May 2016 subject reports

Mathematics HL

Overall grade boundaries

Discrete

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 25	26 - 39	40 - 51	52 - 64	65 - 76	77 - 100

Calculus

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 25	26 - 37	38 - 49	50 - 61	62 - 73	74 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 26	27 - 38	39 - 51	52 - 63	64 - 74	75 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 26	27 - 39	40 - 51	52 - 63	64 - 74	75 - 100

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2016 session the IB has produced time zone variants of Mathematics HL Paper 1 and Paper 2.

Higher level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 16	17 - 20

The range and suitability of the work submitted

The majority of explorations were generally commensurate with the Mathematics HL content but the quality was varied with some explorations in the top range; these consisted of work that had a very interesting creative approach on the use of Mathematics HL topics. Unfortunately a number of candidates submitted explorations that were a direct extraction from textbooks or online sources with some topics having a high level of complexity. In these cases it was evident that the candidate had not understood the mathematics used. In fact some explorations were so far removed from a teacher's/moderator's expected knowledge base that they were largely incomprehensible and very challenging to moderate. Students need to be reminded that the intended audience consists of peer students. Some explorations still lacked in-text citations; this requirement needs to be made clearly known to all teachers for transmission to students. Some teachers are still allowing students to submit explorations that are far too long. Although there is no strict penalty for explorations that exceed 12 pages, students need to be advised about choosing a focused topic that allows for an exploration to be written within the recommended page length. A number of well-worn topics continue to be submitted. These include the "SIR model", "Texas Hold'em Poker", "Fractals" and "The Golden Ratio". Although fewer in number, explorations were submitted that were extracted from mathematical videos. Although such videos act as a good stimulus at the beginning of the exploration process, students should not merely transcribe the video content and submit it as their own work exploration.

Candidate performance against each criterion

Criterion A

In general this criterion was addressed well by most students, with work being coherent and organized to different extents. As mentioned above there is evidence to suggest that some students are not being well advised by teachers and submit work that is far too long, often in excess of 20 pages. A number of students included appendices to keep the length of the exploration within the 6 to 12 pages, however this rendered the exploration incoherent since the reader needed to refer to the appendix in order to understand the actual work. Some students continued to produce a table of contents and a word count. There is no need for either of these in the Exploration. Some problems with coherence were caused by students attempting to explain things that were beyond their own comprehension.

Criterion B

Most students performed well against this criterion. A number of teachers condoned the misuse of calculator notation within student work resulting in an adjustment of the achievement level awarded by the teacher. In a few other cases the teacher allowed for the student to omit the definition of variables and parameters used in a modeling exploration.

Criterion C

There is still a perception by teachers that this criterion is based on the student's commitment and enthusiasm for the topic. It is very important that teachers and students alike understand the scope of this criterion. Extracting work from a textbook, a website or a video clip does not allow the voice of the student to be heard in the exploration. Students are meant to take ownership of the work by solving some curiosity resulting from the stimulus used. Some explorations bore a clear stamp of originality with the student's enthusiasm coming through in the work submitted.

Criterion D

In general students handled this more effectively in this session. This was seen when the student's reflection was ongoing showing cognitive reflection skills on their work. In most cases students seemed to understand what constitutes meaningful reflection but it continues to be challenging for most students to demonstrate critical reflection. Those students who achieved high levels against this criterion also scored highly against criterion C because as they made an effort to overcome their perceived shortcomings they managed to demonstrate personal engagement with their work.

For a few teachers and students this criterion still caused problems. When students include reflection only within the conclusion and just comment on the scope and limitations of the results obtained it often hinders the student from achieving higher levels. Teachers are advised to refer to the document "Additional notes and guidance on the Exploration" which can be found on the OCC.

Criterion E

Once more the explorations submitted in this session included mathematics that was varied, ranging from very basic mathematics to extensions of the HL course that was well beyond the scope of the Exploration. Achieving a 6 still proved to be elusive on either count. Students who opted to explore more complex concepts were unable to demonstrate their understanding of the mathematics used and often transcribed information collected from researched sources; very often this was cobbled together with missing explanations showing that the student did not fully understand the concept and hence was unable to produce a written paper at a level accessible to a typical HL student. Some students who opted for modeling explorations failed to go beyond the mechanical work of either solving a differential or collecting data and technology based regression analysis.

