May 2015 subject reports

Mathematics HL TZ1

Overall grade boundaries

Discrete

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 28	29 - 40	41 - 53	54 - 65	66 - 76	77 - 100

Calculus

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 28	29 - 39	40 - 52	53 - 64	65 - 75	76 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 27	28 - 38	39 - 50	51 - 62	63 - 72	73 - 100

Stastistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 28	29 - 40	41 - 52	53 - 64	65 - 75	76 - 100

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2015 session the IB has produced time zone variants of Mathematics HL Paper 1 and Paper 2.

Higher level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 16	17 - 20

The range and suitability of the work submitted

A wide range of appropriate topics with mixed quality was submitted. In general the explorations were based on topics chosen by the students. In some cases, however, it was evident that the teacher told the students what topic to choose and possibly offered too much guidance. There was also a wide spectrum of suitability. Some works had a very interesting stamp of creativity on the use of HL mathematics topics whereas others either had minimal mathematical content or were the reproduction of classical text book type problems. An interesting phenomenon that was encountered is the transcribing of mathematical videos found on “Numberphile” or “Khan Academy”. Whereas it is not unusual for such videos to act as stimuli for a topic, students should be reminded that it becomes very difficult to achieve top marks unless the teacher / moderator is able to find evidence of personal engagement and critical reflection in their written response. Many students chose modelling explorations. The most popular were projectile motion, modelling the spread of a disease and logistic models for the growth of tumours. Unfortunately most of the candidates quoted a differential equation to model the phenomenon, defined the variables within the exploration context and integrated to obtain a relevant model without being able to interpret why or how the initial differential equation is valid. Some students chose topics that were well beyond the Mathematics HL course that the work produced was largely inaccessible to a peer group. In fact some explorations were so far removed from a teacher’s / moderator’s expected knowledge base so that they were largely incomprehensible and very challenging to moderate. At the other end of the spectrum there were a number of superficial explorations that were not commensurate with the level of the course. Some of these were reports on historical mathematics researched by the students with almost no mathematical content.

A number of students used technology to develop regression functions in an attempt to model data. Some of this work was supported by mathematics that demonstrated good knowledge and understanding of the model. Most of the time the regression model was simply created and applied via technology with very little understanding shown.

A major concern remains the problem of citations. The importance of citations at every point of reference needs to be clearly made to students. It is recommended that teachers provide students with the document “Academic Honesty in the Diploma Programme” and discuss the possible consequences of malpractice.

Mostly students wrote explorations that were within the recommended number of pages, but some responses were too long.

Candidate performance against each criterion

Criterion A

This criterion was addressed well by most of the students, with work being coherent and organized to different extents. It was noted that a number of students included appendices to keep the length of the exploration within the 6 to 12 pages, however this rendered the exploration incoherent since the reader needed to refer to the appendix in order to understand the actual work.

Some teachers guided their students to produce a table of content and a word count and included this in their assessment rubric. There is no need for either of these in the Exploration. Some problems in this criterion were caused by students attempting to explain things that were beyond their own comprehension because the topic chosen was well beyond the level of the course. Teachers should remember that students are not to be penalized twice for the same shortcoming. As mentioned above, it is absolutely essential that students cite any borrowed information at every point of reference.

Criterion B

Most students performed well against this criterion. However a number of students produced pages of irrelevant graphs that were not labelled or spreadsheet information that was not necessary. A number of teachers condoned the misuse of calculator notation within student work resulting in a change of the achievement level awarded by the teacher.

Criterion C

Once more this criterion proved to be the more difficult for teachers to interpret although there was a general improvement over May 2014. It is very important that teachers and students alike understand the scope of this criterion. Transcribing work that can easily be found in a textbook, on a website or a video clip does not allow the voice of the student to be heard in the exploration. Students are meant to solve some curiosity resulting from the stimulus used. It was unfortunate to note that some teachers did not look for personal engagement in the students' work but assessed their students subjectively against this criterion. This often led to inconsistent levels being awarded. A common comment to justify a low level being awarded was "the student was not sufficiently engaged". On the other hand some explorations were very original, revealing student enthusiasm for the topic with this energy coming through in the written work.

Criterion D

This criterion was often not understood well by all teachers and students. A number of students who presented an exploration based on modelling thought that general reflections on mathematics and the application thereof to a real life context were expected. There was evidence to suggest that teachers guided candidates to discuss the scope and limitations as if they were still working on old IA Portfolio tasks. It should be noted that critical reflection has a metacognitive aspect to it that involves isolating a problem, looking at it from different perspectives and analysing the findings. It may also include linking their work to other problems or raising other questions that were not apparent at the beginning of the process. Again teachers are advised to refer to the document "Additional notes and guidance on the Exploration" which can be found on the OCC.

It is interesting to note that those students who achieved high levels against this criterion also scored highly against criterion C because as they made an effort to overcome their perceived shortcomings they managed to demonstrate personal engagement with their work.

