MATHEMATICS HL TZ2

(IB Africa, Europe & Middle East & IB Asia-Pacific)

Overall grade boundaries

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 40	41 - 51	52 - 64	65 - 75	76 - 100

Series and differential equations

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 41	42 - 53	54 - 65	66 - 77	78 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 41	42 - 53	54 - 65	66 - 76	77 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 39	40 - 51	52 - 63	64 - 75	76 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2013 examination session the IB has produced time zone variants of Mathematics HL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 18	19 - 23	24 - 29	30 - 34	35 - 40

The range and suitability of the work submitted

There were many well-produced portfolios this session that provided strong evidence of self-directed mathematical investigations. Generally, good writing skills were noted in the presentation of the students' work. However, there appeared to be an increase in the number of documents that were extremely lengthy, containing an excess of repetitive screen captures and un-annotated spreadsheet listings extending over several pages.

The assessment criteria appear to have been well understood by teachers, who often produced their own marking rubrics to assess their candidates' work in a consistent manner. Observations made by the moderating team are summarised below.

The tasks:

Nearly all of the portfolios contained tasks that were taken from the publication, "Tasks for use in 2012 and 2013", with the two most popular being "Shadow Functions" and "Dice Games". It would appear that the Subject Reports from the past two sessions have been ignored by some teachers, as the task, "Patterns from Complex Numbers", was included in many samples with some students (and teachers) still misreading the instruction and producing a familiar and trivial consequence of De Moivre's theorem: that consecutive n-th roots of unity, when connected, form a regular polygon. Significant deductions were made for such conclusions. Please note the advice given in the Subject Reports for May 2012 and November 2012.

There were only a very small number of teacher-designed tasks submitted. As it was noted in the previous Subject Report, the continued and repeated use of some teacher-designed tasks is of some concern. Tasks such as "Lionel (the dog)", "Ziggurats", and "Pipelines" have been reused time and time again over the past decade. Schools that continue to use the same tasks each year, regardless of their presumed non-expiring status, are ignoring the risk of plagiarism as solutions are easily available, not only online, but through their former students.

Candidate performance against each criterion

The candidates performed well against criterion A, although the occasional student used calculator notation for multiplication or exponentiation, or did not use standard subscript notation as required. The use of correct probability notation was not always present.

Most students produced very well written pieces of work with thorough explanations. However, missing introductions and unlabelled graphs were still in evidence. Some student work, though correct, was far from concise, with a number of portfolios whose lengths well exceeded those of Extended Essays! As it was noted in the previous Subject Report, moderators were overwhelmed and under-impressed with the repetitive and excessive number of pages of similar graphs and seemingly unending spreadsheet listings.

The candidates generally produced good work and the teachers have assessed the work well against criteria C and D. However, it was disappointing to note that in some type I work, the investigation was extremely limited, with little evidence of a pattern to warrant a conjecture, let alone a generalisation. It would appear that some students started with a prior notion of the end result without investigation.

Some candidates did not pay enough attention in their type II task to the requirements in criterion D, and generated results to an inappropriate degree of accuracy, such as in the winnings in a casino game or the cost of petrol to six decimal places.

The use of technology varied somewhat. Full marks were often given much too generously for the use of a large collection of similar graphs. Also, should a task such as “Dice Games” assume that the probabilities be limited to positive values, the graphs should have been so adjusted to reflect those values.

Resourceful uses of technology were noted in the form of graphs with sliders to adjust parameter values, and the use of conditional statements in dynamic spreadsheets. Some students used graphing software such as GeoGebra to show graphs of complex numbers.

There were many pieces of good and complete work; however, the awarding of full marks in criterion F requires evidence of mathematical sophistication that extends beyond the mere completion of the task.

Recommendations and guidance for the teaching of future candidates

Perhaps a look at transformational geometry may be in order prior to approaching a task like “Complex Numbers”, wherein the basic understanding that distances are invariant under rotation would have simplified the students’ approach to their analysis greatly.