Recommendations for the teaching of future candidates

There was evidence to suggest that some teachers do not dedicate enough teaching time to the Exploration process. It is imperative that 10 hours of teaching time are used to guide the students and help them understand the requisites for this Internal Assessment and the Achievement Criteria. One way of achieving this would be to have students read and mark a couple of explorations that can be found on the Teacher Support Material. On the reverse side of the 5/EXCS form there is space to enter background information. The teacher and not the student should fill out this section. It should also include mathematical background knowledge of the class at the time the exploration was assigned and not information about the individual and their commitment to the topic. It is also mandatory that teachers show evidence of marking explorations with tick marks indicating where the mathematics used is correct and identifying errors. Annotations and comments should be written directly on the student's written response. The teacher assesses the work and the role of the moderator is to confirm the achievement levels awarded by the teacher and not to mark the work. Cryptic comments on student work, like "C+" or "D+" do not help the moderator when trying to verify the achievement level awarded. Teachers should avoid sending photocopies of student work. The original annotated work of the student (printed in colour when appropriate) should be sent for moderation.

Higher level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 25	26 - 38	39 - 53	54 - 67	68 - 82	83 - 120

The areas of the programme and examination which appeared difficult for the candidates

- Probability
- Calculus and simple applications of the chain rule
- Binomial theorem
- Graph sketching

The areas of the programme and examination in which candidates appeared well prepared

- Mathematical induction
- Complex number manipulation (in relation to products and sums of roots)
- More advanced trigonometric equations

The strengths and weaknesses of the candidates in the treatment of individual questions

Section A

Question 1

This provided a generally easy start for many candidates. Most successful candidates obtained their answer through row reduction of a suitable matrix. Those choosing an alternative method often made slips in their algebra.

Question 2

Another standard question. On this occasion, specific coordinates were asked for, so some otherwise good candidates missed out on a couple of marks which they would have gained through greater care.

Question 3

This was generally well done. Some weaker candidates tried to solve part (b) through use of a substitution, though the standard result $\arctan x$ was well known. A small number used $\arctan x + c$ and went on to obtain an incorrect final answer.

Question 4

This was generally answered very well. A small minority attempted to 'prove' the result by substituting specific values into the identity and thus gained little or no credit. Some started by assuming the result to be correct, then manipulated both sides until they derived an obvious identity. Reluctantly, they gained credit for this, though such an approach should be discouraged.

Question 5

This question was generally very well done and posed few problems except for the weakest candidates.

Question 6

This was another question that was very well answered by most candidates. Some struggled in part (b) by attempting to find an expression for d , a common difference, then substituting this in to further equations, where algebra tended to falter. The most fruitful technique was to apply $u_3 - u_2 = u_4 - u_3$. Good presentation often helped candidates reach the final result. Correct factorisation was more often seen than not in the final section, though a small number thought it judicious to guess the correct answer(s) here.

Question 7

Part (a) posed few problems. Part (b) was possibly a good discriminator for the 4/5 candidates. Some were aware of an alternative (useful) form for the conditional probability, but were unable to interpret $P\left(A\cap\left(A\cup B\right)\right)$. Large numbers of fully correct answers were seen.

Question 8

This proved to be a good discriminator. The average candidate seemed able to work towards $P\left(k+1\right)=k^3+3k^2+8k+6$, and a number made some further progress.

Unfortunately, even otherwise good candidates are still writing down incorrect or incomplete induction statements, such as ‘Let $n=k$’ rather than ‘Suppose true for $n=k$’ (or equivalent).

It was also noted than an increasing number of candidates this session assumed ‘$P\left(n\right)$ to be true’ before going to consider $P\left(n+1\right)$.Showing a lack of understanding of the induction argument, these approaches scored very few marks.

Question 9

This question proved to be the most problematic question in the paper.

Part (a) was generally well done, with competent fraction and surd manipulation seen successfully in leading to the given answer.

The number of scripts seen where part (b) was tackled with complete success numbered in the single figures; solutions were rarely if ever seen. Some candidates scored one mark by finding, or using, the common denominator $\sin x\cos x$.

Section B

Question 10

Parts (a) and (b) were often well done, though a small number of candidates were clearly puzzled when trying to demonstrate $\begin{pmatrix}1\\1\\3\end{pmatrix}\neq k\begin{pmatrix}p\\2\\1\end{pmatrix}$, with some scripts seen involving needlessly convoluted arguments.

Part (c) often proved problematic, as some candidates unsurprisingly used the sine (or cosine) of an incorrect angle, and few consequent marks were then available. Some good clear solutions were seen, occasionally complete with diagrams in the cases of the thoughtful candidates who were able to 'work through' the question rather than just apply a standard vector result.