Criterion E

The mathematical content was very varied, ranging from very basic mathematics to extensions of the HL course that was well beyond the scope of the Exploration. Achieving a 6 still proved to be elusive on either count. Students who opted to explore more abstract concepts were unable to demonstrate their understanding of the mathematics used and some students who opted for modelling explorations failed to go beyond the mechanical work of solving a differential equation and hence did not demonstrate thorough understanding. There was a larger number of high scoring explorations in this session than there were in May 2014.

Recommendations for the teaching of future candidates

- There was evidence to suggest that some teachers were not dedicating the stipulated hours to the Exploration. It is imperative that 10 hours of teaching time are used to guide the students during the exploration process.
- Students need to tell the story of the development of their exploration. A clear and focused aim that is referred to as the exploration develops will help with organization and coherence.
- Candidates should ask themselves whether another candidate is likely to reproduce the same exploration. If the answer is yes it is unlikely to achieve high levels against criterion E. The exploration should be something personal to the student and hence the probability of another candidate writing something similar should be minimal.
- Teachers need to refer to the TSM as well as the document "Additional notes and guidance on the Exploration", both of which can be found on the OCC
- Teachers should be strongly discouraged from mandating a particular type of exploration.
- Teachers must show evidence of marking explorations with tick marks indicating where the mathematics used is correct and identifying errors. Annotations and comments should be written directly on the student's written response. The teacher assesses the work and the role of the moderator is to confirm the achievement levels awarded by the teacher and not to mark the work.
- Teachers and students need to be consistent in adhering to academic honesty. Each reference, picture, graph must be cited at the point of reference. Some students merely provided a bibliography without citations within the body of their written response. Failure to do this might result in the work being reviewed by the IB.
- The original student work needs to be sent for moderation. When printing out the work for assessment / moderation, the teacher should be aware that black and white printing may it difficult for the moderator to differentiate between graphs or charts.
- Teachers should give feedback to students by annotating their written responses. Comments that reiterate the achievement level descriptors are not helpful.

- One of the aspects of Approaches to Teaching and Learning in the DP is to encourage and stimulate students to develop research and writing skills in mathematics. This can be achieved by assigning mini-tasks, providing opportunities for reading and analyzing different forms of mathematical writing as well as making links to ToK and CAS.
- The lack of annotation and/or comments specific to individual student work remains an issue. A number of schools sent clean copies of Explorations with very brief generic comments on the Form 5/EXCS which resulted in the moderator having to mark the Exploration rather than moderate it. Very often moderators find errors in the mathematics that suggest that the teacher did not check the work. This unfortunately results in achievement levels not being confirmed which in turn affects the marks for the whole school cohort.
- A number of schools used the older Form 5/EXCS rather than the newer one which made it easier for teachers to provide more pertinent comments. It was surprising to see some of these forms filled out by the candidates themselves.
- The document " Additional notes and guidance on the exploration" proved very helpful for some schools but there was evidence to suggest that a number of teachers were not aware of this document.

Further comments

- There seemed to be more mediocre explorations this year than in May 14. These were often explorations submitted with a total of 5 marks or below. Teachers are encouraged to talk to students about the importance of internal assessment and its impact on the final IB mark as well as its intrinsic value for an IB learner.
- In general moderators find the explorations much more interesting to moderate than the old tasks. However it seems that some students are choosing safe topics like statistics and projectile motion.
- The general feeling after seeing the variety of explorations is that the Exploration is that its benefits as an independent piece of work on a topic chosen by the student far outweigh its usefulness as a discriminatory assessment tool. The 20% weighting seems to be just right because those students who are well prepared and guided by their teachers tend to do quite well whereas students whose teachers do not provide sufficient guidance seem do poorly.
- Teachers should discourage students from attempting to write an exploration on a topic which is largely inaccessible to them. Such topics are very difficult to write about in a way that makes the exploration readable by a peer. Very often the students cannot demonstrate thorough understanding and can therefore not draw on any critical and meaningful reflection.

Higher level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 33	34 - 44	45 - 58	59 - 73	74 - 87	88 - 120

General comments

The paper proved to be robust and a fair examination. There were some disappointing responses that were mainly due to the candidates not having been taught all the syllabus.

The areas of the programme and examination which appeared difficult for the candidates

All questions that required proof, Induction in particular.

The areas of the programme and examination in which candidates appeared well prepared

Calculus. Inverse of a function. Dot and vector products.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a) Good methods. Some candidates found the larger angle.

(b) Generally good, some forgot the radii.

Question 2

(a) Well done.

(b) Well done.

(c) Both parts well done.

Question 3

(a) Some correct answers but too many candidates had a poor approach and did not use the trig identity.

(b) Same as (a).

Question 4

(a) Well done although some did not use the binomial expansion.

(b) Fine by those who knew what first principles meant, not by the others.

Question 5

(a) Sometimes backwards working but many correct approaches.

(b) Some candidates did not know what odd and even functions were. Correct solutions from those who applied the definition.

(c) Some realised: just apply the definitions. Some did very strange things involving f and g .

Question 6

(a) Well done. Only a few candidates confused inverse with derivative or reciprocal.

(b) Not enough had the method of polynomial division.

(c) Reasonable if they had an answer to (b) (follow through was given) usual mistakes with not allowing for the derivative of the bracket.