The suggestions below have been offered to teachers in the last Subject Report, but may be worth noting again:

Teachers are expected to write directly on the work submitted, not only to provide feedback to their candidates, but information to the moderators as well. The use of Form B is suggested to allow the teacher to indicate more relevant and descriptive comments.

There has been a noticeable improvement in the provision of background information to each portfolio task that is required to accompany each sample, particularly with the use of Form A or through anecdotal comments. Moderators find them very useful in determining the context in which each task was given when confirming the achievement levels awarded.

A solution key for each task in the sample must accompany the portfolios in order that moderators can justify the accuracy of the work and understand the teacher’s expectations. Where there is more than one HL teacher involved in marking the portfolio work, the use of a common marking scheme has been effective.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 33	34 - 49	50 - 62	63 - 74	75 - 87	88 - 120

The areas of the programme and examination which appeared difficult for the candidates

Many candidates were unable to use convincing reasoning to link algebraic manipulations. Often it seemed that examiners were expected to read the mind or guess the intentions of a candidate.

Many candidates struggled with questions involving complex numbers.

The majority of candidates were unable to factorize a cubic polynomial even when there was an ‘obvious’ root of the associated equation.

Where a question asks for a displayed answer, some candidates seem unaware that their working towards the answer needs to be convincing.

The areas of the programme and examination in which candidates appeared well prepared

Manipulation of vectors and matrices; basic calculus; tree diagrams and probability; finding an inverse function; application of the binomial theorem.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Generally well done, although quite a number of candidates were either unable to integrate the sine term or incorrectly evaluated the resulting cosine at the limits.

Question 2

Very well done. Although there were a few candidates who lost a mark or two because of arithmetic mistakes in calculating the matrix product.

Question 3

Generally well done. The majority of candidates obtained a quintic with correct alternating signs. A few candidates made arithmetic errors. A small number of candidates multiplied out the linear expression, often correctly.

Question 4

Generally well done. A few candidates didn’t take account of the fact that Caz ate the chocolate, so didn’t replace it. A few candidates made arithmetic errors in calculating the probability.

Question 5

The majority of candidates earned significant marks on this question. The product rule and the quotient rule were usually correctly applied, but a few candidates made an error in differentiating the denominator, obtaining $-\sin x$ rather than $1-\sin x$.A disappointing number of candidates failed to calculate the correct gradient at the specified point.

Question 6

Almost all candidates obtained the cubic equation satisfied by the common ratio of the first sequence, but few were able to find its roots. One of the roots was $r = 1$.

Question 7

(a) Disappointingly, few candidates obtained the correct argument for the second complex number, mechanically using arctan(1) but not thinking about the position of the number in the complex plane. (b) Most candidates obtained the correct quadratic or its square root, but few knew how to set about minimising it.

Question 8

Most candidates were familiar with the concept of implicit differentiation and the majority found the correct derivative function. In part (b), a significant number of candidates didn't realise that the value of x was required.

Question 9

(a) This is a question where carefully organised reasoning is crucial. It is important to state that both the numerator and the denominator are positive for $x > 0$. Candidates were more successful with part (b) than with part (a).

Question 10

(a) Those candidates who used the addition formula for the tangent were usually successful. (b) Some candidates left their answer as the tangent of an angle, rather than the angle itself.

Question 11

Many candidates confidently tackled most of this many-part question. Part (b)(i) As the answer was given, candidates were required to show they really knew how to work out a vector product in detail, not just by writing down a 3x3 determinant and then the final answer. Part (d) A few candidates failed to realise that the equation of a line is an equation not simply an expression. Part (e) A significant number of candidates did not realise that they could use their result for part (d).

