May 2014 subject reports

MATHEMATICS HL TZ2

Overall grade boundaries

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 26	27 - 38	39 - 50	51 - 62	63 - 73	74 - 100

Calculus

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 40	41 - 52	53 - 65	66 - 76	77 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 25	26 - 38	39 - 50	51 - 62	63 - 73	74 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 26	27 - 38	39 - 50	51 - 62	63 - 73	74 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2014 examination session the IB has produced time zone variants of Mathematics HL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 16	17 - 20

The range and suitability of the work submitted

The majority of explorations were generally commensurate with the Maths HL content but the quality was very mixed with very few explorations in the top range. Unfortunately many explorations lacked citations. This requirement needs to be made clearly known to all teachers; otherwise students will risk a malpractice decision.

Some of the explorations were too long, sometimes because the scope of the exploration was not focused enough. On the other hand a few explorations were too short and included very little mathematical content.

Some repeated topics were seen like "The Monty Hall Problem", "Rubic Cube Mathematics" or "Mathematics behind the Pokemon game". A number of explorations were based on common textbooks problems and demonstrated little or superficial understanding of the mathematical concepts being explored. A few of the students however demonstrated thorough understanding and managed to personalize their explorations. Modelling explorations based on Physics problems were also abundant. The most popular topic explored was the "Parabolic Trajectory" and the "Catenary equation".

Candidate performance against each criterion

A – In general students performed well against this criterion. Some teachers seem to believe that subheadings indicating "Aim", "Rationale" etc., are required in order to achieve top levels. Most explorations were complete and concise, however, some were far too long. Works that were based on typical text book problems and depended a lot on sources tended to be incoherent and were difficult to follow. Any paraphrased information needs to be cited at the point in the exploration where it is used. A footnote referring to the bibliography is not enough and may lead to a decision of malpractice.

B – Students did well in general on this criterion. Graphs and tables were often provided but not commented on. Sometimes graphs lacked labelling, and tables had no headings. The teacher sometimes condoned the misuse of computer notation; this lead to a change in the achievement level awarded. Some explorations lacked the definition of key terms used.

C – This is the criterion that was mostly misinterpreted by teachers with a quite a few students being awarded top levels because of their commitment or enthusiasm for the subject without any of this being evident in the student work. Students who presented explorations based on common textbook problems beyond the HL curriculum, were unable to score highly on this criterion because the mathematics was not understood fully to enable them to take ownership and extend the work beyond the theory presented. Some teachers understood the criterion descriptors well and this was transmitted to students effectively.

D – Some teachers misunderstood this criterion's descriptors and must have conveyed to students that reflection was a summative of the work done. As such some explorations were written as an old "IA Task" with just a narrative about the scope and limitations of the work done and no meaningful or critical reflection. Again students who wrote a "textbook" problem investigation found it difficult to reflect on the process and / or results and their significance. For higher achievement levels in this criterion students need to consider further explorations, implications of results, compare the strengths and weaknesses of the different mathematical approaches of their investigation and also look at the topic from different perspectives.

E – There was a large variety of mathematical content in the exploration, ranging from very basic mathematics to extensions well beyond the HL syllabus. A number of explorations were full of formulae which seemed to be copied from mathematical journals or Wikipedia without appropriate sources. It was not always clear whether the teacher had checked the mathematical content; this made it more difficult to understand how the achievement levels were interpreted and awarded by the teacher. In some explorations the content seemed "forced" and overly sophisticated abstract concepts were added in an attempt to raise the quality of the exploration. Often this created a patchwork of mathematical formulae and equations that were not necessarily understood by the student. Although an exploration may take the form of a research paper, containing mathematics that is found in appropriate sources, the student needs to demonstrate a deep understanding of the mathematics being explored.

Recommendations and guidance for the teaching of future candidates

The exploration should be introduced early in the course and referred to frequently enough to allow students to reflect on an area of Mathematics that best suits their interest and allows them to develop an appropriate exploration.

Students should be provided with material to stimulate ideas for the exploration. These may include movies, short videos, photographs, experiments etc…

Students need to develop research and writing skills through reading and understanding different forms of mathematical writing as well as the possible assignment of mini tasks.

Teachers should discuss the suitability of the topic chosen by students before a first draft is handed in.

Students should use some of the time allocated to the Exploration to explain clearly the expectations when it comes to using borrowed ideas from sources. Teachers need to make it very clear to students that each and every quoted, paraphrased, borrowed or stolen reference must be cited at the point of reference, otherwise the student's work will be referred to the Academic Honesty department that may decide on a possible malpractice (plagiarizism).

The teacher should ensure that the work being submitted is the student's own work.

The teacher must show evidence of checking the mathematics with tick marks, annotations and comments written directly on the students' work. This will help the moderator to confirm the achievement levels awarded by the teacher.