Question 7

(a) Both parts fine if they used the formula, some tried to use the quadratic equivalent formula. Surprisingly some even found all the roots.

(b) Some notation problems for weaker candidates. Good candidates used either of the methods shown in the Markscheme.

Question 8

Many good complete answers. Some did not realise it was arctan. Some had poor understanding of the method.

Question 9

(a) Well done.

(b) Generally well done, some used more complicated methods rather than considering the range of sine.

(c) Fine if they realised the period was π, not if they thought it was 2π.

(d) Typically 3 marks were gained. It was the shift in the x axis of $\frac{\pi}{10}$ that caused the problem.

Question 10

If a script has lots of numbers with the wrong final answer and no explanation of method it is not going to gain many marks. Working has to be explained. The counting strategy needs to be decided on first. Some candidates misunderstood the context and tried to calculate exactly 3 consecutive losses. Not putting a non-loss as 2/3 caused unnecessary work.

Question 11

(a) Well done.

(b) The logic of an induction proof was not known well enough. Many candidates used what they had to prove rather than differentiating what they had assumed. They did not have enough experience in doing Induction proofs.

(c) Good, some forgot to test for min/max, some forgot to give the y value.

(d) Again quite good, some forgot to check for change in curvature and some forgot the y value.

(e) Some accurate sketches, some had all the information from earlier parts but could not apply it. The asymptote was often missed.

Question 12

(a) Method of first part was fine but then some algebra mistakes often happened. The next two parts were generally good.

(b) Given that (a) indicated that there was a common ratio a disappointing number thought it was an AP. Although some good answers in the next parts, there was also some poor notational misunderstanding with the sum to infinity still involving n.

(c) Not enough candidates realised that this was an AP.

Question 13

(a) Most realised that a subtraction was required.

(b) With the dot product method not all candidates dotted the correct vectors. With the cross product method a common mistake was to put it equal to this rather than a multiple of it.

(c) Follow through marks were gained here.

(d) Either good method shown or complete misunderstanding.

(e) Again follow through marks gained here but they required an answer to (d).

(f) Follow through again.

(g) Not enough candidates had the correct answers to be able to comment. Some students did say that the answers should be the same and thus they realised that they had made a mistake.

Recommendations and guidance for the teaching of future candidates

It is vital that all students are taught all parts of the syllabus. It was clear that some parts e.g. Induction, Vectors had not been sufficiently covered. Teachers need to ensure that candidates understand the meaning of the IB command terms. During the course work has to be collected in and correctively marked. Then bad practice, like starting with what you are trying to prove, moving from the particular to the general, can be stamped out. Candidates should practice on past IB questions and their logic checked. A trial exam under exam conditions is essential. Getting candidates to realise what part of the syllabus is being asked for in each question is important. Candidates should realise that they are trying to communicate to the examiners what they are doing so that marks can be given. The importance of reading the question really carefully cannot be stressed enough. The choice of language used has been carefully thought about so that it gives advice and scaffolding to the students.

Higher level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 18	19 - 36	37 - 49	50 - 63	64 - 77	78 - 91	92 - 120

General comments

This Math HL paper assessed the knowledge and understanding of the syllabus topics but also the ability of the candidates to use problem solving techniques in both familiar and unfamiliar contexts often using technology. This was a calculator paper and in several question candidates were expected to use the graphical display calculator to perform several tasks, including solving inequalities, finding values of numerical integrals and value of derivative of a function at a given point. Candidates were also expected to communicate mathematics effectively using appropriate notation and terminology and giving numerical answers to the required level of accuracy.

It was clear that many candidates were well prepared to sit the paper and had been well prepared to answer all the questions, including the ones that required advanced use of GDC features. However, many other candidates seemed unfamiliar with parts of the syllabus and/or use of graphical calculator to tackle problems.

Unfortunately, in some cases it was clear that candidates were not prepared well enough to take any mathematics HL examination.

The areas of the programme and examination which appeared difficult for the candidates

The main difficulties identified during the marking process were

- Questions involving reasoning;
- Providing explanations;
- Algebraic manipulation;
- Effective use of GDC;
- Knowing whether or not it is appropriate to use a GDC;
- The ambiguous case of the sine rule;
- Conditional probability and Complex numbers;
- Kinematics;
- Remembering the constant of integration and integration in general.

Many candidates were not aware of what constitutes a valid response to a question including the command term ‘Show’.

The areas of the programme and examination in which candidates appeared well prepared

Apart from the weakest candidates, questions one to eight were generally well answered, with most candidates scoring at least a few marks in each of these questions. Candidates showed to be familiar and prepared to:

- Answer routine questions on trigonometry, statistics, probability distributions and differentiation;
- Solve problems about parallel and perpendicular vectors;
- Find equations of lines applying slope-intercept formula;
- Obtain an expression for the volume of revolution;
- Obtain graphs and find intersection of graphs with GDC.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates answered this question correctly. Those candidates who attempted to manipulate the function or attempt an integration wasted time and obtained 3/4 marks. The most common errors were an extra factor ‘2’ and a fourth power when attempting to square the function. Many candidates wrote down the correct expression but not all were able to use their calculator correctly.