Question 11

Part (a) was often answered well, though for some reason a minority tended to use the incorrect $\pi\int(3\cos 2y)^2 dy$ and gained few marks thereafter. Incorrect limits were sometimes seen, which led to only method marks being available. A pleasing number were able to deal with the integration of $\cos^2 2y$ through the use of the correct identity.

Part (b) was well answered and did not pose too many problems.

Correct answers to part (c) were rarely seen. Only the very best candidates appreciated the correct use of the chain rule when trying to determine an expression for $\frac{d^2h}{dt^2}$.

Question 12

The majority of candidates scored full marks in part (a), as well as part (b)(i). It was expected to see $w-1\neq 0$ stated for the (b)(ii) mark, though some did appreciate this.

In part (c), the roots were required to be stated in terms of w. This was sometimes ignored, thankfully not too frequently. Clear argand diagrams were not often seen, and candidates' general presentation in this area could be improved. Having said this, most scripts were awarded at least 2 of 3 marks available.

Part (d) proved to be a good discriminator for the better candidates. The product and sum of roots formulae now seem to be better appreciated, and while only the best scored full marks, a good number were able to demonstrate the result $b=1$.

In part (e), of those candidates who reached this far in the paper, most were able to pick up two or three marks, albeit from sometimes following through incorrect work. A correct reason for choosing $i\sqrt{7}$ over $-i\sqrt{7}$ was necessary, but rarely, if ever seen.

Recommendations and guidance for the teaching of future candidates

- General presentation, particularly with regard to induction questions and argand diagrams.
- Emphasis on the understanding of mathematical induction.
- Taking care with limits in the application of definite integrals.

Higher level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 34	35 - 49	50 - 63	64 - 77	78 - 91	92 - 120

General comments

A good number of candidates found this paper reasonably accessible with a pleasing number submitting excellent scripts characterised by correct work, logical reasoning and argumentation. Also pleasing to note was the number of candidates, who when required, used their GDC judiciously to solve an appropriate equation. However, a recurring concern in HL Mathematics Paper 2 is the number of candidates who have their GDC set to degrees mode. This was obvious in the question testing knowledge of the properties of a continuous probability density function (trigonometric function). Another concern was the use of poor vector notation or indeed the absence of vector notation in a geometrical application of vectors context.

The areas of the programme and examination which appeared difficult for the candidates

- Calculating a normal probability in the form $\mathrm{P}\left(\left|X\right| > x\right)$.
- An inability to correctly exploit the symmetry and area under the curve of a normal probability density function.
- Determining the domain and range of an inverse function.
- Finding the expected value (profit in this instance) of a discrete probability distribution.
- In kinematics, recognising that $\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}s}{\mathrm{d}t} \times \frac{\mathrm{d}v}{\mathrm{d}s} = v\frac{\mathrm{d}v}{\mathrm{d}s}$.
- Using vectors with correct vector notation in a geometric context.
- Using a GDC to solve an inequality.
- Formulating and solving a quadratic equation in e^x.
- Logically setting out solutions to ‘show that’ questions.

The areas of the programme and examination in which candidates appeared well prepared

- Applications of trigonometric relationships including the use of the cosine rule.
- Solution of simultaneous equations using various approaches.
- Formulating equations linked to a geometric sequence.
- Performing routine normal and Poisson distribution calculations.
- Finding an inverse function.
- Performing implicit differentiation.

- Using a GDC to calculate the mean, mode and variance of a continuous probability density function.
- Quotient and product rule differentiation.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Well done by most candidates. A small number of candidates did not express the required angle correct to the nearest degree.

Question 2

Part (a) was generally well done. Part (b) was not well done with many candidates not knowing what $\mathrm{P}\left(\left|X\right|>1\right)$ represents. In part (c), a number of candidates did not recognise that $\mathrm{P}\left(X>c\right)=1-\mathrm{P}\left(X\le c\right)$. In each question part, a number of candidates could have benefited from producing a labelled sketch of the situation.

Question 3

Reasonably well done. Candidates who did not obtain the correct solution generally made an error when attempting to apply logarithmic or exponential laws and hence made erroneous substitutions.

Question 4

Reasonably well done. Quite a number of candidates included a solution outside $-1<r<1$.

Question 5

Most candidates were able to find an expression for the inverse function. A large number of candidates however were unable to determine the domain and range of the inverse.

Question 6

Part (a) was reasonably well done. Some candidates calculated $\mathrm{P}\left(X=1\right)$.