The teacher must mark a first draft of the exploration. This should provide students with written feedback. This should also lead to a discussion to ensure that the student understands the mathematics used and demonstrates this in the work.

Students should be discouraged from using difficult Mathematics beyond the HL syllabus if this cannot lead to some creativity or personalized problem.

Students should be reminded that the exploration should be between 6 to 12 pages typed in an appropriate font size (e.g. Arial 12). Diagrams and /or tables which are not significant and do not enhance the development of the exploration should not be included.

Candidates need to understand the difference between describing results and critically reflecting on their results.

Using difficult mathematics that goes well beyond the HL syllabus often results in a lack of thorough understanding and this in turn makes it difficult for the student to demonstrate Personal Engagement or Reflection.

Students should be encouraged to create their own questions based on their own individual interest which may include current social, economic or environmental problems in the community.

Teachers are encouraged to use past explorations (TSM exemplars) and engage students in marking them early on in the process. This will clarify the importance of each criterion and the impact the choice of topic may have on the achievement levels that may be reached.

Further Comments

A number of explorations showed very little work other than paraphrasing entries in Wikipedia. It is the school's responsibility to check for plagiarism before student work is submitted for assessment.
When students choose to present an exploration which is based on a scientific phenomenon, they should be aware that they are writing about mathematics and not reproducing a laboratory report.
It is felt that the new format of the IA has provided students with a great opportunity to explore a topic in Mathematics that they enjoy as well as take up ownership of their mathematical work.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 41	42 - 55	56 - 70	71 - 84	85 - 120

The areas of the programme and examination which appeared difficult for the candidates

Sums and roots of quadratic equations; applications of vectors to geometry; including notation and transformations.

The areas of the programme and examination in which candidates appeared well prepared

Probability; vector equations of lines and planes; differentiation; stationary points and points of inflection; Integration and applications.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This appeared to be a very straight-forward question for the majority of candidates. A very small number of candidates lost the final mark, with a numerical ‘reason’ required, though fully correct solutions were often seen.

Question 2

With several ways of approaching this question, a great many candidates were able to pick up marks for their initial steps, even if they did not get through to the final answer. Minor slips of sign often led to candidates obtaining the incorrect answer, though the majority either obtained the correct final answer, or ‘stopped’ mid-way through their attempt. As such, incorrect work was not often seen.

Question 3

The successful candidates in this question often employed row-reduction to their augmented matrix to obtain a row of zeros, which they then correctly implied infinite solutions.

Some demonstrated that the determinant of $\begin{pmatrix} 1 & 1 & 2 \\ 3 & -1 & 14 \\ 1 & 2 & 0 \end{pmatrix}$ was zero, but neglected to find a further valid point, and so scored one mark out of the two available. A small number were able to apply Cramer’s rule successfully. Correct solutions to part b) were often seen with pleasingly few algebraic slips.

Question 4

There seemed to be issues with this question in that a great many candidates scored either full marks or zero. This is a new topic on the syllabus, and it may be that some centres have not taught sums and products of roots. Of those that were familiar with the topic, this was found to be a straight-forward question of this type. Candidates could be usefully encouraged to explain from where their final answers appeared, as the answer ‘5’ was often seen in part b) with minimal working.

Question 5

This early question still proved to be a good discriminator. A huge range of incorrect sketches were seen in part (a), though there were also some very neatly drawn, correct attempts. Cusps were occasionally unclear, and with questions of this type, it is important they are seen as important characteristics of the curve. Those scoring full marks in (a), often easily went on to score similarly in (b).

Question 6

Persuasively correct vector notation is still proving troublesome for candidates in this type of question. Although part a) was usually correct, marks were lost in b) due to candidates not distinguishing between a vector and its modulus, often leaving examiners to guess. Simply writing $a = b$ often lost the final two marks in part (b),as the crux of the proof depends on recognising, and communicating, that $|a| = |b|$.

Question 7

Candidates' skills in complex number manipulation were often in evidence as part (a) was often answered successfully. A small number were able to obtain $\frac{10}{w} = \frac{5-5i}{13}$ but progressed no further. Sign slips were occasionally, and perhaps inevitably seen, with $13-13i$ given as a final answer in part (a), though follow through marks were still available in (b). Again, part (b) posed few problems, particularly for those who were successful in (a).

Question 8

Recognition that $f(x) = -3$ at the 2 boundary led to most candidates scoring the first two marks.

In part (b), a significant number of candidates thought of a reflection in the y-axis as being $-f(x)$ rather than the correct $f(-x)$.

The translation through $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ was much better understood. Nevertheless, poor attempts were the norm here, and only the very best candidates were able to obtain the correct expressions for $g(x)$ along with its domain(s).

Question 9

Part (b) proved troublesome for a number of candidates, who often neglected to recognise the condition for convergence of a geometric series. Moduli were often absent, and inevitably all three marks for this section were then lost. Clear solutions were also seen, however.

