May 2014 subject reports

MATHEMATICS HL TZ1

Overall grade boundaries

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 26	27- 38	39 - 50	51 - 61	62- 72	73 - 100

Calculus

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 28	29 - 39	40 - 52	53 - 64	65 - 76	77 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 26	27 - 38	39 - 50	51 - 61	62 - 73	74 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 38	39 - 50	51 - 62	63 - 73	74 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2014 examination session the IB has produced time zone variants of Mathematics HL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 16	17 - 20

The range and suitability of the work submitted

The majority of explorations were generally commensurate with the Maths HL content but the quality was very mixed with very few explorations in the top range. Unfortunately many explorations lacked citations. This requirement needs to be made clearly known to all teachers; otherwise students will risk a malpractice decision.
Some of the explorations were too long, sometimes because the scope of the exploration was not focused enough. On the other hand a few explorations were too short and included very little mathematical content.
Some repeated topics were seen like “The Monty Hall Problem”, “Rubic Cube Mathematics” or “Mathematics behind the Pokemon game”. A number of explorations were based on common textbooks problems and demonstrated little or superficial understanding of the mathematical concepts being explored. A few of the students however demonstrated thorough understanding and managed to personalize their explorations. Modelling explorations based on Physics problems were also abundant. The most popular topic explored was the “Parabolic Trajectory” and the “Catenary equation”.

Candidate performance against each criterion

A – In general students performed well against this criterion. Some teachers seem to believe that subheadings indicating “Aim”, “Rationale” etc., are required in order to achieve top levels. Most explorations were complete and concise, however, some were far too long. Works that were based on typical text book problems and depended a lot on sources tended to be incoherent and were difficult to follow. Any paraphrased information needs to be cited at the point in the exploration where it is used. A footnote referring to the bibliography is not enough and may lead to a decision of malpractice.

B – Students did well in general on this criterion. Graphs and tables were often provided but not commented on. Sometimes graphs lacked labelling, and tables had no headings. The teacher sometimes condoned the misuse of computer notation; this lead to a change in the achievement level awarded. Some explorations lacked the definition of key terms used.

C – This is the criterion that was mostly misinterpreted by teachers with a quite a few students being awarded top levels because of their commitment or enthusiasm for the subject without any of this being evident in the student work. Students who presented explorations based on common textbook problems beyond the HL curriculum, were unable to score highly on this criterion because the mathematics was not understood fully to enable them to take ownership and extend the work beyond the theory presented. Some teachers understood the criterion descriptors well and this was transmitted to students effectively.

D – Some teachers misunderstood this criterion's descriptors and must have conveyed to students that reflection was a summative of the work done. As such some explorations were written as an old "IA Task" with just a narrative about the scope and limitations of the work done and no meaningful or critical reflection. Again students who wrote a "textbook" problem investigation found it difficult to reflect on the process and / or results and their significance. For higher achievement levels in this criterion students need to consider further explorations, implications of results, compare the strengths and weaknesses of the different mathematical approaches of their investigation and also look at the topic from different perspectives.

E – There was a large variety of mathematical content in the exploration, ranging from very basic mathematics to extensions well beyond the HL syllabus. A number of explorations were full of formulae which seemed to be copied from mathematical journals or Wikipedia without appropriate sources. It was not always clear whether the teacher had checked the mathematical content; this made it more difficult to understand how the achievement levels were interpreted and awarded by the teacher. In some explorations the content seemed "forced" and overly sophisticated abstract concepts were added in an attempt to raise the quality of the exploration. Often this created a patchwork of mathematical formulae and equations that were not necessarily understood by the student. Although an exploration may take the form of a research paper, containing mathematics that is found in appropriate sources, the student needs to demonstrate a deep understanding of the mathematics being explored.

Recommendations and guidance for the teaching of future candidates

The exploration should be introduced early in the course and referred to frequently enough to allow students to reflect on an area of Mathematics that best suits their interest and allows them to develop an appropriate exploration.