Question 12

(a) Generally well done. In their answers to Part (b), most candidates found the derivative, but many assumed it was obviously positive. Part (d)(i) Generally well done, but some candidates failed to label their final expression as $f^{-1}(x)$. Part (d)(ii) Marks were lost by candidates who failed to mark the intercepts with values. Marks were also lost in this part and in part (e)(i) for graphs that went beyond the explicitly stated domain.

Question 13

(a)(i) A disappointingly large number of candidates were unable to give the correct arguments for the three complex numbers. Such errors undermined their efforts to tackle parts (ii) and (iii). (b) Many candidates were successful in part (i), but failed to capitalise on that – in particular, few used the fact that roots of $z^7 - 1 = 0$ come in complex conjugate pairs.

Recommendations and guidance for the teaching of future candidates

Encourage students to back up their purely algebraic work with commentary and reasoning.

Emphasise the geometrical representation of complex number in the complex plane (Argand Diagram).

Further comments

This was a very accessible paper, and the majority of candidates seemed well prepared for carrying out the mechanical algebraic aspects of the questions, but struggled with reasoning.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 39	40 - 54	55 - 69	70 - 84	85 - 120

General comments

The paper appeared to be accessible, and of appropriate length. The majority of the candidates demonstrated good knowledge of the course material and ability to apply that knowledge to answer the exam questions.

The areas of the programme and examination which appeared difficult for the candidates

GDC-required examination questions, applying the properties of the normal distribution, integration by substitution, analysis of an expression where the independent variable is discrete, solving trigonometric equations in a finite interval, sketching a continuous probability density function, proof by mathematical induction, conditional probability, solving differential equations involving kinematics, representing a sum using sigma notation, verifying a specific solution to a differential equation, finding the general solution to a differential equation, determining the range of a function, formulating and differentiating expressions involving inverse trigonometric functions, sketching graphs accurately and applying the chain rule to rates of change.

The areas of the programme and examination in which candidates appeared well prepared

Calculating the area of a sector and a triangle, using a GDC to solve a system of linear equations, integration by parts, arithmetic and geometric sequences and series, simple applications of the Poisson and binomial distributions, using definite integrals to calculate area and volume and using the first derivative in an optimization problem.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This was generally well done. In part (a), a number of candidates expressed the required angle either in degrees or in radians stated to an incorrect number of significant figures. In part (b), some candidates demonstrated a correct method to calculate the shaded area using an incorrect formula for the area of a sector.

Question 2

This was generally well done. In part (a), some candidates expressed the system of equations in the form $XA = B$. In part (b), the overwhelming majority of candidates who used a direct GDC approach obtained the correct solution. Candidates who attempted matrix methods such as row reduction without a GDC were generally unsuccessful.

Question 3

A large proportion of candidates experienced difficulties with this question. In parts (a) and (b), the most common error was to use $\sigma = 9.5$. In part (a), a large number of candidates used their range of values to then unnecessarily find the corresponding probability of that time interval occurring. In part (b), a large number of candidates used an unrealistic lower bound (a large negative value) for time.

Question 4

In part (a), a large number of candidates were able to use integration by parts correctly but were unable to use integration by substitution to then find the indefinite integral of tan x. In part (b), a large number of candidates attempted to solve the equation without direct use of a GDC's numerical solve command. Some candidates stated more than one solution for m and some specified m correct to two significant figures only.

Question 5

In part (a), most candidates were able to express u_n and v_n correctly and hence obtain a correct expression for $u_n - v_n$. Some candidates made careless algebraic errors when unnecessarily simplifying u_n while other candidates incorrectly stated v_n as $3(1.2)^n$. In parts (b) and (c), most candidates treated n as a continuous variable rather than as a discrete variable. Candidates should be aware that a GDC's table feature can be extremely useful when attempting such question types. In part (c), a number of candidates attempted to find the maximum value of n rather than attempting to find the maximum value of $u_n - v_n$.