Part (c) was generally answered well, with most able to deal with the problematic $\sin\left(\arccos\left(\frac{1}{4}\right)\right)$ in the numerator of the fraction. It should be again emphasised that this is a *show that* question, and candidates should be encouraged to continue to show every step of their working.

Question 10

While many questions of this type have been generally answered well by candidates in the past, this proved to be more of a discriminator. The weakest candidates often achieved little more than differentiating $x = a\sec\theta$, but even here errors were evident as it was not unusual to see $\frac{dx}{d\theta} = a\sec x \tan x$. Better candidates were able to reach the integral of $\cos^2\theta$, and roughly half of these knew to proceed using the identity for $\cos 2\theta$. Slips when substituting limits were occasionally seen, even from very good candidates, and it was disappointing to see a few 'fudges' after so much progress had been previously made.

Question 11

Part a) proved to be very straightforward in all respects.

Part b) appeared also to be accessible to the majority of candidates. The main error seen was treating the question as 'choosing and replacing [transistors]', and so several marks were lost, possibly in some cases due to careless reading from the candidate. The formula for the expectation was known and applied well, though some candidates still seem to be intent on dividing their final answer by the total frequency.

Question 12

Part a) requires the equation of a line, and so should be expressed in the form $r = \ldots$, which was not always seen.
Although slips were seen in part b), the majority of candidates knew how to show two lines were skew, and many obtained the contradictory z values of $\frac{31}{4}$ and $\frac{3}{2}$.
The cross product was widely understood in part c) and usually applied well.
Part d) often found candidates attempting to use $\cos 60$ in their formula rather than the correct $\cos 30$. This led to two (incorrect) values of k being found and candidates then rarely scored further marks. For those that found the correct value of k, most were then able to go on and find the correct point of intersection.

Question 13

This was a high-scoring question for many candidates, and many obtained correct answers in parts a) and b). Algebraic slips were often seen from candidates in part c), which in many cases may have been avoided through more careful work (and presentation). Part e) proved more difficult for those not realising how to split the integral, though correct answers through using a tan substitution were possible, and occasionally seen.

Question 14

This question seemed to have been tackled least well out of the section B questions, there also being indications that some candidates may have been short of time.
Parts a) and b) were answered well, though a surprising number of candidates found trouble in differentiating $\arctan\left(\frac{1}{x}\right)$ in part c). For those that successfully tackled part c) part ii, each of the three methods shown in the published mark scheme were seen and employed with equal success.
Part d) was not particularly difficult in terms of determining whether this function was odd or even, but many candidates found it so due to their being confused as to the exact definition of odd or even functions. In all, this question proved to be a good discriminator at least, with very able candidates scoring close to full marks.

Recommendations and guidance for the teaching of future candidates

- Greater focus on recent changes to the syllabus.
- Correct vector notation might be emphasized, both in terms of equations of lines and in relation to solving geometric problems or demonstrating geometric proofs.
- General presentation, particularly where significant algebra is involved.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 51	52 - 65	66 - 79	80 - 93	94 - 120

The areas of the programme and examination which appeared difficult for the candidates

- Sigma notation.
- Recognising alternative forms of acceleration.
- Differentiation with composite functions.
- Algebraic manipulation.
- Proving the nature of a stationary point using calculus.
- Using the conjugate root theorem.
- Application questions in real life situations.
- Applying rigour and correct steps in proof by mathematical induction.
- Finding equation of a normal at a cusp with vertical tangent.
- Poisson distribution.

The areas of the programme and examination in which candidates appeared well prepared

- Arithmetic sequences and series.
- Integration by substitution and by parts.
- Normal distribution.
- Implicit differentiation.
- Sketching graphs and finding areas using GDC.
- Binomial expansion.
- Inverse functions.
- Related rates.
- Algebra with complex numbers.
- Applied trigonometry and finding the area of sector and segment.
- Using GDC to evaluate definite integrals, solve equations and find probabilities.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Part (a) (i) was generally well done, however some candidates had difficulty determining the correct number of terms as 27. Expressing the sum in sigma notation appeared to be challenging. Many candidates used incorrect notation, some left this part unanswered, some did not recognize that $7+7n$, the general term, was required with the number of terms from $n=1$ to 27.
In part (b) many candidates set up an equation or inequality correctly thus gaining the method mark but some were unable to solve the quadratic inequality. In general the question was well attempted.

Question 2

Part (a) (i) was generally well done. Most candidates were able to determine correctly the mean. Finding standard deviation caused difficulty to some candidates often caused by mistakes in solving simultaneous equations or using incorrect z values.

Part (b) in general well done, however, some candidates did not read the question properly leaving the answer as 0.184 instead of multiplying the probability by 100 to find the expected number.