Part (b) was not as well done as expected with a surprising number of candidates calculating $5\mathrm{P}\left(X=0\right)+3P\left(X\ge 1\right)$ rather than $5\mathrm{P}\left(X=0\right)-3P\left(X\ge 1\right)$.

Part (c) was very well done.

Question 7

Part (a) was generally well done. Some use of partial differentiation accompanied by rudimentary partial derivative notation was observed in a few candidate’s solutions.

In part (b), a large number of candidates knew to use $\frac{dy}{dx}=0$ and seemingly understood the required solution plan but were unable to correctly substitute $x=k$ and $y=\frac{3k^2}{4}$ into the relation and solve for k.

Question 8

In part (a), a large number of candidates thought that $\frac{dv}{dt}=\frac{dv}{ds}$ rather than $\frac{dv}{dt}=\frac{ds}{dt}\times\frac{dv}{ds}=v\frac{dv}{ds}$. In part (b), quite a few of these candidates then went on to find a value of s that was outside the domain $0\le s\le 1$.

Question 9

In part (a), a significant number of candidates either did not use correct vector notation or simply did not use vector notation at all. A large number of candidates who appeared to adopt a scalar product approach, did not use the scalar product 'dot' and represented $\boldsymbol{a}\cdot\boldsymbol{b}$ as $\boldsymbol{ab}$. A few candidates successfully used the cosine rule with correct vector notation. A small number of candidates expressed $\boldsymbol{a}$ and $\boldsymbol{b}$ in general component form. In part (a) (ii), quite a number of candidates expressed $\overrightarrow{AB}$ as $\boldsymbol{a}-\boldsymbol{b}$ rather than as $\boldsymbol{b}-\boldsymbol{a}$.

In part (b), some very well structured proofs were offered by a small number of candidates.

Question 10

This question was generally accessible to the large majority of candidates. A substantial number of candidates were able to neatly and accurately sketch a non-symmetric bimodal continuous probability density function and to calculate its mean, mode and variance. Quite a few candidates unfortunately attempted this question with their GDC set in degrees.

Question 11

This question was generally accessible to a large majority of candidates. It was pleasing to see a number of different (and quite clever) trigonometric methods successfully employed to answer part (a) and part (b).

The early parts of part (c) were generally well done. In part (c) (i), a few candidates correctly found $\frac{d}{dx}(\tan\alpha)$ in unsimplified form but then committed an algebraic error when endeavouring to simplify further. A few candidates merely stated that $\frac{d}{dx}(\tan\alpha)=\sec^2\alpha$.

Part (c) (ii) was reasonably well done with a large number of candidates understanding what was required to find the correct value of α in degrees. In part (c)(iii), a reasonable number of candidates were able to successfully find $\frac{\mathrm{d}^2}{\mathrm{d}x^2}(\tan\alpha)$ in unsimplified form. Some however attempted to solve $\frac{\mathrm{d}^2}{\mathrm{d}x^2}(\tan\alpha)=0$ for x rather than examine the value of $\frac{\mathrm{d}^2}{\mathrm{d}x^2}(\tan\alpha)$ at $x=\sqrt{35}$.

Part (d), which required use of a GDC to determine an inequality, was surprisingly often omitted by candidates. Of the candidates who attempted this part, a number stated that $x \geq 2.55$. Quite a sizeable proportion of candidates who obtained the correct inequality did not express their answer correct to 3 significant figures.

Question 12

Parts (a) and (c) were accessible to the large majority of candidates. Candidates found part (b) considerably more challenging.

Part (a)(i) was reasonably well done with most candidates able to show that $\frac{1}{4f(x)-2g(x)}=\frac{\mathrm{e}^x}{\mathrm{e}^{2x}+3}$. In part (a)(ii), a number of candidates correctly used the required substitution to obtain $\int \frac{\mathrm{e}^x}{\mathrm{e}^{2x}+3}\mathrm{d}x=\int \frac{1}{u^2+3}\mathrm{d}u$ but then thought that the antiderivative involved natural log rather than arctan.

In part (b)(i), a reasonable number of candidates were able to form a quadratic in e^x (involving parameters n and k) and then make some progress towards solving for e^x in terms of n and k. Having got that far, a small number of candidates recognised to then take the natural logarithm of both sides and hence solve $h(x)=k$ for x. In part (b)(ii), a small number of candidates were able to show from their solutions to part (b)(i) or through the use of the discriminant that the equation $h(x)=k$ has two real solutions provided that $k>\sqrt{k^2-n^2+1}$ and $k>\sqrt{n^2-1}$.

