May 2017 subject reports

# Mathematics HL

## Overall grade boundaries

### Discrete

| Grade: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Mark range: | 0 - 13 | 14 - 28 | 29 - 42 | 43 - 55 | 56 - 67 | 68 - 79 | 80 - 100 |

### Calculus

| Grade: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Mark range: | 0 - 12 | 13 - 27 | 28 - 39 | 40 - 52 | 53 - 64 | 65 - 77 | 78 - 100 |

### Sets, relations and groups

| Grade: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Mark range: | 0 - 13 | 14 - 28 | 29 - 41 | 42 - 55 | 56 - 67 | 68 - 79 | 80 - 100 |

### Statistics and probability

| Grade: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Mark range: | 0 - 13 | 14 - 28 | 29 - 41 | 42 - 54 | 55 - 66 | 67 - 79 | 80 - 100 |

# Higher level internal assessment

## Component grade boundaries

| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range**: | 0 - 2 | 3 - 5 | 6 - 8 | 9 - 11 | 12 - 14 | 15 - 16 | 17 - 20 |

# The range and suitability of the work submitted

Some schools presented an interesting variety of explorations on topics of personal interest to students. It was clear that in these cases the students had received clear guidance from the teacher. Such schools should be commended.

On the other hand a significant number of candidates submitted what they themselves called a “research report” which simply consisted of transcribing mathematics and copying images from online sources or textbooks. An exploration that simply relays published findings is not likely to achieve high levels.

There was evidence to suggest that some schools are still advising students to write explorations on Mathematical topics that are well beyond the level of the Maths HL course. It is difficult in such cases for students to write an exploration that meets the aims of the IA within the page limit. As stated in the guide “The final report should be approximately 6 to 12 pages long. Students should be able to explain all stages of their work in such a way that demonstrates clear understanding. While there is no requirement that students present their work in class, it should be written in such a way that their peers would be able to follow it fairly easily.” Some explorations were still far too long.

# Candidate performance against each criterion

Criterion A

This year the highest achievement level in this criterion seemed to be inaccessible for a larger number of students. A number of students produced explorations which were far too long. Some students submitted work with little flow, segmented with sub-headings. For a piece of work of this length there is no need for a table of contents or a research question. In some cases, students produced a research report about mathematics beyond the scope of the course, and in doing so ended up writing a piece of work that read more like a chapter out of a text book rather than an exploration.

Criterion B

Mathematical presentations were generally good. In most cases variables and parameters were defined and graphs were labelled. Unfortunately some candidates still presented work with calculator notation using * for multiplication and ^ for powers.

Criterion C

This criterion continues to present difficulties for some teachers and students. Some teachers award marks in this criterion, when a student simply introduced their exploration by some feigned interest, such as “I have been playing basketball since I was 4 years old”… Often these rationales were not supported by the rest of the work. Research reports of familiar “textbook” derivations cannot be awarded high levels unless the work is personalized and / or the student’s voice can be heard. Simply learning new mathematics does not demonstrate abundant personal engagement.

Criterion D

Some students provided ongoing meaningful and critical reflection throughout the work. However, more students provided a summative reflection at the end of the exploration as part of the conclusion. Although this is not entirely wrong, the hazard in writing a reflection at the very end, is that students end up describing what was done, without providing any arguments about the validity or correctness of their approach. Reflection in explorations should be ongoing, and act as a stepping stone from one part of the exploration to another. Ongoing critical reflection is meant to drive the development of the exploration, by interpreting results, discussing the implications of results and possibly refining the approach taken when recognising shortcomings.

Criterion E

The mathematical content in explorations varied greatly. There still seems to be confusion among teachers regarding what is “commensurate” with the course. The mathematics does not need to be exclusively from the section of the syllabus that is only HL. A student may use simple mathematics but apply it to a topic that is personalized and still obtain a good grade. If the mathematics used is very simple, then it cannot obtain high scores as it cannot be deemed to reflect the sophistication expected. On the other hand, students who choose to write research reports on topics that are well beyond the level of the course, often end up not being able to explain the mathematics from one step to another, making it difficult to gauge the level of understanding. Unfortunately a number of times, errors were found in students’ work that were not identified by the teacher.

## Recommendations for the teaching of future candidates

It is of fundamental importance that students cite any work at the point of reference in the exploration; this also includes any images, charts or diagrams.