Students should be provided with material to stimulate ideas for the exploration. These may include movies, short videos, photographs, experiments etc…

Students need to develop research and writing skills through reading and understanding different forms of mathematical writing as well as the possible assignment of mini tasks.

Teachers should discuss the suitability of the topic chosen by students before a first draft is handed in.

Students should use some of the time allocated to the Exploration to explain clearly the expectations when it comes to using borrowed ideas from sources. Teachers need to make it very clear to students that each and every quoted, paraphrased, borrowed or stolen reference must be cited at the point of reference, otherwise the student's work will be referred to the Academic Honesty department that may decide on a possible malpractice (plagiarizism).

The teacher should ensure that the work being submitted is the student's own work.

The teacher must show evidence of checking the mathematics with tick marks, annotations and comments written directly on the students' work. This will help the moderator to confirm the achievement levels awarded by the teacher.

The teacher must mark a first draft of the exploration. This should provide students with written feedback. This should also lead to a discussion to ensure that the student understands the mathematics used and demonstrates this in the work.

Students should be discouraged from using difficult Mathematics beyond the HL syllabus if this cannot lead to some creativity or personalized problem.

Students should be reminded that the exploration should be between 6 to 12 pages typed in an appropriate font size (e.g. Arial 12). Diagrams and /or tables which are not significant and do not enhance the development of the exploration should not be included.

Candidates need to understand the difference between describing results and critically reflecting on their results.

Using difficult mathematics that goes well beyond the HL syllabus often results in a lack of thorough understanding and this in turn makes it difficult for the student to demonstrate Personal Engagement or Reflection.

Students should be encouraged to create their own questions based on their own individual interest which may include current social, economic or environmental problems in the community.

Teachers are encouraged to use past explorations (TSM exemplars) and engage students in marking them early on in the process. This will clarify the importance of each criterion and the impact the choice of topic may have on the achievement levels that may be reached.

Further Comments

A number of explorations showed very little work other than paraphrasing entries in Wikipedia. It is the school's responsibility to check for plagiarism before student work is submitted for assessment.
When students choose to present an exploration which is based on a scientific phenomenon, they should be aware that they are writing about mathematics and not reproducing a laboratory report.
It is felt that the new format of the IA has provided students with a great opportunity to explore a topic in Mathematics that they enjoy as well as take up ownership of their mathematical work.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 32	33 - 43	44 - 57	58 - 70	71 - 84	85 - 120

The areas of the programme and examination which appeared difficult for the candidates

The sums and products of roots. This is a topic that it is new in the syllabus this year and was unfamiliar to many students.

Some of the vector question (q 12) was poorly done, particularly surprising was how few knew what was required to prove a quadrilateral was a square.

Though calculus was generally well done the difference between $\frac{dv}{ds}$ and $\frac{dv}{dt}$ was not clear to many.

The logarithms question was a fairly straight forward change of base question. Students should be aware of this formula (section 1.2 of the formula book)

Use of the trigonometric identities was poor with only a few knowing how to work with the compound angle identities to find, for example, $\arctan(A)+\arctan(B)$

Knowledge of the remainder theorem

What was apparent was that the questions that required real thinking and understanding were found difficult, which may reflect on how candidates are prepared for the paper. Often students would head off in the wrong direction on a question and a lot of time was wasted for no marks.

The areas of the programme and examination in which candidates appeared well prepared

The real strength of the candidates generally was shown in the calculus questions (with the exception of q 8 as mentioned above). This was particularly apparent in many fully correct answers to q11.

Use of the sine and cosine rules was well done

Straightforward vector techniques, such as finding the intersection of line and plane, were well done.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This question was most easily done using the Remainder Theorem. Many candidates attempted it using long division with various degrees of success.

Question 2

This question required some reasoning to deduce that the median was the mean of the second and third numbers. Those who realized this generally scored full marks on this question.