Question 3

Part (a) was well done by the vast majority of candidates. A few misread the domain thus giving the values outside the specified domain.

Part (b) likewise very well done. Most candidates set the definite integral correctly and used GDC to evaluate the area.

Question 4

Part (a) was in general very well done with the majority of candidates obtaining the correct angle either in radians or in degrees.

Part (b) was not as well done as expected for a rather standard question. Most candidates realized that they needed to find the area of both sector and triangle, subtract and double the result. However, many candidates made errors in their calculations thus not obtaining the required answer of 1.18 cm^2. Another common mistake was incorrect setting of their GDC (degrees instead of radians) or using the combination of radians and degrees in their areas with one single setting on the GDC.

Question 5

This question was reasonably well done. Most candidates were able to expand each binomial correctly. Some candidates chose to work with the required terms only. It was pleasing to see quite a number of correct answers to this not so easy question. Common mistakes involved not considering both terms contributing to x^{-2} term or making a sign error.

Question 6

Many candidates found part (a) difficult. Most recognized binomial distribution in part (a) (ii) however many candidates were unsure how to answer part (a) (i). Many gave the same answer to both parts not being aware of the differences in the wording of the question. Some candidates used sum $\left(0.6^3+0.4^3\right)$ instead of the product of probabilities $\left(0.6^3\times 0.4^3\right)$ in part (a) (i).

Many candidates experienced difficulties with part (b) and were not able to proceed successfully past the initial equation or inequality. Those who obtained $0.6^n < 0.005$ were mostly successful in solving the above inequality using their GDC either graphically or numerically.

Question 7

Both parts were very well done by the majority of candidates often scoring full marks.

In part (a) common mistakes included writing an incorrect equation for the horizontal asymptote or not showing the asymptotic behaviour properly.

In part (b) most candidates obtained the correct equation for the inverse function. Some did not write it in the correct form as $f^{-1}(x)$ and some candidates forgot to list the domain of the inverse function or made a mistake in the domain.

Question 8

This question proved challenging to many candidates.
In part (a) most candidates used the formula booklet to set up an equation for mean but were unable to solve the polynomial equation to find mean or obtained an incorrect value.

Part (b) was done correctly only by very few candidates, with many not recognizing the relationship between the mean and standard deviation in Poisson distribution. Among those who calculated the value of standard deviation, only very few knew how to set up the condition within one standard deviation from the mean and even less candidates managed to use their GDC to obtain the required answer.

Question 9

This related rates question was answered quite well by many candidates with a few different methods seen, most commonly using $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$. Some candidates used implicit differentiation quite successfully. Thus many correct answers were seen. Common mistakes included incorrect implicit differentiation or not recognising that $h = r$. This related rates question was better done than similar questions in the past.

Question 10

There was a good attempt at implicit differentiation in part (a) by most candidates. Many lost the final mark due to errors in simplification.
In part (b) it was surprising how many candidates did not continue when they obtained an undefined result for the value of the derivative at (1, 1). Many candidates correctly calculated the gradient of the tangent as a fraction with zero in the denominator but did not know how to interpret this result geometrically and did not proceed any further.

Question 11

Interestingly, this continuous distribution question was either extremely well done thus often gaining full marks or was not attempted at all. There was rarely anything in-between. In part (a) the majority of candidates recognized the need for integration by parts with only few students trying to use their GDC here. The common error in this part was the sign error in integration by parts.

Part (b) was very well done with most candidates setting a correct definite integral and using their GDC to evaluate the probability.

In part (c) most candidates attempted to find the mode and the median with some still giving the *y*-value instead if the *x*-value for the mode and some being unable to use their GDC to solve for the upper limit.

Part (d) appeared to be the most challenging. While students recognized this problem involved conditional probability, they struggled with identifying what probability was necessary for the numerator.

Question 12

Part (a) was very accessible considering that the answer was given and almost all candidates obtained full marks here.

Part (b) (i) was attempted by the majority of candidates and reasonably well done. Some common mistakes included missing $\frac{1}{2}$ when using the chain rule. Some candidates had difficulty differentiating the second term of the cost function and made mistakes when differentiating $1000k$.

In part (b) (ii) most candidates equated their $\frac{dC}{dx}$ to zero and tried to solve the equation. Those candidates who differentiated correctly were mostly successful in obtaining the correct value of x for the minimum cost to occur. Some candidates gave unreasonable or invalid answers for x. Justification of a minimum by using the second derivative was either not attempted or lacking numerical values thus incomplete. Some candidates chose a graphical approach to show the change of sign of the first derivative but did not include enough detail in their sketches.
Parts (c), (d) and (e) were done well by those candidates who attempted the rest of the problem. Some candidates were unsure how to find the percentage increase in the minimum total cost. A number of candidates left some parts unanswered particularly when expressions involved k .