It is recommended that the exploration is introduced early in the course, but the actual process should be delayed until a fair amount of the syllabus has been covered. Teachers should invest time in going over the criterion descriptors with students to ensure that students thoroughly understand the expectations. One way of doing this would be to use explorations from the Teacher Support Material with students. There was evidence to suggest that students were not always given adequate feedback on a first draft. Students should also be advised to proof-read their work before submission.

Students should be reminded that the work submitted should be in standard format with an appropriate font and at least 1.5 line spacing. Using a small font and single spacing to fit an exploration into less pages should be avoided at all costs. Once the student work has been scanned it should be checked by both teacher and student to ensure that scans are in colour and that the scanned work is complete and legible.

Once the explorations have been submitted teachers need to mark the explorations. Evidence of marking must be shown on the submitted student work. This includes tick marks to indicate correct work, identification of errors, annotations and comments to explain where and how the achievement levels were awarded. The moderator's role is to confirm the teacher's marks but where annotations and comments are missing the moderator will have to mark the work without having any background information and very often it is less likely that a moderator can confirm all the achievement levels awarded. When annotating work digitally it is better for annotations to be made on the student work at the point of reference and not collected as an appendix or preface to the student work. Internal standardisation should take place to ensure consistent marking.

# Higher level paper one

### Component grade boundaries

| **Grade:** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range:** | 0 - 12 | 13 - 25 | 26 - 37 | 38 - 51 | 52 - 64 | 65 - 78 | 79 - 100 |

# General comments

In general, the performance of the candidates on this component was pleasing. Most candidates showed a reasonable understanding of most of the material. Many candidates were able to achieve excellent marks on this component.

# The areas of the programme and examination which appeared difficult for the candidates

Complex numbers seem to have been poorly prepared as was much of the Statistics. Many candidates seemed ill prepared for questions which required them to think in a different context, rather than textbook type questions. The series question was surprisingly poorly done, given that it is one of the easier topics. Candidates often did not follow instructions well, and did not seem to consider that a 1 mark question is unlikely to require considerable work. There often seemed to be a tendency to rush into a known technique, rather than taking time to think. Frequently the layout of solutions could be very poor and candidates seldom gave any words of explanation. Many candidates seemed little understanding of what is meant by a sketch - and given that graph paper is not supposed to be issued, there were many solutions submitted on graph paper. Sums and products of roots, still does not seem to be covered well in all schools. Similarly, few students knew what an interquartile range was. Proof by induction still remains a problem with candidates failing to lay out the proof correctly.

## The areas of the programme and examination in which candidates appeared well prepared

Techniques of algebra and calculus seemed to be well understood, and trigonometric techniques seemed universally well prepared. Candidates seemed to have a sound knowledge of logarithms, and were frequently able to sustain correct working in a longer algebraic question.

## The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1 – There were many good solutions to this question.

Question 2 – It was surprising how many candidates were unable to get started with this question. Many students did not follow instructions and so lost marks unnecessarily.

Question 3 – The question was well answered by the majority of candidates.

Question 4 – There were many good solutions to this question, utilising a variety of different methods. Many candidates who were not able to arrive at the correct final answer, may well have lost potential marks through not articulating their work. Students should be encouraged to explain their reasoning when answering questions.

Question 5 – The responses to this question were quite varied. Some had clearly been taught vectors and could approach the question well, often arriving at good solutions. Some candidates who knew what they should do lost marks in arithmetical errors in finding the cross product. Other candidates seemed to be floundering at the outset. The reasoning in part (b) was frequently sketchy, and so the final marks were not obtained.

Question 6 – There were many good solutions to this question, although its unfamiliar nature left some candidates unaware of how to start. Many students attempted to integrate leading themselves into inevitable problems.

Question 7 – The question was surprisingly poorly done, given that it is a very common "textbook style" question. This was true of both parts of the question.

Question 8 – Still very few candidates are able to obtain full marks on a relatively simple induction question. Few were able to construct the argument correctly, even if they were able to complete the inductive step successfully. It seemed that candidates had seen inductive proofs, but demonstrated little understanding of the meaning of that structure.

Question 9 – For a fairly difficult question with no scaffolding, the question was surprisingly well answered. Many students were able to do the first integration by parts, but failed to find the second integral.

