May 2013 subject reports

MATHEMATICS HL TZ1

(IB Latin America & IB North America)

Overall grade boundaries

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 28	29 - 40	41 - 52	53 - 64	65 - 76	77 - 100

Series and differential equations

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 41	42 - 53	54 - 66	67 - 78	79 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 30	31 - 42	43 - 53	54 - 65	66 - 77	78 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 39	40 - 51	52 - 64	65 - 75	76 - 100

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Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2013 examination session the IB has produced time zone variants of Mathematics HL papers.

Security breach

The original examination paper one was replaced because it is possible that the content of one question may have become known to persons not authorised to have this information. The IB takes rigorous steps to ensure that the content of an examination paper remains secure however on this occasion there was a possible breach in the security of the paper. As a precautionary measure this question was replaced. The most efficient way to do this was to provide all schools with new examination papers. Unfortunately a very small percentage of candidates did not sit the replacement paper. These candidates were identified and procedures followed at the grade award meeting to ensure that they were neither advantaged nor disadvantaged.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 18	19 - 23	24 - 29	30 - 34	35 - 40

The range and suitability of the work submitted

There were many well-produced portfolios this session that provided strong evidence of self-directed mathematical investigations. Generally, good writing skills were noted in the presentation of the students' work. However, there appeared to be an increase in the number of documents that were extremely lengthy, containing an excess of repetitive screen captures and un-annotated spreadsheet listings extending over several pages.

The assessment criteria appear to have been well understood by teachers, who often produced their own marking rubrics to assess their candidates' work in a consistent manner. Observations made by the moderating team are summarised below.

The tasks:

Nearly all of the portfolios contained tasks that were taken from the publication, “Tasks for use in 2012 and 2013”, with the two most popular being “Shadow Functions” and “Dice Games”. It would appear that the Subject Reports from the past two sessions have been ignored by some teachers, as the task, “Patterns from Complex Numbers”, was included in many samples with some students (and teachers) still misreading the instruction and producing a familiar and trivial consequence of De Moivre's theorem: that consecutive n-th roots of unity, when connected, form a regular polygon. Significant deductions were made for such conclusions. Please note the advice given in the Subject Reports for May 2012 and November 2012.

There were only a very small number of teacher-designed tasks submitted. As it was noted in the previous Subject Report, the continued and repeated use of some teacher-designed tasks is of some concern. Tasks such as “Lionel (the dog)”, “Ziggurats”, and “Pipelines” have been reused time and time again over the past decade. Schools that continue to use the same tasks each year, regardless of their presumed non-expiring status, are ignoring the risk of plagiarism as solutions are easily available, not only online, but through their former students.

Candidate performance against each criterion

The candidates performed well against criterion A, although the occasional student used calculator notation for multiplication or exponentiation, or did not use standard subscript notation as required. The use of correct probability notation was not always present.

Most students produced very well written pieces of work with thorough explanations. However, missing introductions and unlabelled graphs were still in evidence. Some student work, though correct, was far from concise, with a number of portfolios whose lengths well exceeded those of Extended Essays! As it was noted in the previous Subject Report, moderators were overwhelmed and under-impressed with the repetitive and excessive number of pages of similar graphs and seemingly unending spreadsheet listings.

The candidates generally produced good work and the teachers have assessed the work well against criteria C and D. However, it was disappointing to note that in some type I work, the investigation was extremely limited, with little evidence of a pattern to warrant a conjecture, let alone a generalisation. It would appear that some students started with a prior notion of the end result without investigation.

Some candidates did not pay enough attention in their type II task to the requirements in criterion D, and generated results to an inappropriate degree of accuracy, such as in the winnings in a casino game or the cost of petrol to six decimal places.

The use of technology varied somewhat. Full marks were often given much too generously for the use of a large collection of similar graphs. Also, should a task such as “Dice Games” assume that the probabilities be limited to positive values, the graphs should have been so adjusted to reflect those values.

Resourceful uses of technology were noted in the form of graphs with sliders to adjust parameter values, and the use of conditional statements in dynamic spreadsheets. Some students used graphing software such as GeoGebra to show graphs of complex numbers.

There were many pieces of good and complete work; however, the awarding of full marks in criterion F requires evidence of mathematical sophistication that extends beyond the mere completion of the task.

Recommendations and guidance for the teaching of future candidates

Perhaps a look at transformational geometry may be in order prior to approaching a task like “Complex Numbers”, wherein the basic understanding that distances are invariant under rotation would have simplified the students’ approach to their analysis greatly.

The suggestions below have been offered to teachers in the last Subject Report, but may be worth noting again:

Teachers are expected to write directly on the work submitted, not only to provide feedback to their candidates, but information to the moderators as well. The use of Form B is suggested to allow the teacher to indicate more relevant and descriptive comments.

There has been a noticeable improvement in the provision of background information to each portfolio task that is required to accompany each sample, particularly with the use of Form A or through anecdotal comments. Moderators find them very useful in determining the context in which each task was given when confirming the achievement levels awarded.