Question 2

This question was well done with many candidates obtaining full marks. On the whole, but quite a number misunderstood what was required in part (b) and 182 minutes was a repeated incorrect answer. It was disappointing that candidates have not noticed that this answer was clearly too small showing that candidates had not appreciated the context of the question.

Question 3

Overall this question was well answered. In part (a) a number of candidates did not mention the binomial distribution or failed to state its parameters although they could go on and do the next parts. In part (b) most candidates could state the expected value. In part (c) many candidates had problems with inequalities due to the discrete nature of the variable. Most candidates that could deal with the inequality were able to use the GDC to obtain the answer.

Question 4

Most candidates scored 4/6 showing that candidates do not have enough experience with the ambiguous case. Very few candidates drew a suitable diagram that would have illustrated this fact which could have helped them to understand the requirement that the answer should be less than 10. In fact many candidates ignored this requirement or used it incorrectly to solve an inequality.

Question 5

This question was well understood and a large percentage appreciated the need for implicit differentiation although some candidates did not recognise the need to treat h as a constant till late in the question. A number of candidates found the answer $\frac{3\pi}{80}$ instead of $\frac{3}{80\pi}$ due to a basic incorrect use of the GDC.

Question 6

The differentiation was normally completed correctly, but then a large number did not realise what was required to determine the type of the original function. Most candidates scored 1/4 and wrote explanations that showed little or no understanding of the relation between first derivative and the given function. For example, it was common to see comments about horizontal and vertical line tests but applied to the incorrect function.In term of mathematical language, it was noted that candidates used many terms incorrectly showing no knowledge of the meaning of terms like ‘parabola’, ‘even’ or ‘odd’ (or no idea about these concepts).

Question 7

Most candidates successfully finished part a with two fundamental errors occurring regularly. Either e was granted powers of 4 and 5 or an attempt to show that the value of m was 10 was made by evaluation.

Part (b) was challenging for many candidates that showed that the idea of conditional probability was poorly understood. There were many incorrect solutions where often candidates only found $P\left(X=6\right)$.

Question 8

Overall most candidates answered parts (a) and (b) very well with many scoring full marks. However some candidates showed some confusion with the concepts of parallel and perpendicular and their answers to parts a and b were reversed. Part (c) was well attempted with most candidates scoring at least 2/4 marks. Many candidates however could not simplify the expression or use the calculator well enough to solve the resulting quadratic on their calculator.

Question 9

Surprisingly many candidates ignored that fact that paper 2 is a calculator paper, attempted an algebraic approach and wasted lots of time. Candidates that used the GDC were in general successful and achieved 7/7. A number of candidates either found the equation of the tangent or used the positive reciprocal for the normal and many did not find the value of y corresponding to $f(0)$.

Question 10

(a) In general part (a) was performed correctly, with the vast majority of candidates stating the correct open intervals as required. In part (b) many candidates scored a few marks by just finding intersection points and equations of asymptotes; many other candidates showed difficulties in manipulating inequalities and ignored the fact that the quantities could be negative. Candidates that used the graph well managed to achieve full marks. Unfortunately many sketches were very crudely drawn hence they were of limited value for assessment purposes.

Question 11

(a) Most candidates sketched the graph correctly. In a few cases candidates did not seem familiar with the shape of the graphs and ignored the fact that the graph represented a pdf. The correct sketch assisted greatly in the rest of the question.

(b) Most candidates answered this question correctly.

(c) A few good proofs were seen but also many poor answers where the candidates assumed what you were trying to prove and verified numerically the result.

(d) Most candidates stated the value correctly but many others showed no understanding of the concept.

(e) Many candidates scored full marks in this question; many others could not apply the formula due to difficulties in dealing with the piecewise function. For example, a number of candidates divided the final answer by two.

(f) Many misconceptions were identified: use of incorrect formula (e.g. formula for discrete distributions), use of both expressions as integrand and division of the result by 2 at the end.

(g) This part was fairly well done with many candidates achieving full marks.

(h) Many candidates had difficulties with this part showing that the concept of conditional probability was poorly understood. The best candidates did it correctly from the sketch.

Question 12

(a) In part (i) many candidates tried to multiply it out the binomials rather than using the binomial theorem. In parts ii) and (iii) many candidates showed poor understanding of complex numbers and made no attempt to equate real and imaginary parts. In a some cases the correct answer to part (iii) was seen although it was unclear how it was obtained.

(b) This question was poorly done. Very few candidates made a good attempt to apply De Moivre's theorem and most of them could not even equate the moduli to obtain r.

(c) This question was poorly done. From the few candidates that attempted it, many candidates started by writing down what they were trying to prove and made no progress.

(d) Very few made a serious attempt to answer this question. Also very few realised that they could use the answers given in part (c) to attempt this part.

Question 13

(a) Parts (i) and (ii) were well answered by most candidates.

In (iii) the constant of integration was often forgotten. Most candidates calculated the displacement and then used different strategies, mostly incorrect, to remove the negative sign from -500.