Question 10 – In general, candidates seemed to find this the most difficult question in section B. Most students were able to set up the integral in part (a) but many missed the $-\frac{6}{\pi}$ when integrating. In part (b), most students ignored the instruction “by considering the graph” and failed to recognise that long calculus methods are unlikely to be expected for one mark. Most that attempted the three parts using calculus, lost their way at some point, and so could not be awarded any marks. In part (c), few students knew what an interquartile range is. Most candidates who were successful through the question, were able to answer part (d) well.

Question 11 – There were a surprising number of errors in completing the square in part (a). Marks were frequently lost in not sketching the curve properly, especially in indicating the features required. In part (d), a number of candidates ignored the “hence”, and consequently encountered difficulties with the integral. Many students were unable to do part (e) correctly and consequently were unable to continue to part (f).

Question 12 – There were many good solutions to this question, although many were overly long-winded. Division of polynomials was used more than using the sum and product of the roots – indeed, the worst part of the question was part (a). Many students did not recognise in part (e) that $f'(1) = 0$.

## Recommendations and guidance for the teaching of future candidates

It is important that all parts of the syllabus are covered. It was clear in the papers that for many students, large parts of the syllabus have been omitted. Candidates should be made aware of the command terms and encouraged to take note of exactly what is being asked, before embarking on any question. In a similar way, students should be encouraged to think of how the question parts are linking together to help them answer the various parts. Candidates should be aware that sketches of graphs, should be just that (and not on graph paper), and important features are clearly shown.

Candidates should be prepared to apply their mathematical knowledge in unfamiliar situations, and will need to have experience of this throughout the teaching of the course.

Candidates also should be encouraged to develop greater clarity in their reasoning shown in mathematical solutions.

## Higher level paper two

### Component grade boundaries

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Grade:** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| **Mark range:** | 0 - 16 | 17 - 33 | 34 - 44 | 45 - 56 | 57 - 67 | 68 - 79 | 80 - 100 |

## General comments

It made a noticeable difference having less questions. Candidates in general attempted all questions and their performance was better than in previous sessions when they struggled for time to complete the examination. As mentioned by many, the paper seemed to be quite straightforward, but it was surprising how few candidates were able to score high marks.

## The areas of the programme and examination which appeared difficult for the candidates

- Transformations of trigonometric functions.
- The ambiguous case of the sine rule.
- Binomial expansions.
- Probability of non-independent events.
- Related rates of change in an unfamiliar context.
- Finding the area between two graphs using a definite integral.
- Inverse circular functions.
- Recognising binomial distribution.
- Solving normal distribution problems with the unknown standard deviation.
- Differentiation using the combination of the quotient and chain rule.
- The main area that students seemed to find difficult was interpretation of problems set in an unfamiliar way.
- Students seemed to have little conceptual understanding of set notation.

## The areas of the programme and examination in which candidates appeared well prepared

- Implicit differentiation.
- The Poisson distribution.
- Kinematics.
- Calculus.
- Using their GDC for graphs, probabilities, areas.
- Inverse functions.
- Using the cosine rule and the area of triangle formulae.

## The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Part (a) was in general well answered with most candidates giving one answer when solving a quadratic equation. Too many tried to solve by hand when it was more efficient to use their GDC.

Part (b) caused more problems with many candidates assuming the independence of events thus scoring no marks.

Question 2

Part (a) in general very well done; with some mistakes when rearranging the equation or forgetting to differentiate 1.

Part (b) also very well done; however; some candidates only finding the gradient of the tangent instead of the equation of the tangent.

Question 3

Conceptually well understood what needed to be done, however; many numerical errors prevented candidates from obtaining full marks. Not enough candidates chose the general term method to solve the problem; most attempted full expansions, which was unnecessary.

Question 4

Part (a) was attempted by most candidates taking various approaches. Limits were often incorrect and showed that many did not fully understand how to approach this question.

Part (b) was well done by those who had the correct expression in part (a); again, too many tried to integrate by hand instead of using their GDC.

Question 5

Part (a) was very well done.

Part (b) caused more problems with recognising the binomial distribution and then being able to use their GDC to calculate it.

Question 6

Part (a) was in general well done; some candidates lost marks by attempting the general equations of asymptotes rather than giving the three required asymptotes in the given domain.

Part (b) was poorly done; most candidates recognised vertical translation correctly, however, dilation, reflection and horizontal translation caused serious difficulties. Only very few candidates scored full marks for this part.