A solution key for each task in the sample must accompany the portfolios in order that moderators can justify the accuracy of the work and understand the teacher's expectations. Where there is more than one HL teacher involved in marking the portfolio work, the use of a common marking scheme has been effective.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 30	31 - 44	45 - 58	59 - 73	74 - 87	88 - 120

The areas of the programme and examination which appeared difficult for the candidates

The areas that seemed to cause most difficulties to candidates were complex numbers, matrices, induction, logs (in particular changing a base), sketching graphs, and series using summation notation.

Problem solving also caused difficulties, for example candidates could easily deal with the use of the compound angle formula in straightforward situations, but did not see it was needed when its use was not stated explicitly.

As previously there were also indications that a number of the candidates were not familiar with all topics of the Mathematics HL course.

The areas of the programme and examination in which candidates appeared well prepared

Much of the paper was well done by the candidates. Particular strength was in the questions involving calculus in which several candidates who scored poorly elsewhere were very successful.

Binomial probabilities and distribution, solving trigonometric equations (and knowledge of the main trigonometric ratios) were also successfully completed.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Those who tackled this question were generally very successful. A few, with varying success, tried to work out the powers of the complex numbers by multiplying the Cartesian form rather than using de Moivre's Theorem.

Question 2

Candidates showed a good understanding of the vector techniques required in this question.

Question 3

This was well done by most candidates. The most common error otherwise was to assume that matrix multiplication was commutative.

Question 4

It was pleasing to note how many candidates recognised the expression that needed to be integrated and successfully used integration by parts to reach the correct answer.

Question 5

There was a large variety of methods used in this question, with most candidates choosing to implicitly differentiate the expression for volume in terms of r .

Question 6

Quite a few candidates only managed the first mark for checking the truth of the statement for $n = 1$ and the second for assuming the truth of the statement for $n = k$.

These and other candidates seemed unfamiliar with the necessary structure of an induction proof after this point.

Candidates should be aware that the statement for $n = k$ needs to make the assumption of truth clear and that *k* is a particular value of n .

Question 7

The implicit differentiation was generally well done. Some candidates did not realise that they needed to substitute into the original equation to find *y*. Others wasted a lot of time rearranging the derivative to make $\frac{dy}{dx}$ the subject, rather than simply putting in the particular values for x and y .

Question 8

There were plenty of good answers to this question. Those who realised they needed to make each log have the same base (and a great variety of bases were chosen) managed the question successfully.

Question 9

There is a great variety of ways to approach this question and there were plenty of very good solutions produced, all of which required an insight into the structure of conditional probability. A few candidates unfortunately assumed independence and so did not score well.

Question 10

There were a pleasing number of candidates who answered part (a) correctly. Fewer were successful with part (b). It was expected by this stage of the paper that candidates would be able to just write down the value of the integral rather than use substitution to evaluate it.

There were disappointingly few correct answers to part (c) with candidates not realising that it was necessary to combine the previous two parts in order to write down the answer.

Question 11

This question was very well done by many candidates. Generally candidates were aware that in a section B question the answers to the previous parts are often used in later parts of the questions.

This was emphasised by the word ‘hence’ being used three times in the question and its importance was generally realised.

Question 12

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae.

There were many good solutions to parts $(a)-(e)$ but the following points caused some difficulties

(b) Exam technique would have helped those candidates who could not get part (a) correct as any solution of the form given in the question could have led to full marks in part (b). Several candidates obtained expressions which were not of this form in (a) and so were unable to receive any marks in (b) Many missed the fact that if a vertical translation is performed before the vertical stretch it has a different magnitude to if it is done afterwards. Though on this occasion the markscheme was fairly flexible in the words it allowed to be used by candidates to describe the transformations it would be less risky to use the correct expressions.

(c) Generally the sketches were poor. The general rule for all sketch questions should be that any asymptotes or intercepts should be clearly labelled. Sketches do not need to be done on graph paper, but a ruler should be used, particularly when asymptotes are involved.

(e) and (f) were well done up to the final part of (f), in which candidates did not realise they needed to use the compound angle formula.

Question 13

This question linked the binomial distribution with binomial expansion and coefficients and was generally well done.

(a) Candidates need to be aware how to work out binomial coefficients without a calculator

(b) (ii) A surprising number of candidates chose to work out the values of all the binomial coefficients (or use Pascal’s triangle) to make a total of 64 rather than simply putting 1 into the left hand side of the expression.

(d) This was poorly done. Candidates were not able to manipulate expressions given using sigma notation.

Recommendations and guidance for the teaching of future candidates

More practice at sketching graphs should help improve the quality of these.

Review the required reasoning for proof by mathematical induction.

Ensure that candidates consider multiple approaches to problem solving in order they are better able to select an appropriate method in unfamiliar situations, and recognise in a timely manner when the first approach is not working (in common with many IB papers there were several questions in this paper which could be successfully approached in different ways - including three which have three different methods reproduced in the mark scheme).