Question 3

The key formula was in the formula booklet. A good policy is that when logarithms are given in different bases, the change of base formula is likely to be the way forward.

Question 4

Several good candidates left out this question or tried to do it by inappropriate methods. A possible explanation is that some schools were not aware that the syllabus change included this as a new topic.

Question 5

This was found to be a difficult question. (a) Students need to be aware of the rigour required when asked to 'prove' an identity. In this particular case almost all lost a mark through failing to justify only considering the positive solution.

(b) the phrase "similar expression" was often overlooked;

(c) Most candidates who got the first two parts correct, managed to make the necessary links to solve part c also. Most of the errors came from algebraic slips, rather than not knowing how to integrate the expression.

Question 6

This was a question that expected the candidates to apply their understanding of the links between an integral and areas. Though many were unable to start the question it was pleasing to see that plenty of candidates scored full marks.

Question 7

This was one of the better done questions. Several correct approaches were used to find AD. A common mistake was to assume that the angle at D was a right angle.

Question 8

Many incorrectly used $a = \frac{dv}{ds}$. Another common mistake was to substitute 50cm rather than 0.5m

Question 9

Most candidates recognized that this was implicit differentiation in (a). A common error was to give the derivative of $\arctan\left(1+x^2\right)$ as $\frac{2x}{1+x^2}$

(b) Hardly any realised that the value of *y* had to found in (b). Few realized they needed to use the formula for tan(A+B), and even when this was done few of these managed to complete the algebra successfully.

Question 10

Those who spotted they needed to square the given expression often managed to use trigonometric identities correctly to achieve full marks on this question.

Question 11

There was plenty of good wok to be seen in this question, which was often well presented and easy to follow, and most candidates coped well with both logs and exponentials. In (e) many were able to find the integral of $\frac{\ln x}{x}$ either by substitution (and changing the limits) or by parts.

Question 12

Part (a). A large number of candidates did not realise all the conditions which were needed to prove the quadrilateral was a square, while others spent a page showing everything which they could think of - this is one of several occasions in the paper where thought before starting the question is needed;

(b) was well done;

(c) most attempted the vector product approach, but some forgot they needed to show that the equation was equal to zero.

(d) Many students began their answer with ‘ $L =$ ‘, which lost one of the available marks

(e) This is a standard technique and was well done;

(f) This part was generally poorly done. The majority of those who were successful calculated the parameter needed in the equation of the line to find the image, but others used the fact the coordinates of the mid-point are the average of those of the point and its image.

(g) This was a straight forward question and was largely well done. A common error was to find the angle between $\overrightarrow{OD}$ and $\overrightarrow{AD}$

Question 13

(a) was often well done and manipulation of complex numbers was generally sound;

(b) candidates managed to substitute into the correct formula but struggled to find the value of $(1+i)^{20}$

(c)(i) and (d) In these parts many candidates tried to prove the sequence was geometric by considering the first few terms, rather than the general term, and so scored no marks.

In (d) the modulus sign was often ignored.

Recommendations and guidance for the teaching of future candidates

All schools need to be fully aware of the syllabus changes.

Candidates need to be aware that spending more time on the earlier questions is often more profitable than rushing in order to attempt all the questions. The later questions in each section are intended to be discriminators for the level 6/7 candidates.

Students should realize that proving a sequence to be geometric should either entail finding a formula of the form $u_n = u_1 r^{n-1}$ or showing the result of dividing two successive general terms, for example u_n and u_{n+1}, is a constant value.

Students frequently did well in the standard parts of the paper but failed to adapt to unfamiliar situations. Teachers should emphasise the teaching of thinking rather than simply doing past paper and text book questions.

There was evidence in the papers that some schools were giving a lot more time to certain parts of the syllabus (calculus in particular) at the expense of other parts. All sections of the syllabus should be taught in line with the guidance in the Higher Level guide.