Question 7

This question was attempted by most candidates with varying degree of success. Most candidates were able to find vector AB and attempted to calculate the cross product; not all correctly. After which they proceeded to substitute the point and find the Cartesian equation of the plane. Some candidates chose a different method with forming simultaneous equations but mostly were unable to arrive at the correct answer.

Question 8

Part (a) was done very well by many candidates; some forgetting to multiply the area of segment by 10. However, some candidates chose a very complicated method with half-angles and were rarely successful in obtaining the correct answer.

In part (b) many correct answers were seen; some were awarded FT marks; most attempted related rates or implicit differentiation.

Question 9

Parts (a) and (b) were well done.

Part (c) was a mixture of all correct and full marks or nothing correct and no marks.

Question 10

Part (a) extremely well done, only a handful of really weak candidates not scoring full marks here.

Part (b) very well done, with mostly correct answers seen. Some candidates did not follow the ‘hence’ and attempted the sine rule which was much more time consuming.

Part (c) equally well done.

Part (d) was often left blank with most candidates unable to recognise the ambiguous case of the sine rule. Some used the discriminant correctly thus gaining partial marks for the lower limit but no or partial marks for the upper limit.

Question 11

Probably the best answered question on the paper with nearly all candidates scoring full marks on parts (a) and (b). Part (c) was also done extremely well with most gaining full marks here as well.

Some candidates tried to use the constant acceleration formulas throughout which was incorrect. Some were unable to use their GDC to calculate correctly the value of the definite integral.

Question 12

Part (a) was well done by most but interval notation was often incorrectly used.

In part (b) most candidates scored the shape mark but many missed on the asymptotes and axes intercepts.

Parts (c) and (d) were very well done with most candidates reasoning properly about even and one to one functions.

Part (e) was mainly well done, most candidates getting 2 or 3 marks; some missing the domain mark, some missing marks due to incorrect algebraic manipulation.

Part (f) was rather poorly done with many candidates unable to handle the combination of the quotient and chain rules.

Part (e) (i) some good attempts here but reasoning from the incorrect derivative in many cases.

Part (e) (ii) as the last part on the paper often unattempted. Some candidates confused which function to differentiate in parts (e) and (f).

## Recommendations and guidance for the teaching of future candidates

It would assist candidates if teachers would emphasize the following to their students:

- Candidates should be encouraged to check the feasibility of their answers – for example, finding the probability that is greater than 1.
- Candidates need to recognise when they need to use algebra and calculus to gain marks and when using their GDC is a more effective and errorless way.
- Candidates need to be shown good practice how to provide correct reasoning and proper justification.
- Interval notation was often incorrect and requires good examples how to state domain, and range and determine vertical asymptotes of functions.
- More care needs to be taken when copying a graph from GDC, particularly with the correct shape, approaching the asymptotes and marking all important features as required.
- Teach statistics with relevance of how to distinguish between different distributions and to show a method more clearly.
- Encourage use of cosine rule as an effective way of dealing with the ambiguous case.
- Draw more diagrams to illustrate solutions and refer to them in solutions.
- To teach students how to re-evaluate methods when something goes wrong and how to check if their solution is reasonable.

## Higher level paper three: discrete

### Component grade boundaries

| **Grade:** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range:** | 0 - 7 | 8 - 15 | 16 - 25 | 26 - 30 | 31 - 34 | 35 - 39 | 40 - 50 |

## General comments

A good proportion of candidates seemed to find this to be an accessible paper with ample opportunities to demonstrate their reasoning skills and knowledge of the course algorithms.

## The areas of the programme and examination which appeared difficult for the candidates

Finding the general solution to a Diophantine equation.

Articulating clearly what is meant by a circuit and an Eulerian circuit.

Reasoning whether or not the complement of a graph *G* that has six vertices and an Eulerian circuit can also have an Eulerian circuit.

Verifying that a given second degree (order) recurrence relation is satisfied by $u_n = A\alpha^n + B\beta^n$.

## The areas of the programme and examination in which candidates appeared well prepared

Using the Euclidean algorithm to find the greatest common divisor of two numbers.

Finding a particular solution to a Diophantine equation.

Expressing two numbers as products of their prime factors and then determining the lowest common multiple of those two numbers.

Applying the nearest neighbour algorithm to find an upper bound for the travelling salesman problem.

Applying the deleted vertex algorithm in attempting to find a lower bound for the travelling salesman problem.

Using an auxiliary equation and its complex roots to solve a second degree (order) recurrence relation.