Question 6

Part (a) was generally well done. Some candidates did not follow instructions and express their final answer correct to the nearest degree. A large number of candidates successfully employed a graphical approach. Part (b) was not well done. Common errors included attempting to solve for x rather than for sec x, either omitting or not considering $\sec x = -\sqrt{2}$, not rejecting $\sec x = \pm\sqrt{\frac{2}{3}}$ and not working with exact values.

Question 7

Part (a) was generally well done. Common errors usually involved not recognizing that the sum of the two integrals was equal to one, premature rounding or not showing full working to conclusively show that $a = 9.6$. Part (b) was not well done with many graphs poorly labelled and offering no reference to domain and range. Part (c) was reasonably well done. The most common error involved calculating an incorrect probability from an incorrect definite integral.

Question 8

This proof by mathematical induction challenged most candidates. While most candidates were able to show that P(1) = 0, a significant number did not state that zero is divisible by 576. A few candidates started their proof by looking at P(2). It was pleasing to see that the inductive step was reasonably well done by most candidates. However many candidates committed simple algebraic errors. The most common error was to state that $5^{2(k+1)} = 5(5)^{2k}$. The concluding statement often omitted the required implication statement and also often omitted that P(1) was found to be true.

Question 9

Part (a) was generally well done although a number of candidates added the two probabilities rather than multiplying the two probabilities. A number of candidates specified the required probability correct to two significant figures only. Part (b) challenged most candidates with only a few candidates able to correctly employ a conditional probability argument.

Question 10

Most candidates experienced difficulties with this question. A large number of candidates did not attempt to separate the variables and instead either attempted to integrate with respect to v or employed constant acceleration formulae. Candidates that did separate the variables and attempted to integrate both sides either made a sign error, omitted the constant of integration or found an incorrect value for this constant. Almost all candidates were not aware that this question could be solved readily on a GDC.

Question 11

In part (a) (i), a large number of candidates were unable to correctly use sigma notation to express the sum of the first n positive odd integers. Common errors included summing $2n - 1$ from 1 to n and specifying sums with incorrect limits. Parts (a) (ii) and (iii) were generally well done.

Parts (b) (i) and (iii) were generally well done. In part (b) (iii), many candidates unnecessarily simplified their quadratic when direct GDC use could have been employed. A few candidates gave $n >$ 1416 as their final answer. While some candidates displayed sound reasoning in part (b) (ii), many candidates unfortunately adopted a ‘proof by example’ approach.

Part (c) was generally well done. In part (c) (ii), some candidates multiplied the two probabilities rather than adding the two probabilities.

Question 12

Part (a) was not well done and was often difficult to mark. In part (a) (i), a large number of candidates did not know how to verify a solution, $y(x)$, to the given differential equation. Instead, many candidates attempted to solve the differential equation. In part (a) (ii), a large number of candidates began solving the differential equation by correctly separating the variables but then either neglected to add a constant of integration or added one as an afterthought. Many simple algebraic and basic integral calculus errors were seen. In part (a) (iii), many candidates did not realize that the solution given in part (a) (i) and the general solution found in part (a) (ii) were to be equated. Those that did know to equate these two solutions, were able to square both solution forms and correctly use the trigonometric identity $\sin 2x = 2\sin x \cos x$. Many of these candidates however started with incorrect solution(s).

In part (b), a large number of candidates knew how to find a required area and a required volume of solid of revolution using integral calculus. Many candidates, however, used incorrect expressions obtained in part (a). In part (b) (ii), a number of candidates either neglected to state 'π' or attempted to calculate the volume of a solid of revolution of 'radius' $f(x) - g(x)$.