Surprisingly part (b) was not well done as the question stated the method. Many candidates simply wrote down $\frac{\mathrm{d}v}{\mathrm{d}t}$ while others seemed unaware that $\frac{\mathrm{d}v}{\mathrm{d}t}$ was the acceleration.

Part (c) was not always well done as it followed from (b) and at times there was very little to allow follow through. Once again some candidates started with what they were trying to prove. Among the candidates that attempted to integrate many did not consider the constant of integration properly. In part (d) many candidates ignored the answer given in (c) and attempted to manipulate different expressions. Part (e) was poorly answered: the constant of integration was often again forgotten and some inappropriate uses of Physics formulas assuming that the acceleration was constant were used. There was unclear thinking with the two sides of an equation being integrated with respect to different variables. Although part (e) was often incorrect, some follow through marks were gained in part (f).

Recommendations and guidance for the teaching of future candidates

- The whole of the syllabus has to be taught, including their applications (eg kinematics) and prior knowledge topics (e. g. algebraic manipulation, number facts, properties and names of polyhedra).
- Teacher are encouraged to take students work in and mark it so that they can identify

misconceptions and difficulties in communication.

- Provide opportunities for students to do past papers exam under exam conditions, mark them according to IB markschemes and go over questions that cause difficulties.
- Provide opportunities for students to use their GDC and advice about when and when not to use it. Remember when using GDCs to work with a large number of significant figures throughout the entire problem; rounding intermediate calculations can lead to errors in final answers.
- Show students a variety of methods and ensure that they are able to quickly find the most appropriate method for each question. Many students are using inefficient or inappropriate methods, such as trying to differentiate complex expressions or trying to solve equations by hand, both of which can be immediately solved with a GDC.
- Remind students of the need of showing individual steps when working problems; if answers are incorrect with no working and/or skipped steps, students may not get any credit for it.
- Provide a wide variety of problems related to topics in the syllabus and develop problem solving skills. Many students had trouble working with concepts that were presented in unfamiliar contexts or in less straightforward ways. Teachers should ensure that candidates are thoroughly prepared, not only for the topics that are covered as part of the HL course, but for the variety of contexts through which these topics may be encountered.

Higher level paper three: discrete

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 16	17 - 27	28 - 34	35 - 41	42 - 48	49 - 60

General comments

A substantial number of candidates seemed well-prepared for this examination paper and generally found it to be quite accessible. Many candidates exhibited good content knowledge and often displayed sound reasoning skills. Naturally, a good number of candidates were challenged by the last question part in each of the five questions which suggests a good hierarchical level of difficulty was established within each question.

The areas of the programme and examination which appeared difficult for the candidates

Construction of a proof and argumentation skills.

Recognising why a given weighted graph has no solution to the classical Travelling Salesman Problem.

Precise recall of definitions for the Travelling Salesman Problem, the fundamental theorem of arithmetic and the handshaking lemma.

Understanding properties of simple, connected planar graphs and important inequalities linking the number of edges, vertices and faces in such graphs.

Applying the rules of modular arithmetic to establish a mathematical result.

Using the fundamental theorem of arithmetic to determine the lcm and gcd of a pair of numbers.

The areas of the programme and examination in which candidates appeared well prepared

Constructing a weighted graph from an adjacency table.

Applying Kruskal's algorithm to find the minimal spanning tree of a weighted graph.

Finding a solution to the Chinese Postman Problem for a weighted graph.

Solving first-degree and second-degree linear homogeneous recurrence relations with constant coefficients.

Drawing $K_{2,2}$ in planar form, drawing a spanning tree for $K_{2,2}$ and drawing the complement of $K_{2,2}$.

Drawing a simple, connected planar graph.

Applying Euler's formula and related corollaries to planar graphs.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Part (a) was generally very well answered. Most candidates were able to correctly sketch the graph of H and apply Kruskal's algorithm to determine the minimum spanning tree of H. A few candidates used Prim's algorithm (which is no longer part of the syllabus).

Most candidates understood the Chinese Postman Problem in part (b) and knew to add the weight of PQ to the total weight of H. Some candidates, however, did not specify a solution to the Chinese Postman Problem while other candidates missed the fact that a return to the initial vertex is required.

In part (c), many candidates had trouble succinctly stating the Travelling Salesman Problem. Many candidates used an 'edge' argument rather than simply stating that the Travelling Salesman Problem could not be solved because to reach vertex P, vertex Q had to be visited twice.

Question 2

Part (a) was generally well done with a large number of candidates drawing a correct planar representation for $K_{2,2}$. Some candidates, however, produced a correct non-planar representation of $K_{2,2}$. Parts (b) and (c) were generally well done with many candidates drawing a correct spanning tree for $K_{2,2}$ and the correct complement of $K_{2,2}$.

Part (d) tested a candidate's ability to produce a reasoned argument that clearly explained why the complement of $K_{m,n}$ does not possess a spanning tree. This was a question part in which only the best candidates provided the necessary rigour in explanation.