Question 13

Part (a), proof by mathematical induction, was not done well and only very few candidates obtained full marks for this part. Many candidates applied the properties of multiplication of complex numbers in modulus-argument form to complete the inductive step of the proof. It was unclear to some students that they were actually required to prove De Moivre's theorem and could not use it in the proof. Furthermore, candidates often lost the first reasoning mark for not showing clearly that $P(1)$ is true, without any reference to LHS and RHS. Likewise the language needed in the assumption step was a problem for many with many claiming $P(k)$ was proved true, or saying only assume $n = k$ instead of assume $P(k)$ is true.

Part (b) was very well done, however some candidates gave an incorrect argument for v without checking where point B is located in the Argand diagram. Some candidates left their answer to (b) (ii) unsimplified, which was fine this time as equivalent forms were accepted.

Plotting points A and B in part (c) was often inaccurate and lacked scales and appropriate labels.

Part (d) occurred to be quite challenging with only few candidates showing correct rotations. Many candidates used reflections instead or rotated in the wrong direction.

Most candidates made a start in part (e) recognizing conjugate pairs and attempting to find each quadratic. However, many candidates made mistakes in algebraic expansion thus not being able to obtain full marks. In general this part was not very well done.

Question 14

Part (a) was not done too well with many candidates sketching the graph over an incorrect domain or copying incorrectly the *y*-coordinate of the maximum as 0.884 instead of 0.0844. Most candidates who provided the sketch showed the correct shape with asymptotic behaviour.

Part (b) was reasonably well answered. Since the substitution was given, most candidates were successful in expressing the integral in terms of *u*. Most candidates recognized that it was an $\arctan$ type of integral. Common errors involved missing $\frac{1}{2}$ in front of the integral and not replacing *u* with t^2 in the final step.

In part (c) it was well understood what was needed to be done to calculate distance. Most candidates wrote down the correct definite integral in terms of t and tried to use their answer from (b) to obtain the exact answer. A number of candidates used a GDC to provide the numerical value thus not gaining the remaining two marks.

Part (d) was not accessible to many candidates and showed the lack of knowledge of alternate forms of acceleration. It was very rare to see the correct answer to this part. Few candidates attempted to find $\frac{dv}{ds}$ but were unable to progress beyond this point. Some substituted numbers into $\frac{dv}{ds}$ and gave this as the acceleration without multiplying by v .

Recommendations and guidance for the teaching of future candidates

- Emphasize the need for setting out proofs by mathematical induction correctly and applying rigour by showing each step logically. The formality of the proof needs to be adhered to and candidates need to be aware that they cannot use what they are asked to prove.
- Encourage candidates to store numerical answers obtained with the GDC or show how to carry work through using enough significant figures so that the final answers are expressed correct to the appropriate degree of accuracy.
- Students need to remember that final answers should be fully accurate or given to 3 significant figures and that intermediate working needs to use more than 3 significant figures.
- Encourage candidates to use the GDC to solve equations and integrate numerically; provide a wide range of problems that allow students to explore more advanced features of the GDC.
- There are still too many candidates who erroneously think they will gain more credit for laborious, error prone algebraic approaches. These candidates are penalized by wasting precious time with routine questions and often are unable to answer all the questions on the paper.
- Raise awareness of the importance of basic graph sketching skills including careful consideration of realistic or prescribed domain, range, key features and asymptotic behavior.
- Remind students to read questions very carefully. 0.184 is not a good number of bear cubs!
- Clarify the meaning of each of the command terms in the Mathematics HL guide.
- Emphasize what it means to convincingly answer ‘show that’ examination questions and provide many examples of mathematical proofs.
- Explain to candidates that if the ‘show that’ question involves an exact answer, the GDC cannot be used in those situations.

- Students should be encouraged to pay attention to mathematical notation and appropriate terminology.
- Students need to cover the entire syllabus.
- Teachers should provide a variety of non-routine problems set in real life situations where candidates need to use their thinking and problem solving skills. It is important that answers to these types of questions are checked for being reasonable, valid and within given restrictions.

Paper three - Discrete mathematics

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 14	15 - 21	22 - 26	27 - 32	33 - 37	38 - 60

The areas of the programme and examination which appeared difficult for the candidates

The candidates did not seem particularly comfortable with recurrence relations. As this is new to the syllabus, I thought that teachers would have made a point of covering it well. As in the past candidates found it more difficult to come up with proofs themselves rather than just applying algorithms that they knew.

The areas of the programme and examination in which candidates appeared well prepared

The upper and lower bound algorithms for the travelling salesman problem were well known, as were the methods to convert to different bases.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a)This was well drawn. (b) Generally answered quite well. There was too much confusion with the twice a minimal spanning tree upper bound method. Some candidates forgot to go back to D. (c) Generally good answers. Some candidates forgot to add on the 2 smallest edges into A.