## The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1:

Question 1 was generally well answered. Most candidates were able to determine that 3 is the greatest common divisor of 264 and 1365 and that 120120 is the lowest common multiple of 264 and 1365. However, in attempting to determine the lowest common multiple, some candidates either did not complete the prime factorisation process or ignored the given instruction in the question and instead attempted to use $\gcd(264,1365)\text{lcm}(264,1365) = 264 \times 1365$. Some candidates thought that 91 was a prime factor of 1365.

In part (b) (i), most candidates accurately worked backwards to obtain a particular solution ( $x = 212, y = 41$) to the Diophantine equation $264x - 1365y = 3$. However, a number of candidates either did not attempt to find the general solution ( $x = 212 + 455N,\ y = 41 + 88N$ ) or gave a general solution that contained a sign error(s) due to not noticing the negative sign in the equation.

In attempting to find the general solution to the Diophantine equation in part (b) (ii) ( $264x - 1365y = 6$ ), candidates generally knew to multiply by 2 although some did it incorrectly and stated $x = 424 + 910N,\ y = 82 + 176N$ as their general solution rather than $x = 424 + 455N,\ y = 82 + 88N$ (or equivalent). In part (b) (ii), some candidates lost time by again working backwards rather than referring to the general solution obtained in part (b) (i). Follow through marks were often awarded in part (b) (ii).

Question 2:

In part (a), most candidates were able to apply the nearest neighbour algorithm to find an upper bound for the travelling salesman problem for graph *G*. Errors included not returning to the starting vertex (A), inventing the ‘furthest neighbour algorithm’ or doubling the weight of a minimal spanning tree (which is not mentioned on the syllabus).

In part (b), a good proportion of candidates were able to apply the deleted vertex algorithm in attempting to find a lower bound for the travelling salesman problem for graph *G*. When applying Kruskal’s algorithm, marks were lost either by not indicating the edge order or by simply selecting an incorrect edge towards the end of the algorithm. Most candidates knew to reconnect vertex A with the two edges of least weight, namely, AB and AF. Some candidates lost time by repeating the method with other vertices deleted.

Question 3:

Responses to part (a) were quite varied with many candidates either not including all the required information in their definitions of a circuit and an Eulerian circuit, not expressing their definitions with sufficient clarity and precision or using the word ‘path’ instead of ‘walk’ or ‘trail’.

Part (b), which required candidates to display sound reasoning and argumentation skills, was well answered by a reasonable number of the cohort. Candidates that started their proof by stating that the degree of all the vertices in *G* are even often progressed well. However, many responses were not precise enough with these candidates often confusing vertices and edges. A number of candidates gained partial credit for basing their argument on a specific graph *G* rather than developing a general argument. In such questions, it is important to realise that drawing one particular example of *G* does not constitute a complete and convincing argument.

In part (c), a pleasing number of candidates produced pairs of graphs with five vertices that have an Eulerian trail but not an Eulerian circuit. Errors included producing one correct graph and an incorrect complement or producing two incorrect graphs.

Question 4:

Part (a) was poorly answered by most candidates with only a small number even attempting to substitute for $u_n$, $u_{n+1}$ and $u_{n+2}$ into the LHS of the recurrence relation. Many candidates seemingly did not understand what was required by the command term ‘verify’ and often just re-iterated the form that they knew the solution must have. Of the small number of candidates who correctly substituted into the LHS of the recurrence relation, most unfortunately developed solutions that were set to zero throughout and ended with the conclusion that 0=0.

In contrast, part (b), which required the solving of a second degree (order) recurrence relation was quite well answered by a good number of candidates with many pleasingly obtaining full credit. Most candidates were not put off by the roots being complex. Errors committed were mostly of a computational nature rather than not knowing the solution method. Candidates who were awarded partial credit often found the correct auxiliary equation and its roots and then frequently obtained method marks for subsequent work.

## Recommendations and guidance for the teaching of future candidates

- The OR method in the markscheme is a neat way of tracking the linear combinations when applying the Euclidean algorithm.
- Encourage students to use technology to check the correctness of a particular solution to a Diophantine equation.
- Explain to students the importance of using the correct terminology and definitions when studying the graph theory part of the course.
- Provide students with questions, particularly in graph theory, that help hone their reasoning skills and provide practice in developing mathematical arguments.
- Provide students with opportunities to work on past IB papers and to discuss mark allocations with them.