Question 3

In part (a), a good number of candidates were able to 'see' the solution form for u_n and then (often in non-standard ways) successfully obtain $u_n = 4\times 7^n + 1$. A variety of methods and interesting approaches were seen here including use of the general closed form solution, iteration, substitution of $u_n = 4\times 7^n + 1$, substitution of $u_n = An + B$ and, interestingly, conversion to a second-degree linear recurrence relation. A number of candidates erroneously converted the recurrence relation to a quadratic auxiliary equation and obtained $u_n = c_1(6)^n + c_2(1)^n$.

Compared to similar recurrence relation questions set in recent examination papers, part (b) was reasonably well attempted with a substantial number of candidates correctly obtaining $v_n = 4(11)^n$. It was pleasing to note the number of candidates who could set up the correct auxiliary equation and use the two given terms to obtain the required solution. It appeared that candidates were better prepared for solving second-order linear recurrence relations compared to first-order linear recurrence relations.

Most candidates found part (c) challenging. Only a small number of candidates attempted to either factorise $11^n - 7^n$ or to subtract 7^n from the expansion of $(7+4)^n$. It was also surprising how few went for the option of stating that 11 and 7 are congruent mod 4 so $11^n - 7^n \equiv 0 (\text{mod}\, 4)$ and hence is a multiple of 4.

Question 4

In part (a) (i), many candidates tried to prove $2e \geq 3f$ with numerical examples. Only a few candidates were able to prove this inequality correctly. In part (a) (ii), most candidates knew that K_5 has 10 edges. However, a number of candidates simply drew a diagram with any number of faces and used this particular representation as a basis for their 'proof'. Many candidates did not recognise the 'hence' requirement in part (a) (ii).

In part (b) (i), many candidates stated the 'handshaking lemma' incorrectly by relating it to the 'handshake problem'. In part (b) (ii), only a few candidates determined that $v = e$ and hence found that $f = 2$.

In part (c), a reasonable number of candidates were able to draw a simple connected planar graph on 6 vertices each of degree 3. The most common error here was to produce a graph that contained a multiple edge(s).

Question 5

In part (a), most candidates omitted the 'uniquely' in their definition of the fundamental theorem of arithmetic. A few candidates defined what a prime number is.

In part (b), a substantial number of candidates used the Euclidean algorithm rather than the fundamental theorem of arithmetic to calculate $\gcd(5577, 99099)$ and $\text{lcm}(5577, 99099)$.

In part (c), a standard proof that has appeared in previous examination papers, was answered successfully by candidates who were well prepared.

Recommendations and guidance for the teaching of future candidates

It is important that the whole syllabus is taught and that students are aware of the various definitions given in the syllabus.

Teachers need to continue to highlight the importance of proof and discuss what constitutes sound logical argumentation and reasoning. It is important that students work on the precision of their explanations in questions involving proof and reasoning. Looking at the structure of proofs on mark schemes of previous exams should serve to assist in mastering these important attributes of mathematical communication as 'waffle' rarely gains many marks. It is also important that the beginning of a proof should not start with what is trying to be proved and that using numerical examples does not constitute proof.

Teachers need to continue to highlight question wording that specifically asks for a particular method or a specific result to be used. For example, in Question 5 (b), candidates were asked to use the fundamental theorem of arithmetic on 5577 and 99099 in turn and use these factorized results to then determine the lcm and gcd of these numbers. It is important to warn candidates that marks can be lost by not reading carefully enough what a question actually says and ensure to highlight hints present in question wording.

Although this option involves graphs and trees, there is no need for candidates to use graph paper to display some of their responses. With examination papers being scanned, it can be very difficult to read candidate answers that are produced on graph paper.

Higher level paper three: calculus

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 18	19 - 24	25 - 31	32 - 38	39 - 45	46 - 60

General comments

This seemed to be an accessible paper to the vast majority of candidates. There was good syllabus coverage and students had been well prepared by their schools. As always with a calculus paper a lack of rigour was sometimes present, with students occasionally getting into circular arguments.

The areas of the programme and examination which appeared difficult for the candidates

The comparison / limit comparison test caused difficulties, as did the Mean Value Theorem.

The areas of the programme and examination in which candidates appeared well prepared

The candidates were well prepared for the topics of, L'Hôpital's Rule, Maclaurin's Theorem, the Ratio and Integral tests and the convergence of indefinite integrals.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most students had a good understanding of the techniques involved with this question. A surprising number forgot to show $f(0)=0$. Some candidates did not simplify the second derivative which created extra work and increased the chance of errors being made.

Question 2

(a) This question allowed for several different approaches. The most common of these was the use of the integrating factor (even though that just took you in a circle). Other candidates substituted the solution into the differential equation and others multiplied the solution by x and then used the product rule to obtain the differential equation. All these were acceptable.

(b) This was a straightforward question. Some candidates failed to use the hint of ‘hence’, and worked from the beginning using the integrating factor. A surprising number made basic algebra errors such as putting the $+C$ term in the wrong place and so not dividing it by x.