It was pleasing to see the number of candidates who attempted part (c). In part (c)(i), a large number of candidates were able to correctly apply either the quotient rule or the product rule to find $t'(x)$. A smaller number of candidates were then able to show equivalence between the form of $t'(x)$ they had obtained and the form of $t'(x)$ required in the question. A pleasing number of candidates were able to exploit the property that $f'(x)=g(x)$ and $g'(x)=f(x)$. As with part (c)(i), part (c)(ii) could be successfully tackled in a number of ways. The best candidates offered concise logical reasoning to show that $t'(x)>0$ for $x\in\mathbb{R}$

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Recommendations and guidance for the teaching of future candidates

- Discuss with students what is required in a show that or proof question.
- Discuss with students appropriate GDC mode settings and make sure they understand how to use their GDC to make degrees/radians conversions.
- Discuss with students what effective GDC use looks like in Paper 2. Also discuss when use of a GDC will either be more efficient or appropriate than analytical approaches.
- Discuss with students the need to express final answers correct to three significant figures unless otherwise stated in the question.
- Discuss with students the importance of not rounding too early in a GDC active examination.
- Encourage students to sketch labelled diagrams when attempting questions involving the normal probability distribution.
- Encourage students to use correct vector notation.
- Make the link between the domain and range of a function and the range and domain of its inverse.
- Give students plenty of opportunity to practice calculus and probability knowledge in context.
- Provide questions that could be solved in a variety of ways and ensure that students are given the opportunity to discuss these various approaches.
- Kinematics ideas require further consolidation, particularly the common misconception that $\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}s}$.

Higher level paper three: discrete

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 16	17 - 27	28 - 34	35 - 42	43 - 49	50 - 60

General comments

Candidates should know before the exam what to expect for the format of the paper. The instructions on page one and at the top of page two e.g. unless otherwise stated in the question all numerical answers should be given exactly or correct to three significant figures, start each question on a new page, answers must be supported by working and/or explanations, are often being ignored.

The areas of the programme and examination which appeared difficult for the candidates

The candidates found it difficult with work on graphs and trees when they had to think rather than just apply an algorithm. Induction proved to be a good discriminator.

The areas of the programme and examination in which candidates appeared well prepared

Candidates were very good at applying Euclid's algorithm and quite good at working backwards with it. Graph algorithms were known but sometimes confused.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a) Very well answered. Some candidates lost the final mark by not saying that their working showed that the greatest common divisor (gcd) was 1.

(b) The working backwards method was generally well known but there were arithmetic mistakes. Some candidates did not realise that their aim was to keep 1 as a combination of *two* remainders. The final answer could have been checked with the calculator as could intermediary steps. What was sadly less well known was the linear combinations format of laying the work out. See the OR method in the mark scheme. This makes the numerical work much less tedious and deserves to be better known.

Question 2

(a) Generally good use of the nearest neighbour algorithm. Some candidates showed no knowledge of it and there was some confusion with the twice the weight of a minimal spanning tree method. Some candidates forgot to go back to A and thus did not have a Hamiltonian cycle.

(b) The method was generally known. Some candidates used nearest neighbour instead of Kruskal's algorithm to find a minimal spanning tree. Some forgot to add in the two edges connected to A. Some with the right method made mistakes in not noticing the correct edge to choose.

Question 3

(a) Well answered.

(b) The fact that this gave an identity was managed by most. Then some showed their misunderstanding by saying any real number. Few noticed that the digit 7 means that the base must be greater than 7.

(c) The cubic equation was generally reached but many candidates then forgot what type of number n had to be. To justify that there are no positive integer roots you need to write down what the roots are. There were a couple of really neat solutions that obtained a contradiction by working modulo n.

Question 4

(a) This was either done well and completely correct or very little achieved at all (working out v_0 for some reason). As expected a few candidates forgot what to do for a repeated root. The varied response to this question was surprising since it is just standard book work.

(b) Strong induction proved to be a very good discriminator. Some candidates knew exactly what to do and did it well, others had no idea. Common mistakes were not checking $n = 1$ and 2, trying ordinary induction and worse of all assuming the very thing that they were trying to prove.

(c) Most candidates that had the 2 expressions, knew how to get rid of the minus sign in the 2 cases. Some candidates could not attempt this as they had not completed part (a) although when it was wrong, follow through marks could be obtained.

Question 5

Generally in this question, good candidates thought their way through it whereas weak candidates just wrote down anything they could off the formula booklet or drew pictures of particular graphs. It was important to keep good notation and not let the same symbol stand for different things.

(a) If they considered the complete graph they were fine.