Further comments

- Think about what is wanted in a question before embarking on the solution to a question - encourage some sort of logical thought and presentation.
- Know what information is available in the Formula Booklet
- Don't argue from particular cases to the general e.g. Q13.
- Correct method must be shown before any answer marks can be gained e.g. Q3; likewise M marks can be picked up even if a question is not completed e.g. Q10

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 48	49 - 63	64 - 77	78 - 92	93 - 120

The areas of the programme and examination which appeared difficult for the candidates

It was noted that in general the students seemed better prepared for this exam than in previous years. There were, however, certain areas where their performance was surprisingly poor. This was particularly noticeable with statistics questions, where many students were unable to answer any of the statistics questions. It was also noted that a number of students had not been prepared for some of the new additions to the syllabus – most notably the sum and product of roots. Students also seemed ill prepared for answering questions where reasoning needed to be shown. They often seemed unable to explain their reasoning adequately. It was also noted that, more than in previous years, the neatness and laying out of work was poor, often leading to unnecessary errors. Although we were quite lenient on this occasion, the care with which some graphs were sketched was often quite poor, failing to show features which could be important.

The areas of the programme and examination in which candidates appeared well prepared

Students generally appeared well prepared for calculus questions, usually able to successfully perform differentiation and integration tasks. There appeared to be a significant improvement in the use of calculators evident on this paper, although a surprising number of candidates did not realise that the use of a calculator would have been more appropriate for question 5(b).

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

A large proportion of students obtained full marks on this question. However, it was often not efficiently answered, with candidates often taking a longer algebraic route than necessary.

Question 2

Whilst many students could easily get full marks on this question, there were a surprising number that were attempting to use the formula for a normal distribution function or not able to start the question at all. It was clear that in these cases, the topic had not been taught.

Question 3

Many good attempts at this question and many students had full or partial marks. It was common to add rather than to multiply combinations.

Question 4

Many very poor attempts at this question. Many students who did work through part (a) correctly were unable to reason the second part.

Question 5

Part (a) was generally well done although many marks were lost in part (b). The usual errors were to subtract and square the functions, rather than square and subtract. A surprising number of students having written down the correct integral were unable to successfully obtain the correct answer from their calculators.

Question 6

Part (a) caused a lot of problems for the candidates, who frequently started by dividing by x. A variety of strategies were possible for answering this question, but students were frequently unclear as to how they might begin. Many students obtained full marks for part (b) although far too often by the laborious method of multiplying out the brackets.

Question 7

Many students lost a mark for only attempting to prove for $n \in \mathbb{Z}^+$. Otherwise there were many errors in both the proof, and the procedures followed.

Question 8

Both parts (a) and (b) had a good selection of good answers, although surprisingly few that did both parts successfully. The Binomial coefficients were sometimes missing from either part. Very few were able to see the connection between the two parts.

Question 9

There were many good answers to part (a) whereas, unsurprisingly, only the best candidates were able to obtain full marks on part (b).

Question 10

There were many good answers to all parts of this question, with the worst responses being for (b)(ii). It was common for candidates not to appreciate the reasoning required to show that there was only one solution. It needs to be made clear that a graph showing part of a function would be insufficient to show that there are no further solutions. For part (a) most students were able to find the vertical asymptote, but there were far fewer correct answers for the horizontal.

Question 11

This question was frequently well answered although it was quite common to be making arithmetical errors. There were some students who did not know how to approach the question at all.

Question 12

There were very many pleasing responses to this question. Candidates were successful in their approach to this unfamiliar situation.

Recommendations and guidance for the teaching of future candidates

Firstly it must be made clear that all parts of the syllabus are adequately covered. This is especially true of new additions to the syllabus that might not appear in the text book or previous year's curriculum. Calculator use needs to be developed, so that where there is a quick and easy calculator method to solve a problem, students are accustomed to do that, rather than embark on lengthy algebraic procedures. This is perhaps part of a more general recommendation that students should be given more opportunities for choosing methods of approaching problems so that they are more accustomed to selecting the most efficient method to a solution, rather than embarking on lengthy procedures. This was a problem evident in different ways in most questions in the paper.
Teachers should also help students to keep their work neat and orderly. It was very evident that many candidates were not used to providing clear solutions to problems.