Question 2

(a) (i) Most candidates knew one of the two methods. Some did not realise that 11 was B in base 13. A few very weak candidates thought that the numbers given were already in base 13. (ii) This was badly answered as most candidates ignored the “Hence” in the question and just applied Euclid’s algorithm to the original base 10 numbers. A few candidates did read carefully and saw what to do.

(b) The responses were variable and some candidates ignored it. A common mistake was to assume particular numbers for the elements of set L. Those candidates whose first thought was the pigeonhole principle were quite reasonable.

(c) (i) This was well done with just minus sign slips from the candidates with poor algebra. (ii) Answers were variable. Too many candidates did not read about mod 2. Initial conversion made the system of equations easy. Not enough candidates realised that if they were not initially working mod 2 then they could solve the system with their calculator rather than slogging it out. Often the answer given was not converted to mod 2.

(d) (i) Reasonably well done. Some explanations could have been clearer. Unfortunately a few candidates thought that a few examples would suffice. (ii) This was well done. (iii) Either candidates saw the counterexample to select or they did not.

Question 3

(a) This was reasonably well done but too many candidates did not read “Draw a spanning tree” and thus just drew K_4 and $K_{4,4}$.

(b) This was not well answered. Insufficient candidates realised that you had to apply the pigeon-hole principle. It was unfortunate that candidates thought that a few examples would suffice. Others just wrote down things that they knew about graphs and claimed that these proved the result.

(c) This was very badly answered indeed. Candidates either just gave some examples or said that it was true because it was obvious. It required careful thinking to describe how you obtained the spanning tree.

Question 4

Since solving a recurrence relation is essentially standard bookwork for the syllabus I was surprised that candidates did not do better in this question.

(a)(i) This could either have been done by realising it was a geometric progression or using the auxiliary equation. (ii) Far too many candidates did not use the suggested solution and just substitute it in. (iii) Not too many marks were gained here as many candidates had gone wrong earlier.

(b) Solving the auxiliary equation should have been standard but too many candidates did not achieve this. Putting the answer into the format required was more challenging as you would expect for the last part of the last question. I like the thinking of one of the candidates that did achieve this who then wrote “that was cool”.

Recommendations and guidance for the teaching of future candidates

Although this option involves graphs and trees there was no need for candidates to use graph paper for some of their answers! It made it more difficult to read the answers of candidates that did this with the papers being scanned. Candidates lost marks by not reading carefully enough what the question actually said and using the hints in the wording of the questions. If a candidate introduces a variable that is not given in the question then they need to say what it stands for so that the examiner can follow their working. They have to remember that they are trying to communicate to the examiner so careful use of words and diagrams can only assist them. Candidates need to be prepared for proofs as well as algorithms and know that “waffly” words rarely gain many marks. Looking at the structure of proofs on the mark-schemes of previous exams will help. For example, you cannot start with what you are trying to prove and examples are not proofs. I cannot really emphasis those last two points enough and we should all be getting this message across. With many of the points mentioned above, careful corrective marking of a trial exam should have assisted the candidates, if they were prepared to learn. It is important that the whole syllabus is covered in the teaching.

Paper three - Calculus

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 19	20 - 26	27 - 33	34 - 40	41 - 47	48 - 60

The areas of the programme and examination which appeared difficult for the candidates

Most of the candidates faced difficulties in Rolle's theorem. In some cases it was clear that the statement of this theorem was unknown to the students. Other candidates knew that they should consider functional values at the endpoints of a given interval but nothing more. Overall there was evidence of lacking of understanding of the Rolle's theorem and ability of its application.

Other areas that caused difficulties were the use of integrating factors to solve a linear differential equation and testing end points of the interval for convergence of the power series. Many of the candidates also faced difficulties in the evaluation of the improper integral. Most candidates also seemed unaware of the need of adjusting the limits of integration when changing variables.
Surprisingly many candidates had difficulties in using the graphics calculator to produce a reasonable sketch graph with the required information on it.

In general the mathematical communication was poor and many candidates seemed unfamiliar how to deal with command terms ‘show’ and ‘hence’.

The areas of the programme and examination in which candidates appeared well prepared

Students seemed to do well with basic concepts such as derivatives, limits, integrals, sequences and series. Most candidates attempted questions 1, 2 and 3 showing that they were familiar with the topics and could at least start the questions. Maclaurin expansion seemed a well prepared topic on the whole and most candidates were familiar with the use the ratio test to find the radius of convergence.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Generally many candidates could attempt Q1(a) but many students wrote down the results that were already given in the question and did not show any relevant step.

Part (b) was attempted by almost all candidates and was well done by most of them. Most candidates deduced the expansion from first principles but in a few cases candidates used the expansion of the exponential function to deduce the result. This latter approach resulted in more calculation errors.