Question 13

Part (a) was reasonably well done. While many candidates exhibited sound trigonometric knowledge to correctly express θ in terms of x, many other candidates were not able to use elementary trigonometry to formulate the required expression for θ. In part (b), a large number of candidates did not realize that θ could only be acute and gave obtuse angle values for θ. Many candidates also demonstrated a lack of insight when substituting endpoint x-values into θ. In part (c), many candidates sketched either inaccurate or implausible graphs. In part (d), a large number of candidates started their differentiation incorrectly by failing to use the chain rule correctly. For a question part situated at the end of the paper, part (e) was reasonably well done. A large number of candidates demonstrated a sound knowledge of finding where the maximum value of θ occurred and rejected solutions that were not physically feasible. In part (f), many candidates were able to link the required rates, however only a few candidates were able to successfully apply the chain rule in a related rates context.

Recommendations and guidance for the teaching of future candidates

- Raise awareness of the importance of basic graph sketching skills including careful consideration of domain, range and key features.
- Raise awareness for the need to support answers with accurate sketches and/or indication of the method even when the answers can be obtained with GDC.
- Encourage candidates to use GDC to solve equations and integrate numerically; provide a wide range of problems that allow students to explore more advanced features of the GDC including a GDC's table feature.
- Encourage candidates to store numerical answers obtained with GDC or show how to carry work through using enough significant figures so that the final answers are correct to the appropriate degree of accuracy.
- Emphasize what it means to convincingly answer 'show that' examination questions and provide many examples of mathematical proofs.
- Clarify the meaning of each of the command terms in the Mathematics HL guide.
- Provide a wide range of probability questions in context and clarify the definition of a continuous probability density function.
- Increase awareness of what constitutes a correct concluding statement to a proof by mathematical induction.
- Provide many examples of past examination questions and teach efficient ways of answering these examination questions; provide timed practice to improve candidates' efficiency in answering examination papers.

Paper three - Discrete mathematics

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 17	18 - 27	28 - 33	34 - 40	41 - 46	47 - 60

The areas of the programme and examination which appeared difficult for the candidates

The candidates were less happy when they had to think and to create proofs for themselves e.g. Q.5, than when they were doing known algorithms. They did not seem to have practiced working in different bases much as shown by Q3.

The areas of the programme and examination in which candidates appeared well prepared

The candidates generally knew how to apply algorithms. The overall standard of the candidates was pleasing.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Part (a) was well answered and (b) fairly well answered. There were problems with negative signs due to the fact that there was a negative in the question, so candidates had to think a little, rather than just remembering formulae by rote. The lay-out of the algorithm that keeps track of the linear combinations of the first 2 variables is recommended to teachers. Part (c) was reasonably well done, not all candidates used the hint given and some did not read carefully enough that the smallest value was required.

Question 2

In part (a) the criteria for Eulerian circuits and trails were generally well known and most candidates realised that they must start/finish at G/E. Candidates who could not do (a) generally struggled on the paper. For part (b) the layout varied greatly from candidate to candidate. Not all candidates made their method clear and some did not show the temporary labels. It is recommended that teachers look at the tabular method with its backtracking system as it is an efficient way of tackling this type of problem and has a very clear layout.

Question 3

Part (a) was a good indicator of overall ability. Many candidates did not write both sides of the equation in terms of n and thus had an impossible equation, which should have made them realise that they had a mistake. Part (b) was not well answered and of those candidates that did, some only gave one side of the equation in base 7. The answers that were given in (a) and (b) could have been checked, so that the candidate knew they had done it correctly.

Question 4

Most candidates knew the method for part (a), a few did not. Part (b) was reasonably well answered with a variety of methods. Part (c) had generally good answers, some candidates in (ii) thought that the degree sequence being the same was sufficient for isomorphism. Part (d) was a good indicator of the overall ability of the candidate with good candidates giving a logical proof and poor candidates giving a single example.

Question 5

Only the top candidates were able to produce logically, well thought-out proofs. Too many candidates struggled with the summation notation and were not able to apply Fermat's little theorem. There was poor logic i.e. looking at a particular example and poor algebra. I was amused with the attempts at a proof by induction-starting with the prime $p=1$ and moving from prime k to prime $k+1$!