Paper three - Discrete mathematics

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 14	15 - 21	22 - 26	27 - 32	33 - 37	38 - 60

The areas of the programme and examination which appeared difficult for the candidates

The candidates did not seem particularly comfortable with recurrence relations. As this is new to the syllabus, I thought that teachers would have made a point of covering it well. As in the past candidates found it more difficult to come up with proofs themselves rather than just applying algorithms that they knew.

The areas of the programme and examination in which candidates appeared well prepared

The upper and lower bound algorithms for the travelling salesman problem were well known, as were the methods to convert to different bases.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a)This was well drawn. (b) Generally answered quite well. There was too much confusion with the twice a minimal spanning tree upper bound method. Some candidates forgot to go back to D. (c) Generally good answers. Some candidates forgot to add on the 2 smallest edges into A.

Question 2

(a) (i) Most candidates knew one of the two methods. Some did not realise that 11 was B in base 13. A few very weak candidates thought that the numbers given were already in base 13. (ii) This was badly answered as most candidates ignored the "Hence" in the question and just applied Euclid's algorithm to the original base 10 numbers. A few candidates did read carefully and saw what to do.

(b) The responses were variable and some candidates ignored it. A common mistake was to assume particular numbers for the elements of set L. Those candidates whose first thought was the pigeonhole principle were quite reasonable.

(c) (i) This was well done with just minus sign slips from the candidates with poor algebra. (ii) Answers were variable. Too many candidates did not read about mod 2. Initial conversion made the system of equations easy. Not enough candidates realised that if they were not initially working mod 2 then they could solve the system with their calculator rather than slogging it out. Often the answer given was not converted to mod 2.

(d) (i) Reasonably well done. Some explanations could have been clearer. Unfortunately a few candidates thought that a few examples would suffice. (ii) This was well done. (iii) Either candidates saw the counterexample to select or they did not.

Question 3

(a) This was reasonably well done but too many candidates did not read "Draw a spanning tree" and thus just drew K_4 and $K_{4,4}$.

(b) This was not well answered. Insufficient candidates realised that you had to apply the pigeon-hole principle. It was unfortunate that candidates thought that a few examples would suffice. Others just wrote down things that they knew about graphs and claimed that these proved the result.

(c) This was very badly answered indeed. Candidates either just gave some examples or said that it was true because it was obvious. It required careful thinking to describe how you obtained the spanning tree.

Question 4

Since solving a recurrence relation is essentially standard bookwork for the syllabus I was surprised that candidates did not do better in this question.

(a)(i) This could either have been done by realising it was a geometric progression or using the auxiliary equation. (ii) Far too many candidates did not use the suggested solution and just substitute it in. (iii) Not too many marks were gained here as many candidates had gone wrong earlier.

(b) Solving the auxiliary equation should have been standard but too many candidates did not achieve this. Putting the answer into the format required was more challenging as you would expect for the last part of the last question. I like the thinking of one of the candidates that did achieve this who then wrote "that was cool".

Recommendations and guidance for the teaching of future candidates

Although this option involves graphs and trees there was no need for candidates to use graph paper for some of their answers! It made it more difficult to read the answers of candidates that did this with the papers being scanned. Candidates lost marks by not reading carefully enough what the question actually said and using the hints in the wording of the questions. If a candidate introduces a variable that is not given in the question then they need to say what it stands for so that the examiner can follow their working. They have to remember that they are trying to communicate to the examiner so careful use of words and diagrams can only assist them. Candidates need to be prepared for proofs as well as algorithms and know that "waffly" words rarely gain many marks. Looking at the structure of proofs on the mark-schemes of previous exams will help. For example, you cannot start with what you are trying to prove and examples are not proofs. I cannot really emphasis those last two points enough and we should all be getting this message across. With many of the points mentioned above, careful corrective marking of a trial exam should have assisted the candidates, if they were prepared to learn. It is important that the whole syllabus is covered in the teaching.