(b) Some confusion here if they were not clear about which graph they were applying Euler's formula to. If they were methodical with good notation they obtained the answer.

(c) Again the same confusion about applying the inequality to both graphs. Most candidates realised which inequality was applicable. Many candidates had the good exam technique to pick up the last two marks even if they did not obtain the quadratic inequality.

Recommendations and guidance for the teaching of future candidates

There were some candidates who did not put in any explanation or comments or reasoning and this lost them marks. It is always important that candidates read and re-read the question as carefully as possible. For example there were candidates in question 2 that did the nearest neighbour algorithm starting at each of the vertices in turn and used the deleted vertex method again deleting all the different vertices in turn. This lost them time. There are specific words used in the questions with specific meanings. The examiner is trying to guide them through the question. They should always try to see the point of the question and how one part can assist in a latter part. It is good to consider which part of the syllabus each question is testing. It is vital that all candidates do a trial exam that is marked correctively by their teacher and given back for them to study. Then they understand more exactly what they are expected to do. It is always beneficial to work on past IB papers and see how the marks are given to show them the standard that is required. As this is a calculator allowed paper, candidates should be taught to use their calculator efficiently and to save time e.g. solving polynomial equations on PolySmlt on the TI. Candidates should be taught to realise that you cannot prove anything in Maths by starting with it and thus non-proofs that end in $0=0$ are always going to be treated with disfavour by examiners. If a candidate themselves introduces a symbol that is not given in the question then they need to explain what it stands for. Question 5 is an example of this, where good labelling of variables would have both helped the candidate themselves and the examiner. It is good to teach students to check at the end of each part of a question, is the answer that they have given the type of entity that is required e.g. is it an integer, or a real to 3 significant figures, or a tree, or an expression etc.

Higher level paper three: calculus

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 15	16 - 21	22 - 27	28 - 34	35 - 40	41 - 60

General comments

Most candidates attempted all the questions although, in many cases, the answers revealed some unfamiliarity with the content.

The areas of the programme and examination which appeared difficult for the candidates

All questions which required justification seemed to appear difficult to candidates. Lines of reasoning were often incomplete. Lagrange error formula, Fundamental theorem of calculus corollaries, transformation of differential equations using substitution and use of partial sums to establish upper bound of alternating convergent series caused difficulties. Many candidates were also unfamiliar with the Mean value theorem or had a vague idea about it and the ones that could quote it showed difficulties in using it to establish a given inequality.

The areas of the programme and examination in which candidates appeared well prepared

Derivation of Maclaurin series from first principles; determining an integration factor to obtain an exact differential equation; simple integration.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a) This part of the question was well answered by most candidates. In a few cases candidates failed to follow instructions and attempted to use known series; in a few cases mistakes in the determination of the derivatives prevented other candidates from achieving full marks; part (b) was also well answered using both the Maclaurin expansion or L'Hôpital rule; again in most cases that candidates failed to achieve full marks were due to mistakes in the determination of derivatives. Part (c) was poorly answered with few candidates showing familiarity with this part of the option. Most candidates quoted the formula and managed to find the 4$^{\text{th}}$ derivative of f but then could not use it to obtain the required answer; in other cases candidates did obtain an answer but showed little understanding of its meaning when answering (c)(ii).

Question 2

Many candidates answered this question well. Many others showed no knowledge of this part of the option; candidates that recognized the Fundamental Theorem of Calculus answered this question well. In general the scores were either very low or full marks.

Question 3

Although many candidates achieved at least a few marks in this question, the answers revealed difficulties in setting up a proof. The Mean value theorem was poorly quoted and steps were often skipped. The conditions under which the Mean value theorem is valid were largely ignored, as were the reasoned steps towards the answer.

There were inequalities everywhere, without a great deal of meaning or showing progress. A number of candidates attempted to work backwards and presented the work in a way that made it difficult to follow their reasoning; in part (b) many candidates ignored the instruction 'hence' and just used GDC to find the required values; candidates that did notice the link to part a) answered this question well in general. A number of candidates guessed the answer and did not present an analytical derivation as required.

Question 4

(a) Several misconceptions were identified that showed poor understanding of the chain rule. Although many candidates were successful in establishing the result the presentation of their work was far from what is expected in a show that question. Part (b) was well attempted using both method 1 (integration factor) and 2 (separation of variables). The most common error was omission of the constant of integration or errors in finding its value. Candidates that used method 2 often had difficulties in integrating $\frac{1}{(1-z)}$ correctly and making z the subject often losing out on accuracy marks.