Recommendations and guidance for the teaching of future candidates

Although this option involves graphs and trees there was no need for candidates to use graph paper for some of their answers! It made it more difficult to read the answers of candidates that did this with the papers being scanned. If a candidate introduces a variable that is not given in the question then they need to say what it stands for so that the examiner can follow their working. Candidates need to be prepared for proofs as well as algorithms and know that "waffly" words rarely gain many marks. Looking at the structure of proofs on the mark-schemes of previous exams will help. For example, you cannot start with what you are trying to prove and examples are not proofs. With many of the points mentioned above, careful corrective marking of a trial exam should have assisted the candidates, if they were prepared to learn. It is important that the whole syllabus is covered in the teaching.

Paper three - Series and differential equations

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 11	12 - 22	23 - 30	31 - 37	38 - 45	46 - 52	53 - 60

The areas of the programme and examination which appeared difficult for the candidates

On this paper a significant number of candidates found difficulty in correctly applying Taylors Theorem. Candidates also found it difficult to sum an alternating series correctly to a given degree of accuracy. Similarly candidates were not confident in summing a telescoping series and managing the algebra to find a sum of such a series.

The areas of the programme and examination in which candidates appeared well prepared

A significant majority of candidates showed good knowledge of the conditions and technique required to find a limit using L'Hopital's rule. Most candidates were also confident in solving differential equations using Euler's methods and many candidates were able to solve a differential equation using an integration factor. Many candidates were able to find a radius of convergence and to verify the associated interval of convergence. Most candidates were able to make progress on a problem involving Riemann's Sum.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Many candidates achieved full marks on this question but there were still a large minority of candidates who did not seem familiar with the application of Taylor series. Whilst all candidates who responded to this question were aware of the need to use derivatives many did not correctly use factorials to find the required coefficients. It should be noted that the formula for Taylor series appears in the Information Booklet.

Question 2

Most candidates knew Euler's method and were able to apply it to the differential equation to answer part (a). Some candidates who knew Euler's method completed one iteration too many to arrive at an incorrect answer. Surprisingly few candidates were able to efficiently use their GDCs to answer this question and this led to many final answers that were incorrect due to rounding errors.

Most candidates were able to correctly derive the Integration Factor in part (b) but some lost marks due to not showing all the steps that would be expected in a "show that" question. The differential equation was solved correctly by a significant number of candidates but there were errors when candidates multiplied by $\sec x$ before the inclusion of the arbitrary constant.

Question 3

It was pleasing that most candidates were aware of the Radius of Convergence and Interval of Convergence required by parts (a) and (b) of this problem. Many candidates correctly handled the use of the Ratio Test for convergence and there was also the use of Cauchy's n^{th} root test by a small number of candidates to solve part (a). Candidates need to take care to justify correctly the divergence or convergence of series when finding the Interval of Convergence. The summation of the series in part (c) was poorly handled by a significant number of candidates, which was surprising on what was expected to be quite a straightforward problem. Again efficient use of the GDC seemed to be a problem. A number of candidates found the correct sum but not to the required accuracy.

Question 4

Nearly all candidates were able to correctly find the partial fractions required by part (a). In part (b) whilst a majority of candidates were able to recognise the telescoping series using partial fractions many were not able to manipulate them so as to correctly find the sum of the series. It was not clear whether this was a lack of algebraic fluency to a lack of familiarity with telescoping series. When the sum was found most candidates were able to find the limit as $n \to \infty$.

Question 5

Many candidates made progress with this problem. This was pleasing since whilst being relatively straightforward it was not a standard problem. There were still some candidates who did not use the definite integral correctly to find the area under the curve in part (a) and part (b). Also candidates should take care to show all the required working in a "show that" question, even when demonstrating familiar results. The ability to find upper and lower bounds was often well done in parts (a) (ii) and (b) (ii).

Recommendations and guidance for the teaching of future candidates

- Candidates need to use their GDC correctly to apply Euler's method and the summation of series.