Paper three - Calculus

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 19	20 - 26	27 - 33	34 - 40	41 - 47	48 - 60

The areas of the programme and examination which appeared difficult for the candidates

Most of the candidates faced difficulties in Rolle's theorem. In some cases it was clear that the statement of this theorem was unknown to the students. Other candidates knew that they should consider functional values at the endpoints of a given interval but nothing more. Overall there was evidence of lacking of understanding of the Rolle's theorem and ability of its application.

Other areas that caused difficulties were the use of integrating factors to solve a linear differential equation and testing end points of the interval for convergence of the power series. Many of the candidates also faced difficulties in the evaluation of the improper integral. Most candidates also seemed unaware of the need of adjusting the limits of integration when changing variables.
Surprisingly many candidates had difficulties in using the graphics calculator to produce a reasonable sketch graph with the required information on it.

In general the mathematical communication was poor and many candidates seemed unfamiliar how to deal with command terms ‘show’ and ‘hence’.

The areas of the programme and examination in which candidates appeared well prepared

Students seemed to do well with basic concepts such as derivatives, limits, integrals, sequences and series. Most candidates attempted questions 1, 2 and 3 showing that they were familiar with the topics and could at least start the questions. Maclaurin expansion seemed a well prepared topic on the whole and most candidates were familiar with the use the ratio test to find the radius of convergence.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Generally many candidates could attempt Q1(a) but many students wrote down the results that were already given in the question and did not show any relevant step.

Part (b) was attempted by almost all candidates and was well done by most of them. Most candidates deduced the expansion from first principles but in a few cases candidates used the expansion of the exponential function to deduce the result. This latter approach resulted in more calculation errors.

In part (c) many candidates ignored the instruction ‘hence’ and tried to find the limit using L’Hôpital rule. The wrong answer of $\frac{1}{2}$ was seen very often due to calculation errors, namely incorrect removal of brackets.

Part (d) caused difficulties to many candidates that failed to spot an adequate substitution. Among the students that did use substitution, very few recognized the need of calculating the lower limit of integration leading in many cases to incorrect an answer. This part of the question also showed the difficulties candidates had in setting out their work properly, using correct notation. It was surprising to see how many candidates failed to recognize the improper nature of the integral and made no attempt to study its convergence.

Question 2

Overall, this question was well attempted. In part (a) most candidates could find the derivative but some failed to use the chain rule correctly in part (ii) A surprising number of candidates failed to relate increasing function with positive gradient function in the given interval.

In part (b) (i) most candidates who could identify the differential equation to be linear wrote it in standard form, many could find the integrating factor correctly and then solve the exact equation obtained, although a few candidates lost marks at the end due to careless errors. A number of candidates however treated the differential equation as homogeneous and wasted time trying to solve it using substitution.

In part (b) (ii) a number of candidates lost marks because they simply verified that the function given was a solution of the differential equation satisfying the initial conditions given.

In part (b)(iii) most candidates managed to sketch the graph but in many cases the sketches were not well labeled, showed extraneous asymptotes and incorrect values for the minimum point. Many candidates also ignored the domain and sketched the graph of the expression for $\mathrm{x} < 1$.

Question 3

In part (a) the majority of the candidates could find the expression for b(n) and c(n) correctly. Unfortunately, several candidates lost a mark in part a) because they did not answer the question asked, stating an interval instead of the radius of convergence.

In general part (b) was well answered with almost all knowing how to start and what to do. However, some candidates did not use the ratio test properly to determine the convergence of the power series and a few of them apparently did not realize that convergence of the series depend upon the values of x.