Question 3

(a) In this part the required test was not given in the question. This led to some students attempting inappropriate methods. When using the comparison or limit comparision test many candidates wrote the incorrect statement $\frac{1}{n^2}$ converges, (p-series) rather than the correct one with $\sum$. This perhaps indicates a lack of understanding of the concepts involved.

(b) There were many good, well argued answers to this part. Most candidates recognised the importance of the result in part (i) to find the limit in part (ii). Generally a standard result such as $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)=1$ can simply be quoted, but other limits such as $\lim_{n\to\infty}\left(\frac{\ln n}{\ln(n+1)}\right)=1$ need to be carefully justified.

(c)(i) Candidates need to be aware of the necessary conditions for all the series tests.

(c)(ii) The integration was well done by the candidates. Most also made the correct link between the integral being undefined and the series diverging. In this question it was not necessary to initially take a finite upper limit and the use of ∞ was acceptable. This was due to the command term being ‘determine’. In q4b a finite upper limit was required, as the command term was ‘show’. To ensure full marks are always awarded candidates should err on the side of caution and always use limit notation when working out indefinite integrals.

Question 4

(a) This question was done successfully by most candidates.

(b) A few errors with the signs but candidates largely worked through the integration by parts successfully. In this question it was important to use limit notation to show the integral converged to 2.

Question 5

(a)(i) This was well done by most candidates.

(a)(ii) This was generally poorly done, with many candidates failing to draw the curve correctly as they did not appreciate the importance of the given domain. Another common error was to draw the graph of the derivative rather than the function.

(b)(i) This was very poorly done. A lot of the arguments seemed to be stating what was being required to be proved, eg ‘because the derivative is equal to 0 the line is flat’. Most candidates did not realise the importance of testing a point inside the interval, so the most common solutions seen involved the Mean Value Theorem applied to the end points. In addition there was some confusion between the Mean Value Theorem and Rolle's Theorem.

(b)(ii) It was pleasing that so many candidates spotted the link with the previous part of the question. The most common error after this point was to differentiate incorrectly. Candidates should be aware this is a 'prove' question, and so it was not sufficient simply to state, for example, $f(0) = \pi$.

Recommendations and guidance for the teaching of future candidates

Students need to be exposed to a wider range of the uses of the Mean Value Theorem.

Students should practice spotting the correct technique for convergence of series questions.

The command 'hence' is there largely to alert students to the need to use the result of a previous part.

Higher level paper three: sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 20	21 - 25	26 - 31	32 - 36	37 - 60

General comments

The mathematics in the Sets, Relations and Groups option differs from that in the other three options in that it deals with very abstract concepts rather than being based on the application of mechanical rules. Proof and strict logical reasoning play a very significant role in this option.

The areas of the programme and examination which appeared difficult for the candidates

Working with arithmetic modulo an integer.

Many candidates mistook the use of mathematical synonyms as constituting a proof. For example, simply saying that a function is one-to-one does not amount to a proof that the function is injective.

Many candidates were uncomfortable working with infinite groups and infinite discrete sets.

The areas of the programme and examination in which candidates appeared well prepared

The definition of a group and associated concepts: Cayley tables; inverses; the order of an element; permutations.

The definition of an equivalence relation.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a) The majority of candidates were able to complete the Cayley table correctly. Unfortunately, many wasted time and space, laboriously working out the missing entries in the table - the identity is p and the elements q, r and s are clearly of order two, so 14 entries can be filled in without any calculation. A few candidates thought t and u had order two.

(b) Generally well done. A few candidates were unaware of the definition of the order of an element.

(c) Often well done. A few candidates stated extra, and therefore incorrect subgroups.

Question 2

Many candidates were not sufficiently familiar with modular arithmetic to complete this question satisfactorily. In particular, some candidates completely ignored the requirement that solutions were required to be found modulo 7, and returned decimal answers to parts (a) and (b). Very few candidates invoked Lagrange's theorem in part (b)(ii). Some candidates were under the misapprehension that a group had to be Abelian, so tested for commutativity in part (b)(ii). It was pleasing that many candidates realised that an identity had to be both a left and right identity.

Question 3

(a) A surprising number of candidates thought that an example was sufficient evidence to answer this part.

(b) Again, a lack of confidence with modular arithmetic undermined many candidates' attempts at this part.

(c) and (d) Most candidates started these parts, but some found solutions as fractions rather than integers or omitted zero and/or negative integers.

(e) Some candidates regarded R as an operation, rather than a relation, so returned answers of the form $aRb \neq bRa$.

Question 4

(a) Those candidates who formulated the questions in terms of the basic definitions of injectivity and surjectivity were usually sucessful. Otherwise, verbal attempts such as ' f is one-to-one $\Rightarrow f$ is injective' or ' g is surjective because its range equals its codomain', received no credit. Some candidates made the false assumption that f and g were mutual inverses.

(b) Few candidates gave completely satisfactory answers. Some gave functions satisfying the mutual identity but either not defined on the given sets or for which g was actually a bijection.

Question 5

(a) This part was generally well done. Where marks were lost, it was usually because a candidate failed to choose two different elements in the proof of closure.

(b) Only a few candidates realised that they did not have to prove that H is a group - that was stated in the question. Some candidates tried to invoke Lagrange's theorem, even though G is an infinite group.