## Higher level paper three: calculus

### Component grade boundaries

| **Grade:** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range:** | 0 - 5 | 6 - 11 | 12 - 17 | 18 - 23 | 24 - 28 | 29 - 34 | 35 - 50 |

## General comments

This proved to be an accessible paper with the majority of candidates able to score good marks on questions 1 to 4.  Question 5 proved to be more difficult and the reasons for this are discussed below.

## The areas of the programme and examination which appeared difficult for the candidates

The following topics caused most difficulties for the candidates: Riemann sums, manipulation of series and homogeneous differential equations.

## The areas of the programme and examination in which candidates appeared well prepared

The following topics were well done by the majority of candidates: Maclaurin series, L'Hôpital's rule, and the integral test

## The strengths and weaknesses of the candidates in the treatment of individual questions

Q1 Most candidates knew how to apply L'Hôpital's rule, and were able to use product and chain rule successfully to obtain the correct derivatives. In most cases it was correctly deduced that the rule needed to be applied a second time. Sometimes errors were made when trying to simplify the expression obtained which could have been avoided by substitution of 0 earlier.

Q2 a(i) was well done. A few students tried to obtain the expansion directly using Maclaurin's, this error could have been avoided if they had read the question carefully. In part a(ii) some students were let down by their algebra, and chose to square each of the individual terms of the expansion for tan*x* rather than the whole expansion.

a(ii) was an easy application of the results in a(i), and those who scored full marks in the earlier part invariably achieved all marks in the second.

Q3 Several students spent a lot of time trying to prove that $f(x)=\frac{1}{x\sqrt{\ln x}}$ was a decreasing function. In general a result like this could simply be stated unless the question asks for it to be proved. In addition when the question gives the test that is to be used it may be assumed that the necessary conditions for the test are satisfied.

Candidates should be warned that they should not use the same variable in their integral as is being used in the series.

Most candidates were able to correctly integrate the expression. A few took the longer route of using integration by parts, but several of these were successful in their attempts.

In general, on the calculus paper, candidates should use limit notation when evaluating improper integrals.

To conclude the integral test candidates must not forget to comment that because the integral diverges then the series must do as well. Several simply wrote 'hence diverges' without making it clear if they were referring to the integral or the series.

Q4 (a) Most candidates were successful in differentiating $y = vx$. Some candidates failed to provide any evidence of integrating both sides in a “show that” question resulting in loss of final accuracy mark.

(b) This was done well. Many candidates scored some of the marks in this part. Evaluating $\int \frac{1}{1+2v+v^2}\,\mathrm{d}v$ was found to be the most difficult aspect.

Q5 (a) This part should have been very familiar to candidates via their work on Riemann Sums. Many though were unable to make the connection and so were unable to score any marks here.

(b) A correct solution to part (a) was not necessary for part (b) which relied on using the result given in the question. Many candidates did not take the hint of 'hence' and so did not sum the relevant parts of the result. Those that did write down the sum were usually successful in manipulating the log expression to obtain the correct result.

Part (ii) required careful thought. Changing the limits in the sum proved to be difficult for most of the candidates.

(c) Once again the candidates who took careful note of 'hence' were able to link the result in part (b) to the proof required in (c) (i).

Part (c) (ii) required linking back to part (a) which some of the students missed.

(d) Common errors here were to try and use results from tests for the convergence of series, or to say the sequence must converge to zero. It was not necessary for candidates to use formal language in their description of why it must converge

## Recommendations and guidance for the teaching of future candidates

Teachers should advise candidates to read the questions carefully to improve the quality of response to the problem and write legibly as this will greatly help examiners with their marking.

Teachers must emphasize the need to show appropriate reasoning and clear methods/steps leading to the answer. Teachers should continue to emphasize the importance of command terms such as "show that", "Hence", “Explain” in the classroom and make sure candidates understand the meaning and expectation of these terms in the context of problem solving.

It was clear that some candidates were not familiar with the idea of Riemann sums, it is important to cover all parts of the syllabus.

Question 5 involved linking different parts and applying known knowledge in unfamiliar situations, it would be helpful to students to practice these types of questions under test conditions.

