at all well done.

4. Part (a) was easy if the candidate used symmetry to get the value 65 but even if this method was overlooked the part proved relatively straight forward.

Part (b)(i) (Amanda) was done easily but part(Roger) proved more problematical. Part (b)(ii) was found by many candidates to be difficulty free.

Section B:

1. (a) should have been easy but there were some strange answers that used very odd notations. Even so a large number of candidates were successful. Part (b) could have been done starting at either the left or right side and was not found to be difficult. Unfortunately there are still too many scripts offering a Venn diagram ‘proof’. This is not acceptable!

2. This question was quite well done except for parts (b) (ii) and (iii). The idea of equivalence classes still eludes too many candidates.

3. Part (a) was often done using the calculator which is not really acceptable; a more formal proof like the one in the mark scheme was required. The last R1A1 marks were usually easy to come by.

Part (b) proved to be problematical given that there was an incorrect definition of a codomain in the paper. However adequate measures were taken to protect the candidates who did this option.

4. Part (a) was found to be easy but many candidates, whilst finding one of the two generators, did not show that $S,*$ formed a group. Part (b) was well done except for a few problems with (iii).

Section C:

1. Some candidates confused domain and range in (a) (i) and had no small difficulty in finding the successive derivatives for the Maclaurin series.

The phrase 'Use the small angle approximation……..' was too often ignored so that some candidates produced irrelevant work that gained no marks.

The intricacies of part (c)(i) were beyond not a small number of candidates.

2. This question proved to be quite easy for most candidates.

3. Part (a) was well done generally but the final line showing that the differential equation in X and Y had homogeneous form was often omitted. The second use $Y = vX$ of led to all sorts of wrong turns although an encouraging number of candidates were successful.

4. 'By considering the graph….'was completely ignored by many candidates. A quick sketch of the graph was what was expected (and needed) to set the solution in the right direction. This process must have been seen before in the beginnings of learning calculus so the question was in no way esoteric. Part (c) was found to be difficult and there were not many acceptable solutions to it.

Section D:

1. Most candidates found this question to be accessible with only (a) (ii) presenting some difficulty.

2. This question was largely well done with many correct solutions and answers.

3. . Part (b) caused difficulty for some candidates since a proof (logical argument) is required. Most worked through part (c) but missed (4,3) not being a possible answer

4.This was largely well done although part (a) proved to be tricky.

Recommendations and guidance for the teaching of future candidates

It seems that sometimes candidates cover a great deal of material but not enough of it thoroughly. Perhaps this is a problem of time management when teaching the course. If this is the case it might be better to consider reorganizing the scheme (i.e. order of topics) used to better make better use of the time available. Linking topics in the option with core material is a profitable approach and to some extent the chosen option can be taught as an integral part of that option.

Ideas of proof and the specific nature of proof required in a particular option would be worth thinking about.

More efficient use of the GDC is needed. Devoting lessons specifically to this is not a waste of time.

Too great a reliance on the formula book possibly tends to encourage the idea that results need not be remembered.