Many candidates could not discuss mathematically the convergence at $x = \pm 1$. Also in part (c), many candidates did not fully justify the use of the alternating series test

Question 4

Part (a) was well attempted and there was a range of marks scored**.** Some candidates did not realize that exact answers were required and attempted to use GDC to answer the question and lost marks for accuracy This was another question where it was evident the difficulty that many candidates had in setting out their work in a logical way.

Part (b) was a challenging question for most candidates. Some just did not understand or know how to use Rolle's theorem. Many left it blank or made random attempts to use theorems from the course. Attempts to use Bolzano’s and Mean value theorem were seen very often. Very few candidates obtained full marks in this part of the question.

Part (b) also showed that many candidates were not aware of the implications of the continuity and differentiability of a function on the behavior of its graph over the interval given. Like in previous questions many candidates ignored the instructions ‘prove that’ and ‘hence’ and attempted to answer the questions using GDC.

Recommendations and guidance for the teaching of future candidates

- Require that students set out all their work using appropriate notation and terminology and that course work includes answering the questions in a logical and clear way.
- Emphasise the need of showing working out and presenting it clearly and neatly. Many students arguments were difficult to follow and very untidy.
- Simple algebraic/numerical errors can have serious grade allocation consequences and students do need to be reminded that they must double check their steps to avoid simple careless errors.
- Recognise and follow instructions associated to IB command terms (eg ‘hence’, ‘prove that’ and ‘show that’).
- Teachers should ensure that their students are very comfortable in differentiating and integrating when starting this option, including knowing and understanding well integration by parts and by substitution and recognize easily when to apply these techniques.
- Provide a wide range of examples about the relations between behavior of functions and their derivatives, including piecewise functions and functions with restricted domains rather than the largest possible domain of their expressions.
- Teach students how to approach improper integrals in a proper way.
- Clarify methods to solve differential equations: students need to be able to recognise the type of equation before trying to apply specific methods to solve them.
- Emphasise the need of studying the endpoints in detail when establishing the convergence interval for a power series and ensure students know the difference between a radius and interval of convergence.
- Explore in more detail continuity and differentiability and associated theorems and stress the importance of fully justifying the conditions of a theorem before applying the theorem.
- Whilst there were some very well prepared students, it was also evident that some candidates scored very few marks. Teachers need to clarify the expectations of the DP mathematics courses and guide students to choose the appropriate level.

Paper three - Sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 21	22 - 27	28 - 32	33 - 38	39 - 60

The areas of the programme and examination which appeared difficult for the candidates

This is an option where proof and justification are most important. There was little evidence of an appreciation of this except at the higher levels. Many candidates had difficulty deciphering the standard terminology defining a subset in terms of a condition imposed on the elements of an overall set. This was evident both in understanding questions and in expressing answers. Many candidates were very hazy about the new syllabus concepts of ‘homomorphism’, ‘kernel’ and ‘cosets’. Many candidates had difficulty with the notion that a Cartesian product could involve both continuous and discrete factors.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates were happy working with Cayley tables and extracting the required information. The definition of a group was well understood. The generalities of equivalence relations was well understood.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates scored well on this question.

(a) Hard to go wrong.

(b) Generally well done. A very small minority confused commutativity with associativity.

(c) Generally well done, but one was sometimes left with the lingering doubt whether the candidate really got it. There was sometimes an invalid argument that was based on cancellation – we do not have a group, so one cannot cancel at will.

(d) Generally well done.

(e) Generally well done, but sometimes the examiner was expected to extract the answer from a mass of data.

Question 2

(a) (i) All examiners commented on the astonishing inability of many of the candidates to correctly answer this part. Twice a number is an integer means that the number is half an integer. Clearly a misunderstanding of set notation is an issue.