(c) Many candidates showed that the mapping is injective. Most attempts at proving surjectivity were unconvincing. Those candidates who attempted to establish the homomorphism property sometimes failed to use two different elements.

Recommendations and guidance for the teaching of future candidates

The notion of 'proof' and well-based logical arguments is very important in mathematics, but particularly for students taking the Sets, Relations and Groups option. The earlier students are exposed to these ways of thinking the better.

Although this option deals with very abstract mathematics, that is best supported by means of a wide range of concrete examples: discrete and continuous number sets; modular arithmetic; permutations; transformations of sets, including symmetries of plane figures.

Encourage students to work mathematically rather than in terms of verbal explanations. Too often such work appears to be tautological or meaningless.

Higher level paper three: statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 16	17 - 25	26 - 31	32 - 37	38 - 43	44 - 60

The areas of the programme and examination which appeared difficult for the candidates

This examination showed that the majority of candidates are unable to interpret confidence intervals correctly. It is important for candidates not only to calculate confidence intervals but also to explain the meaning of their result.

Many candidates seemed to be unaware that they could use their calculators to carry out procedures in inferential statistics. It was fairly common to see candidates using the appropriate formula to calculate test statistics instead of reading them directly from the calculator.

Many candidates were unable to define estimators correctly although this may be due to an inability to write a verbal explanation rather than a lack of understanding.

Probability generating functions cause problems for some candidates.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates are able to carry out statistical tests even though the methods used are often inefficient.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates solved (a) correctly. In (b) and (c), however, many candidates made the usual error of confusing $\sum_{i=1}^{n} X_i$ and nX . Indeed some candidates even use the second expression to mean the first. This error leads to an incorrect variance and of course an incorrect answer. Some candidates had difficulty in converting the verbal statements into the correct probability statements, particularly in (c).

Question 2

Almost every candidate gave the correct estimate of the mean but some chose the wrong variance from their calculators to estimate σ^2. In (b)(i), the hypotheses were sometimes incorrectly written, usually with an incorrect symbol instead of μ, for example d , $\overline{x}$ and 'mean' were seen. Many candidates failed to make efficient use of their calculators in (b)(ii). The intention of the question was that candidates should simply input the data into their calculators and use the software to give the p -value. Instead, many candidates found the p -value by first evaluating t using the appropriate formula. This was a time consuming process and it gave opportunity for error. In (b)(iii), candidates were expected to refer to the claim so that the answers 'Accept H_0' or 'Reject H_1' were not accepted.

Question 3

The intention in (a) was that candidates should input the data into their calculators and use the software to give the confidence interval. However, as in Question 2, many candidates calculated the mean and variance by hand and used the appropriate formulae to determine the confidence limits. Again valuable time was used up and opportunity for error introduced. Answers to (b) were extremely disappointing with the vast majority giving an incorrect interpretation of a confidence interval. The most common answer given was along the lines of 'There is a 99% probability that the interval [9.761,9.825] contains μ'. This is incorrect since the interval and μ are both constants; the statement that the interval [9.761,9.825] contains μ is either true or false, there is no question of probability being involved. Another common response was 'I am 99% confident that the interval [9.761,9.825] contains μ'. This is unsatisfactory partly because 99% confident is really a euphemism for 99% probability and partly because it answers the question 'What is a 99% confidence interval for μ' by simply rearranging the words without actually going anywhere. The expected answer was that if the sampling was carried out a large number of times, then approximately 99% of the calculated confidence intervals would contain μ. A more rigorous response would be that a 99% confidence interval for μ is an observed value of a random interval which contains μ with probability 0.99 just as the number $\bar{x}$ is an observed value of the random variable $\bar{X}$. The concept of a confidence interval is a difficult one at this level but confidence intervals are part of the programme and so therefore is their interpretation. In view of the widespread misunderstanding of confidence intervals, partial credit was given on this occasion for interpretations involving 99% probability or confidence but this will not be the case in future examinations. Many candidates solved (c) correctly, mostly using Method 2 in the mark scheme.

Question 4

In general, solutions to (a) were extremely disappointing with the vast majority unable to give correct explanations of estimators and unbiased estimators. Solutions to (b) were reasonably good in general, indicating perhaps that the poor explanations in (a) were due to an inability to explain what they know rather than a lack of understanding.

Question 5

Solutions to (a) were often disappointing with some candidates simply writing down the answer. A common error was to forget the possibility of X being zero so that $G(t) = pt$ was often seen. Explanations in (b) were often poor, again indicating a lack of ability to give a verbal explanation. Very few complete solutions to (c) were seen with few candidates even reaching the result that $(q_1 + p_1t)(q_2 + p_2t)$ must equal $(q + pt)^2$ for some p.

Recommendations and guidance for the teaching of future candidates

In general, candidates are able to calculate confidence intervals but it is important for them to be able to give a correct interpretation of their result.

Candidates need to be more familiar with the statistical software on their calculators.

More time should perhaps be spent on probability generating functions which appear to cause difficulties for some candidates.