In part (c) many candidates ignored the instruction ‘hence’ and tried to find the limit using L’Hôpital rule. The wrong answer of $\frac{1}{2}$ was seen very often due to calculation errors, namely incorrect removal of brackets.

Part (d) caused difficulties to many candidates that failed to spot an adequate substitution. Among the students that did use substitution, very few recognized the need of calculating the lower limit of integration leading in many cases to incorrect an answer. This part of the question also showed the difficulties candidates had in setting out their work properly, using correct notation. It was surprising to see how many candidates failed to recognize the improper nature of the integral and made no attempt to study its convergence.

Question 2

Overall, this question was well attempted. In part (a) most candidates could find the derivative but some failed to use the chain rule correctly in part (ii) A surprising number of candidates failed to relate increasing function with positive gradient function in the given interval.

In part (b) (i) most candidates who could identify the differential equation to be linear wrote it in standard form, many could find the integrating factor correctly and then solve the exact equation obtained, although a few candidates lost marks at the end due to careless errors. A number of candidates however treated the differential equation as homogeneous and wasted time trying to solve it using substitution.

In part (b) (ii) a number of candidates lost marks because they simply verified that the function given was a solution of the differential equation satisfying the initial conditions given.

In part (b)(iii) most candidates managed to sketch the graph but in many cases the sketches were not well labeled, showed extraneous asymptotes and incorrect values for the minimum point. Many candidates also ignored the domain and sketched the graph of the expression for $\mathrm{x} < 1$.

Question 3

In part (a) the majority of the candidates could find the expression for b(n) and c(n) correctly. Unfortunately, several candidates lost a mark in part a) because they did not answer the question asked, stating an interval instead of the radius of convergence.

In general part (b) was well answered with almost all knowing how to start and what to do. However, some candidates did not use the ratio test properly to determine the convergence of the power series and a few of them apparently did not realize that convergence of the series depend upon the values of x.

Many candidates could not discuss mathematically the convergence at $x = \pm 1$. Also in part (c), many candidates did not fully justify the use of the alternating series test

Question 4

Part (a) was well attempted and there was a range of marks scored**.** Some candidates did not realize that exact answers were required and attempted to use GDC to answer the question and lost marks for accuracy This was another question where it was evident the difficulty that many candidates had in setting out their work in a logical way.

Part (b) was a challenging question for most candidates. Some just did not understand or know how to use Rolle's theorem. Many left it blank or made random attempts to use theorems from the course. Attempts to use Bolzano’s and Mean value theorem were seen very often. Very few candidates obtained full marks in this part of the question.

Part (b) also showed that many candidates were not aware of the implications of the continuity and differentiability of a function on the behavior of its graph over the interval given. Like in previous questions many candidates ignored the instructions ‘prove that’ and ‘hence’ and attempted to answer the questions using GDC.

Recommendations and guidance for the teaching of future candidates

- Require that students set out all their work using appropriate notation and terminology and that course work includes answering the questions in a logical and clear way.
- Emphasise the need of showing working out and presenting it clearly and neatly. Many students arguments were difficult to follow and very untidy.
- Simple algebraic/numerical errors can have serious grade allocation consequences and students do need to be reminded that they must double check their steps to avoid simple careless errors.
- Recognise and follow instructions associated to IB command terms (eg ‘hence’, ‘prove that’ and ‘show that’).
- Teachers should ensure that their students are very comfortable in differentiating and integrating when starting this option, including knowing and understanding well integration by parts and by substitution and recognize easily when to apply these techniques.
- Provide a wide range of examples about the relations between behavior of functions and their derivatives, including piecewise functions and functions with restricted domains rather than the largest possible domain of their expressions.
- Teach students how to approach improper integrals in a proper way.
- Clarify methods to solve differential equations: students need to be able to recognise the type of equation before trying to apply specific methods to solve them.
- Emphasise the need of studying the endpoints in detail when establishing the convergence interval for a power series and ensure students know the difference between a radius and interval of convergence.
- Explore in more detail continuity and differentiability and associated theorems and stress the importance of fully justifying the conditions of a theorem before applying the theorem.
- Whilst there were some very well prepared students, it was also evident that some candidates scored very few marks. Teachers need to clarify the expectations of the DP mathematics courses and guide students to choose the appropriate level.

Paper three - Sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 21	22 - 27	28 - 32	33 - 38	39 - 60

The areas of the programme and examination which appeared difficult for the candidates

This is an option where proof and justification are most important. There was little evidence of an appreciation of this except at the higher levels. Many candidates had difficulty deciphering the standard terminology defining a subset in terms of a condition imposed on the elements of an overall set. This was evident both in understanding questions and in expressing answers. Many candidates were very hazy about the new syllabus concepts of ‘homomorphism’, ‘kernel’ and ‘cosets’. Many candidates had difficulty with the notion that a Cartesian product could involve both continuous and discrete factors.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates were happy working with Cayley tables and extracting the required information. The definition of a group was well understood. The generalities of equivalence relations was well understood.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates scored well on this question.