Question 5

(a) Very few candidates presented a valid reason to justify the alternating nature of the series. In most cases candidates just reformulated the wording of the question by saying that it changed signs and completely ignored the interval over which the expression had to be integrated to obtain each term.

(b)(i) Most candidates achieved 1 or 2 marks for attempting the given substitution; in most cases candidates failed to find the correct limits of integration for the new variable and then relate the expressions of the consecutive terms of the series. In part (ii) very few correct attempts were seen; in some cases candidates did recognize the conditions for the alternating series to be convergent but very few got close to establish that the limit of the general term was zero.

(c) A few good attempts to use partial sums were seen although once again candidates showed difficulties in identifying what was needed to show the given answer. In most cases candidates just verified with GDC that in fact for high values of n the series was indeed less than the upper bound given but could not provide a valid argument that justified the given statement.

Recommendations and guidance for the teaching of future candidates

- Teach all aspects of the option listed on the syllabus;
- Provide opportunities for candidates to test their understanding of the results of the course and do many examples of proofs; when a result is to be proved students should be encouraged to make sure that they are really justifying each step of their argument;
- Provide a wide variety of examples of differential equations that can be transformed into the standard examples listed in the syllabus using given substitutions; make candidates aware that some differential equations can be solved in more than one way;
- Provide many examples of applications of the theorems about continuous and differentiable functions, namely the Mean value theorem and its corollaries. Go over application conditions of the Mean value theorem in detail.
- Provide examples of use if Lagrange error formula and discuss its meaning.
- Teachers should continue to emphasize the importance of command terms such as "show that", "Hence", "Deduce" in the classroom and make sure candidates understand the meaning and expectation of these terms in the context of problem solving.
- Some candidates are clearly not suitable for the mathematics HL course and it would be helpful if schools ensure appropriate placement of candidates at the start of the diploma program. Knowledge of basic differentiation and integration techniques are essential when covering the calculus option. Teachers should consider carefully the option topic chosen when teaching very weak students.

Higher level paper three: sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 18	19 - 26	27 - 32	33 - 39	40 - 45	46 - 60

General comments

The mathematics in the Sets, Relations and Groups option differs from that in the other three options in that it deals with very abstract concepts rather than being based on the application of mechanical rules. Proof and strict logical reasoning play a very significant role in this option.

Most candidates attempted all of the questions, although in some cases their responses to the last three questions had little relation to what was required.

The areas of the programme and examination which appeared difficult for the candidates

- Many candidates mistook the use of mathematical synonyms as constituting a proof. For example, simply saying that a function is one-to-one does not amount to a proof that the function is injective.
- Finding equivalence classes.

The areas of the programme and examination in which candidates appeared well prepared

- The definition of a group and associated concepts: Cayley tables; inverses; the order of an element; subgroups.
- The definition of an equivalence relation.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a) The majority of candidates were able to complete the Cayley table correctly.

(b) Generally well done. However, it is not good enough for a candidate to say something along the lines of 'the operation is closed or that inverses exist by looking at the Cayley table'. A few candidates thought they only had to prove commutativity.

(c) Often well done. A few candidates stated extra, and therefore incorrect subgroups.

(e) The majority found only one solution, usually the obvious $x = 2$, but sometimes only the less obvious $x = 7$.

Question 2

(a) Most candidates were familiar with the terminology of the required conditions to be satisfied for a relation to be an equivalence relation. The execution of the proofs was variable. It was grating to see such statements as 'R is symmetric because $aRb = bRa$ or $aRa = a^n - a^n = 0$, often without mention of mod p, and such responses were not fully rewarded.

(b) This was not well answered. Few candidates displayed a strategy to find the equivalence classes.

Question 3

A surprising number of candidates wasted time and unrewarded effort showing that the mapping *f*, stated to be a bijection in the question, actually was a bijection. Many candidates failed to get full marks by not properly using the fact that the group was stated to be Abelian. There were also candidates who drew the graph of $y = \frac{1}{x}$ or otherwise assumed that the inverse of x was its reciprocal - this is unacceptable in the context of an abstract group question.

Question 4

(a)(b)(i) Those candidates who formulated their responses in terms of the basic mathematical definitions of injectivity and surjectivity were usually successful. Otherwise, verbal attempts such as 'f is one-to-one $\Rightarrow$ f is injective' or 'g is surjective because its range equals its codomain', received no credit.

(b)(ii) It was surprising to see that some candidates were unable to relate what they had done in part (b)(i) to this part.

Question 5

This is an abstract question, clearly defined on a subset. Far too many candidates almost immediately deduced, erroneously, that the full group was Abelian. Almost no marks were then available.