(b) (i) There was the feeling that many candidates cannot appropriately translate a concept into simple algebra. So aRb => bRa becomes aRb = bRa, which makes no sense. The notion of a symmetric relation was poorly handled.

Question 3

Many candidates were not comfortable with the concept of a Cartesian product, and certainly not with the ability of visualizing and handling such sets.

Question 4

This was a bookwork question straight off the syllabus. Many candidates were not familiar with the concepts of kernel and coset.

Recommendations and guidance for the teaching of future candidates

This is an option where concepts and understanding are more important than the manipulative ability that is required. Ensure that candidates know this and are up for that challenge. The set notation is key to this option, so make clear, by way of many examples, the various ways that sets can be defined both finite, infinite and several dimensional. Structured proofs are important, so emphasize this feature. Ensure that candidates write clearly, particularly when diagrams are involved. The examiner cannot read the mind of the candidate, so the candidate must make clear that they understand what they are writing in response to the question.

Ensure that all items in the syllabus are covered.

Paper three - Statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 15	16 - 21	22 - 27	28 - 33	34 - 39	40 - 60

The areas of the programme and examination which appeared difficult for the candidates

Many candidates made parts of Question 2 much longer than necessary by not using the calculator software to the full. It makes no sense to find correlation coefficients, p-values and equations of regression lines by using the calculator to find Σx etc and then calculating these other quantities using the appropriate formulae. Candidates need to be aware of the full capability of the statistics menu on their calculator.

Some candidates seemed unsure about handling probability generating functions. It is important to be aware of the several definitions of the probability generating function so that the most appropriate one can be chosen to solve a particular problem. The notion of unbiased estimation seems not to be understood by many candidates.

The areas of the programme and examination in which candidates appeared well prepared

Despite the comments in the section above, candidates seem to understand the concepts of correlation and regression fairly well.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Part (a)(i) was correctly answered by most candidates. In (a)(ii), however, a not uncommon error was to state that $\mathrm{P}(5 \le X \le 8) = \mathrm{P}(X \le 8) - \mathrm{P}(X \le 5)$. Part (b) was well answered by many candidates. Part (c)(i) was well answered in general with almost all the candidates using the Central Limit Theorem. It was surprising to note that very few candidates converted the probability to

$\mathrm{P}(284 < \Sigma X < 340)$ which could then be evaluated as a Poisson probability. Many candidates failed to see how to solve (c)(ii).

Question 2

Most candidates stated the hypotheses correctly although candidates who failed to mention p were penalised. It was disappointing to see that many candidates, by choosing the wrong menu on their calculator, involved themselves in lines of arithmetic in answering (b), (c) and (d). A correct choice of software would have given the required results immediately. Part (f) was poorly answered in general with many candidates having no idea how to proceed. Many candidates wrote the regression line of x on y as $y = 0.409x - 12.2$ instead of $x = 0.409y - 12.2$ so that their gradient was incorrect. The incorrect answer $38°$ was therefore seen more often than the correct answer $7°$.

Question 3

Part (a) was not well answered in general with many solutions not even containing any expectation signs. Part (b) was reasonably well answered although not many candidates ended up with the correct expression for $\mathrm{E}(Y)$. Surprisingly, very few candidates realised that the algebra could be made easier by using the substitution $t = y - \lambda$. It was disappointing to note in (b)(i) that, although most candidates realised that $\int_{-\infty}^{\infty} f(y)\mathrm{d}y$ had to equal 1, very few candidates realised that they also had to show that $f(y)$ had to be non-negative over the appropriate range.

Question 4

Parts (a) and (b) were well answered by many candidates. Parts (c) and (d), however, proved difficult for most candidates with only a minority taking the easier route of defining a probability generating function in the form $\mathrm{E}(t^X)$ as opposed to $\sum p_x t^x$.

Recommendations and guidance for the teaching of future candidates

Candidates should be made aware of the full capability of the statistics menu on their calculator. Candidates should be familiar with the definitions and applications of probability generating functions.