- Candidates need to be aware of the degree of accuracy required by a problem.
- Candidates need to apply the necessary rigour and level of communication required by a problem asking a student to prove a statement or show that it is true.
- Candidates need to be aware of techniques such as using partial fractions to find a telescoping series and using algebraic techniques efficiently to find the sum of such series.

Paper three - Sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 11	12 - 23	24 - 31	32 - 37	38 - 43	44 - 49	50 - 60

The areas of the programme and examination which appeared difficult for the candidates

In this paper candidates found difficulty presenting well constructed proofs. There was generally a lack of rigour evident in both poor terminology and notation as well as proof techniques. Candidates often started with what they were trying to prove and then used circular reasoning. In referring to bijections candidates often just stated the definition of injection and bijection and did not follow the instructions which required them to refer to the graphed function.

The areas of the programme and examination in which candidates appeared well prepared

Many candidates demonstrated a good knowledge of the syllabus throughout the examination, making a reasonable attempt at all questions. The candidates had an excellent knowledge of group theory in terms of considering the properties of the binary operations, completing a Cayley table and finding order of the elements.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

For the commutative property some candidates began by setting $a*b=b*a$. For the identity element some candidates confused $e*a$ and ea stating $ea=a$. Others found an expression for an inverse element but then neglected to state that it did not belong to the set of natural numbers or that it was not unique.

Question 2

There were no problems with parts (a), (b) and (d) but in part (c) candidates often failed to state that the set was associative under the operation because multiplication is associative. Likewise they often failed to list the inverses of each element simply stating that the identity was present in each row and column of the Cayley table. The majority of candidates did not answer part (d) correctly and often simply listed all subsets of order 2 and 3 as subgroups.

Question 3

For the most part the piecewise function was correctly graphed. Even though the majority of candidates knew that it is required to establish that the function is an injection and a surjection in order to prove it is a bijection, many just quoted the definition of injection or surjection and did not relate their reason to the graph. The majority of candidates found the inverse of the first part of the piecewise function but some struggled with the algebra of the second part. In finding the inverse of the quadratic part of the function some candidates omitted the plus or minus sign in front of the square root. Others who had it often forgot to eliminate the negative sign and so did not gain the reasoning mark. Most did not state the correct domain for either part of the inverse function.

Question 4

Candidates knew the properties of equivalence relations but did not show sufficient working out in the transitive case. Others did not do the modular arithmetic correctly, still others omitted the mod(5) in part or throughout.

Question 5

This question presented the most difficulty for students. Overall the candidates showed a lack of ability to present a formal proof. Some gained points for the proof of the identity element in the intersection and the statement that the associative property carries over from the group. However, the vast majority gained no points for the proof of closure or the inverse axioms.

Recommendations and guidance for the teaching of future candidates

- This option necessitates some preliminary work on the nature of mathematical proof. Teachers would do well to spend some time on the different kinds of mathematical proofs, e.g. proof by contradiction and direct proof.
- The need to expose candidates to problems requiring sophisticated mathematics reasoning and communication continues to exist. Candidates need to practice clearly communicating their ideas and arguments in a logical and legible manner.
- In problems involving inverse functions candidates need to be reminded to find the domain of the inverse function.

Paper three - Statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 16	17 - 24	25 - 31	32 - 38	39 - 45	46 - 60

The areas of the programme and examination which appeared difficult for the candidates

It was disturbing to note that so many candidates were unable to find an unbiased estimate of the variance in Question 1. Candidates at this level should know that division by $(n-1)$ and not n is required to find an unbiased estimate. Many candidates appear to be unfamiliar with the cumulative distribution function. Even those who know the definition often write it as $F(x)=\int_a^x f(x)\mathrm{d}x$. The double use of *x* is, of course, an abuse of notation and candidates should be encouraged to use the definition $\int_a^x f(u)\mathrm{d}u$.