Recommendations and guidance for the teaching of future candidates

The notion of 'proof' and well-based logical arguments is very important in mathematics, but particularly for students taking the Sets, Relations and Groups option. The earlier students are exposed to these ways of thinking the better.

Although this option deals with very abstract mathematics, that is best supported by means of a wide range of concrete examples: discrete and continuous number sets; modular arithmetic; permutations; transformations of sets, including symmetries of plane figures.

Encourage students to work mathematically rather than in terms of verbal explanations. Too often such work appears to be tautological or meaningless.

Higher level paper three: statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 19	20 - 29	30 - 34	35 - 40	41 - 45	46 - 60

General comments

The areas of the programme and examination which appeared difficult for the candidates

Many candidates had no more than a superficial understanding of estimation theory.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates are confident in using their graphical display calculator.

Most candidates are able to solve problems involving linear combinations of normal random variables.

There has been an improvement over the last few years in the understanding and use of probability generating functions.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Part (a) was very well answered with only a very few weak candidates using 0.8 instead of 0.841...

Part (b) was well answered with only a few candidates calculating the variance incorrectly.

Part (c) was again well answered. The most common errors, not often seen, were writing the variance of $Y - 2X$ as either $\mathrm{Var}(Y) + 2\mathrm{Var}(X)$ or $\mathrm{Var}(Y) - 2(or\ 4)\mathrm{Var}(X)$.

Question 2

Part (a) was well answered with only a few candidates using inappropriate symbols, for example r or μ. Also, only very few candidates failed to realise that the wording of the question indicated that a two-tailed test was required.

The test in (b) was generally well carried out and the p-value found correctly. The most common errors were using incorrect degrees of freedom and evaluating a one-tailed p-value instead of a two-tailed p-value.

In (c), many realised that the earlier work meant that the regression line should not be used because the variables had been found to be independent. Incorrect reasons, however, were not uncommon, for example the suggestions that either the regression line of x on y should be used or that there were insufficient data.

Question 3

Solutions to (a) were often disappointing with some candidates seeming to be confused by the notation used.

In (b)(i), many candidates evaluated the sample mean as 5.1 but some failed to convert this to the estimate 10.2 even if they had correctly found the value of k.

In (b)(ii), very few candidates realised that θ = 10.2 was not a feasible estimate when one of the sample values was 10.3.

Solutions to (c) were generally poor.

In (c)(i), many good answers were seen although some candidates failed to take account of the difference between $\mathrm{Var}(X)$ and $\mathrm{Var}(\bar{X})$.

In (c)(ii), many candidates thought that $\mathrm{E}(\bar{X}^2) = [\mathrm{E}(\bar{X})]^2$ although this had the unfortunate consequence of showing that U^2 is an unbiased estimator for θ^2. Few candidates realized that an expression for $\mathrm{E}(U^2)$ could be found by considering the standard result that $\mathrm{Var}(U) = \mathrm{E}(U^2) - [\mathrm{E}(U)]^2$ or the equivalent expression for $\mathrm{Var}(\bar{X})$. Part (c)(iii) was inaccessible to candidates who were unable to solve (ii).

Question 4

Most candidates stated the correct hypotheses in (a).

In (b)(i), the mean was invariably found correctly, although to find the variance estimate, quite a few candidates divided by 20 instead of 19. Incorrect variances were followed through in the next part of (b)(i). The t-test was generally well applied and the correct conclusion drawn. It was, however, surprising to note that many candidate used the appropriate formula to find the value of t and hence the p-value as opposed to using their GDC software.

Part (c) was generally well answered.

Question 5

In (a), it was disappointing to find that very few candidates realised that $P(Y=y)$ could be found by integrating $f(x)$ from y to $y+1$. Candidates who simply integrated $f(x)$ to find the cumulative distribution function of X were given no credit unless they attempted to use their result to find the probability distribution of Y.

Solutions to (b)(i) were generally good although marks were lost due to not including the $y=0$ term.

Part (b)(ii) was also well answered in general with the majority of candidates using the GDC to evaluate $G'(1)$.

Candidates who tried to differentiate $G(t)$ algebraically often made errors.

Recommendations and guidance for the teaching of future candidates

Although candidates are generally confident in the use of their GDC, some candidates are still using longhand methods to evaluate statistics and p-values which can be found more efficiently using the GDC.

It would seem that more time should be spent on ensuring that estimation theory is better understood.

Candidates should be strongly advised to take note of the rubric on the examination paper which states that ‘Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures’.