# Higher level paper three: sets, relations and groups

## Component grade boundaries

| Grade: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Mark range: | 0 - 7 | 8 - 14 | 15 - 23 | 24 - 29 | 30 - 34 | 35 - 40 | 41 - 50 |

# General comments

Although this paper was very accessible, surprisingly, some candidates showed lack of familiarity with even the most basic ideas contained in this option. A significant proportion of candidates were very careless in the process of manipulation. This was especially evident in question 1. There were careless mistakes in parts of the question that based solely on prior knowledge. The layout of the proof of equivalence relation in Question 2 was sloppy. The algebraic manipulation in Question 3 was also weak. This is disappointing given this is such an accessible paper.

# The areas of the programme and examination which appeared difficult for the candidates

Candidates had some difficulty in applying the learned definitions to specific examples in 'show' and 'show that' questions. Although the definition of an equivalence relation and the properties of groups were well known, at times the definitions and properties were not interpreted correctly within the given examples.

Some candidates showed difficulties in using correct mathematical notation, particularly as pertains to equivalence relations and congruence. Some candidates also did not know how to determine the symmetric difference of two given sets.

Many candidates showed some difficulty in determining equivalence classes of a given equivalence relation.

Some candidates had difficulty in the algebraic manipulation necessary to show that a function in two variables was a bijection and determining its inverse.

Most candidates had difficulty in finding a proper subgroup of a given group, and in most cases were not even aware of the necessary conditions.

## The areas of the programme and examination in which candidates appeared well prepared

Candidates had good awareness of key definitions contained in this option. They generally showed good ability in answering questions on sets and set operations. They were familiar with properties of equivalence relations, definition of homomorphism and properties of groups, and could satisfactorily show that a given relation on a set was an equivalence relation, and a binary operation on a given set satisfied the group properties.

## The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a)(i) Many candidates included the number 1 as a prime number in set B but surprisingly several candidates did not include 2 as a prime number. Some candidates struggled with the definition of congruence.

(ii) Some candidates obviously did not know the definition of the symmetric difference of two sets.

(b)(i) Students generally satisfactorily answered the questions in this part.

(ii) Some students did not verify the distributive property in the given case, as stated in the question, but rather attempted to justify the property in the general case, either using Venn diagrams or the double inclusion method.

Question 2

(a)(i) Candidates were very familiar with the properties of an equivalence relation but in many cases they exhibited sloppiness at times in their notation for divisibility and congruence.

(ii) Some candidates showed difficulty understanding what an equivalence class is, and hence could not answer the question.

(b) Few candidates showed any working out for this question, although some did answer this satisfactorily. Many candidates stated that there were 7 equivalence classes.

Question 3

(a) Many candidates could not apply the definitions of surjective and injective to functions in two variables. Among the candidates that could answer the question partially, the proof of injectivity was better attempted than surjectivity. At times candidates failed to state that both of these conditions are necessary to show that a function is a bijection. A number of candidates failed to score some of the R marks as their answers had no conclusion.

(b) Most candidates understood that they already found the inverse function in showing that the function is surjective, but sometimes they did not use correct notation. A number of candidates showed the misconception that they should interchange the variables *x* and *y* to obtain the inverse.

Question 4

This question was attempted satisfactory by most candidates.

(a) This part of the question was well attempted even for some weak candidates. The majority of them knew the definition of an Abelian group. Most candidates were able to show that the given binary operation on the given set satisfied the group properties. At times their notation was sloppy, and sometimes definitions were quoted without a correct interpretation in the given problem. A few candidates stated that the given binary operation was associative and commutative due to the properties of addition of real numbers. Some wrote down 0 as the identity and $-a$ as the inverse since they inherited those from addition. A couple of candidates only proved commutativity of the binary operation instead of checking the general group properties.

(b) On this part most candidates knew the meaning of the order of an element and could make a start. Quite a lot of them jumped to the conclusion without completing their argument. The common reasons were no elements could be self-inverse and the only element with the given condition was the identity, hence contradiction. Some candidates failed to mention that the identity has order 1.

(c) Very few candidates answered this question successfully, and quite a few omitted it entirely. This was the worst attempted within the whole paper. Only a few managed to write down a proper subgroup of the given group. Among those who got the right answer, only a couple went on to justify their answer.

(d) Most candidates answered this question successfully. Even the weak candidates knew the definition of isomorphism although they struggled with the algebraic manipulation thereafter. A significant number of candidates wasted their time attempting to prove the function was a bijection without noticing that was given in the question.