Many candidates are unfamiliar with the Central Limit Theorem. This is a major concern since it is widely regarded as the most important theorem in Statistics and candidates taking the Statistics Option should be familiar with it. The most common fallacy is that the sampled distribution instead of the sample mean tends to normality as the sample size increases.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates are extremely competent in the use of the calculator to solve problems involving statistical inference although in some cases, it would be advisable to explain more fully what is being done.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

In (a), most candidates estimated the mean correctly although many candidates failed to obtain a correct unbiased estimate for the variance. The most common error was to divide Σx^2 by 20 instead of 19. For some candidates, this was not a costly error since we followed through their variance into (b) and (c). In (b) and (c), however, since the variance was estimated, the confidence interval and test should have been carried out using the t-distribution. It was extremely disappointing to note that many candidates found a Z-interval and used a Z-test and no marks were awarded for doing this. Candidates should be aware that having to estimate the variance is a signpost pointing towards the t-distribution.

Question 2

The most common error in (a) was to define hypotheses including the sample mean 2.3. It is important for candidates to realise that testing for a Poisson goodness of fit is different from testing for a Poisson goodness of fit with specified mean. Indeed the two tests are carried out with different degrees of freedom. In (b), many candidates worked with a '5' cell instead of a '≥ 5' cell. Consequently, the sum of their expected frequencies was less than the sum of the observed frequencies which led to an invalid test. Some candidates failed to realise that, since the parameter was estimated, the degrees of freedom would be two less than the number of cells.

Question 3

This question was well answered by many candidates. The most common error was to attempt to use a normal approximation to find approximate probabilities instead of the Poisson distribution to find the exact probabilities. Some candidates appeared not to be familiar with the term 'Type II error probability' which made (b)(ii) inaccessible. Another fairly common error was to believe that the complement of $x \leq 25$ is $x \geq 25$.

Question 4

Solutions to (a)(i) were disappointing in general, suggesting that many candidates are unfamiliar with the concept of the cumulative distribution function. Many candidates knew that it was something to do with the integral of the probability density function but some thought it was $\int_1^2 f(x)\mathrm{d}x$ which they then evaluated as 1 while others thought it was just $\int f(x)\mathrm{d}x = \frac{\left(x^2 + x^3\right)}{10}$ which is not, in general, a valid method. However, most candidates solved (a)(ii) correctly, usually by integrating the probability density function from 1 to *m*. In (b)(i), the statement of the central limit theorem was often quite dreadful. The term 'sample mean' was often not mentioned and a common misconception appears to be that the actual distribution rather than the sample mean tends to normality as the sample size increases. Solutions to (b)(ii) often failed to go beyond finding the mean and variance of X. In calculating the variance, some candidates rounded the mean from 1.5916666.. to 1.59 which resulted in an incorrect value for the variance. It is important to note that calculating a variance usually involves a small difference of two large numbers so that full accuracy must be maintained.

Question 5

Part (a) was well answered in general although some candidates were unable to distinguish between the binomial and negative binomial distributions. In (b)(ii), most candidates knew what to do but algebraic errors were not uncommon. Candidates often used equal instead of inequality signs and this was accepted if it led to $x = 23.5$. The difficulty for these candidates was whether to choose 23 or 24 for the final answer and some made the wrong choice. Some candidates failed to see the relevance of the result in (b)(ii) to finding the most likely value of X and chose an 'otherwise' method, usually by creating a table of probabilities and selecting the largest.

Recommendations and guidance for the teaching of future candidates

As indicated earlier, many candidates were unfamiliar with what is regarded as fairly basic knowledge. With the new material on estimation in the programme, candidates should be more aware of the need to divide by $(n-1)$ rather than n to find an unbiased estimate of the variance. Candidates should be encouraged to explain more fully the methods used when they use a calculator to solve a problem. If their final answer is incorrect, there is always the possibility of method marks being awarded if a method is indicated.