(a) Hard to go wrong.

(b) Generally well done. A very small minority confused commutativity with associativity.

(c) Generally well done, but one was sometimes left with the lingering doubt whether the candidate really got it. There was sometimes an invalid argument that was based on cancellation – we do not have a group, so one cannot cancel at will.

(d) Generally well done.

(e) Generally well done, but sometimes the examiner was expected to extract the answer from a mass of data.

Question 2

(a) (i) All examiners commented on the astonishing inability of many of the candidates to correctly answer this part. Twice a number is an integer means that the number is half an integer. Clearly a misunderstanding of set notation is an issue.

(b) (i) There was the feeling that many candidates cannot appropriately translate a concept into simple algebra. So aRb => bRa becomes aRb = bRa, which makes no sense. The notion of a symmetric relation was poorly handled.

Question 3

Many candidates were not comfortable with the concept of a Cartesian product, and certainly not with the ability of visualizing and handling such sets.

Question 4

This was a bookwork question straight off the syllabus. Many candidates were not familiar with the concepts of kernel and coset.

Recommendations and guidance for the teaching of future candidates

This is an option where concepts and understanding are more important than the manipulative ability that is required. Ensure that candidates know this and are up for that challenge. The set notation is key to this option, so make clear, by way of many examples, the various ways that sets can be defined both finite, infinite and several dimensional. Structured proofs are important, so emphasize this feature. Ensure that candidates write clearly, particularly when diagrams are involved. The examiner cannot read the mind of the candidate, so the candidate must make clear that they understand what they are writing in response to the question.

Ensure that all items in the syllabus are covered.

Paper three - Statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 15	16 - 21	22 - 27	28 - 33	34 - 39	40 - 60

The areas of the programme and examination which appeared difficult for the candidates

Many candidates made parts of Question 2 much longer than necessary by not using the calculator software to the full. It makes no sense to find correlation coefficients, p-values and equations of regression lines by using the calculator to find Σx etc and then calculating these other quantities using the appropriate formulae. Candidates need to be aware of the full capability of the statistics menu on their calculator.

Some candidates seemed unsure about handling probability generating functions. It is important to be aware of the several definitions of the probability generating function so that the most appropriate one can be chosen to solve a particular problem. The notion of unbiased estimation seems not to be understood by many candidates.

The areas of the programme and examination in which candidates appeared well prepared

Despite the comments in the section above, candidates seem to understand the concepts of correlation and regression fairly well.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Part (a)(i) was correctly answered by most candidates. In (a)(ii), however, a not uncommon error was to state that $\mathrm{P}(5 \le X \le 8) = \mathrm{P}(X \le 8) - \mathrm{P}(X \le 5)$. Part (b) was well answered by many candidates. Part (c)(i) was well answered in general with almost all the candidates using the Central Limit Theorem. It was surprising to note that very few candidates converted the probability to

$P(284 < \Sigma X < 340)$ which could then be evaluated as a Poisson probability. Many candidates failed to see how to solve (c)(ii).

Question 2

Most candidates stated the hypotheses correctly although candidates who failed to mention p were penalised. It was disappointing to see that many candidates, by choosing the wrong menu on their calculator, involved themselves in lines of arithmetic in answering (b), (c) and (d). A correct choice of software would have given the required results immediately. Part (f) was poorly answered in general with many candidates having no idea how to proceed. Many candidates wrote the regression line of x on y as $y = 0.409x - 12.2$ instead of $x = 0.409y - 12.2$ so that their gradient was incorrect. The incorrect answer $38°$ was therefore seen more often than the correct answer $7°$.

Question 3

Part (a) was not well answered in general with many solutions not even containing any expectation signs. Part (b) was reasonably well answered although not many candidates ended up with the correct expression for $\mathrm{E}(Y)$. Surprisingly, very few candidates realised that the algebra could be made easier by using the substitution $t = y - \lambda$. It was disappointing to note in (b)(i) that, although most candidates realised that $\int_{-\infty}^{\infty} f(y)\mathrm{d}y$ had to equal 1, very few candidates realised that they also had to show that $f(y)$ had to be non-negative over the appropriate range.

Question 4

Parts (a) and (b) were well answered by many candidates. Parts (c) and (d), however, proved difficult for most candidates with only a minority taking the easier route of defining a probability generating function in the form $\mathrm{E}(t^X)$ as opposed to $\sum p_x t^x$.

Recommendations and guidance for the teaching of future candidates

Candidates should be made aware of the full capability of the statistics menu on their calculator. Candidates should be familiar with the definitions and applications of probability generating functions.