## Recommendations and guidance for the teaching of future candidates

Make students aware of basic facts like '1' is not a prime and '0' is not always the identity when addition is the operation.

Candidates should be exposed to different kinds of problems in which they need to interpret the properties of groups and equivalence relations, including how to find equivalence classes.

The use of correct communication and notation should be stressed, as well as all steps necessary in justifying their conclusions.

Candidates should be made aware of the need of being more rigorous in setting out proofs. Be harsh in scoring the details of equivalence relations proofs so that candidates learn the importance of precision.

Expose candidates to more examples of functions with more than one variable and how to prove injection and surjection in these cases.

Expose candidates to more examples of modular arithmetic.

## Higher level paper three: statistics and probability

### Component grade boundaries

| **Grade:** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
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| **Mark range:** | 0 - 7 | 8 - 14 | 15 - 22 | 23 - 28 | 29 - 33 | 34 - 39 | 40 - 50 |

## The areas of the programme and examination which appeared difficult for the candidates

It was evident in Q2 that many candidates seem to confuse the probability density function $f(x)$ and the cumulative distribution function $F(x)$.

Many candidates do not understand the Central Limit Theorem. A common fallacy is that as the sample size increases, the sampled distribution tends to normality. This is of course an impossibility.

## The areas of the programme and examination in which candidates appeared well prepared

Most, but not all, candidates are able to use their graphical calculator to carry out hypothesis tests. Conclusions, however, are often not given in context when required.

## The strengths and weaknesses of the candidates in the treatment of individual questions

Q1 – Many candidates chose to calculate the mean and variance estimates using the formulae and then used the calculator software to carry out the $t$-test, apparently not realising that the software gave these estimates as part of the output. Division by $n$ instead of $n-1$ was often seen so that 0.0066 instead of 0 0072 was often seen. Some candidates calculated the $t$-value using the formula and then found the $p$-value by using the cumulative probability function on the calculator. This is of course a valid method but it is more time consuming than intended. Many candidates failed to give the conclusion in context, as required by the question.

Q2 – It was disappointing to note that many candidates incorrectly integrated the cumulative distribution function to solve (a)(i) and (a)(ii). Some candidates attempted to calculate the median incorrectly by evaluating $F(0.95)$. Some candidates attempted to find the mean and variance of *X* by using integration by parts and some completed this successfully. This often required several pages of algebra. Candidates, however, were expected to evaluate the integrals using the integration facility on their calculator and most did that. Part (c) was poorly answered with many candidates stating that the distribution itself is approximately normal for large samples instead of the sample mean. It is extremely disappointing that what is arguably the most important theorem in Statistics is not understood by the vast majority of candidates.

Q3 – Parts (a) and (b) were well answered in general although in (a) some candidates failed to provide a convincing argument for summing an infinite geometric series. It is important in a 'show that' question not just to write down the answer without justification. The differentiation in (b) was disappointing in some cases with candidates using the method for differentiating quotients, thinking that *p* was a variable with derivative 1. Many candidates were unable to solve (c) successfully. A common error was to write

$$\text{PGF} = E\left(t^{2X+1}\right) = E\left((t^X)^2\right)E(t) \text{ instead of } E\left((t^2)^X\right)E(t)$$

which then enables the result from (a) to be used. Candidates who attempted to write down the series for the probability generating function of *Y* were generally more successful.

Q4 – Parts (a) and (b) were reasonably well answered although a certain amount of carelessness in algebraic manipulation was seen. In (c), some candidates used the calculator to minimise the variance but this method was unable to give the answers as fractions so that marks were lost.

Q5 – This was a slightly unusual question but it was well answered by many candidates. In (a), some candidates used *r* instead of *ρ* which was of course penalised. In (b), as in Q1, some candidates failed to give the conclusion in context as required. In (c), some candidates found the inverse *t*-value of 0.177 instead of 0.823 which gave a negative value of *t* which resulted in a negative value of *r* which was however followed through.

## Recommendations and guidance for the teaching of future candidates

Many candidates seem to be unaware of the instruction on the front of the examination paper which states that 'Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures'. Many candidates lose marks by failing to obey this instruction.

Although candidates are generally competent in using their graphical calculators, not all candidates use them efficiently. Candidates should be aware that the output from carrying out a hypothesis test contains not only the value of the test statistic and the *p*-value but also the means and variances and degrees of freedom.

It would be useful if more time could be devoted to improving candidates' understanding of the Central Limit Theorem.