Candidates need to be aware of the need to provide reasoning in addition to working. A significant number of candidates in section A seemed to just use the space provided for what appeared to be the rough working necessary to get to an answer, rather than an opportunity to show a method. It was often difficult to tell what part to look at first and also hard to tell what some of the calculations referred to. Perhaps increased practise in applying these mark schemes in peer assessment could lead to a greater understanding of the need to show clear working and reasoning in an organised manner.

Candidates need to be aware of the structure of the paper, the questions at the end of section A and the last parts of the questions in section B are likely to be difficult. Some candidates clearly spent too long on the more difficult questions in section A and ran out of time for some of the easier parts in section B.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 32	33 - 46	47 - 59	60 - 73	74 - 86	87 - 120

The areas of the programme and examination which appeared difficult for the candidates

Frequently candidates found surprising difficulties with algebraic techniques – factorisation, quadratics, solution of simultaneous equations etc. Many had difficulties with probabilities and statistics indicating that for many the topic had not been taught completely. Students had difficulties with questions that were set in less familiar terms, and frequently got lost in multi-stage problems. Although the use of GDC's in simple situations seemed to have improved, it was clear that few candidates could use the calculator in a sophisticated way. Many candidates were losing marks with a loss of accuracy during calculations, and simply not giving answers to the correct degree of accuracy.

The areas of the programme and examination in which candidates appeared well prepared

It was good to see an improvement in students' calculator use in simple circumstances. Few were unable to sketch a graph form a calculator, indicating the key points. 3D geometry seemed to have been quite well prepared generally.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

The question was well done generally as one would expect.

Question 2

Candidates who used the determinant method usually obtained full marks. Few students used row reduction and of those the success was varied. However, many candidates attempted long algebraic methods, which frequently went wrong at some stage. Of those who did work through to correctly isolate one variable, few were able to interpret the resultant value of k .

Question 3

There were many good answers to this question. Some students lost accuracy marks by early rounding. Some students struggled with the Binomial distribution.

Question 4

Although it was recognised that the imprecise nature of the wording of the question caused some difficulties, these were overwhelmingly by candidates who were attempting to rotate around the *x*-axis. The majority of students who understood to rotate about the *y*-axis had no difficulties in writing the correct integral. Marks lost were for inability to find the correct value of the integral on the GDC (some clearly had the calculator in degrees) and also for poor rounding where the GDC had been used correctly. In the few instances where students seemed confused by the lack of precision in the question, benefit of the doubt was given and full points awarded.

Question 5

Most students found the value of r, but a surprising number had difficulties finding the height of the rectangle by any one of the many methods possible. Those that did, frequently failed to round their final answer to the required accuracy leading to few students obtaining full marks on this question. A surprising number of students found the area – clearly misinterpreting the meaning of “dimensions”.

Question 6

Whilst most candidates knew that another root was $2-\mathrm{i}$, much fewer were able to find the quadratic factor. Surprisingly few candidates knew that $(x-2)$ must be a repeated factor and less surprisingly many did not recognise that the whole expression needed to be multiplied by $\frac{1}{5}$.

Question 7

Some surprising misconceptions were evident here, using right angled trigonometry in non right angled triangles etc. Those that used the cosine rule, usually managed to obtain the correct answer to part (a). Many students attempted to find the value of the minimum algebraically instead of the simple calculator solution.

Question 8

Some good solutions to part (a) and certainly fewer completely correct answers to part (b). Many candidates were able to access at least partial credit, if they were showing their reasoning.

Question 9

Most candidates were able to find the discriminant (sometimes only as part of the quadratic formula) but fewer were able to explain satisfactorily why there were two distinct roots. Only the better candidates were able to give good answers to part (b).

Question 10

There were some good answers to part (a), although poor calculator use frequently let down the candidates. Very few candidates were able to access part (b).

Question 11

There were many good answers to part (a) showing a clear understanding of finding the vector equation of a line. Unfortunately this understanding was marred by many students failing to write the equation properly resulting in just 2 marks out of the 3. The most common response was of the form

$L_1 = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$ which seemed a waste of a mark. In part (b) many students failed to verify that the lines do indeed intersect. Part (c) was very well done. In part (d) most candidates were able to obtain the first three marks, but few were able to find the second point. There were few correct answers to part (e).

Question 12

Part (a) was generally well done, although correct accuracy was often a problem. Parts (b) and (c) were also generally quite well done. A variety of approaches were seen in part (d) and many candidates were able to obtain at least 2 out of 3. A number missed to consider the $+c$, thereby losing the last mark. Surprisingly few candidates were able to solve part (e) correctly. Very few could recognise the easy variable separable differential equation. As a consequence part (f) was frequently left.

Question 13

There were many good attempts at parts (a) and (b), although in (b) many were unable to give a thorough justification. Few good solutions to parts (c) and (d)(ii) were seen although many were able to answer (d)(i) correctly.

Recommendations and guidance for the teaching of future candidates

Correct layout of students' work should be encouraged. Many papers were very badly presented resulting in needless loss of marks through careless errors. Good calculator use needs to be further developed in many of the candidates and this needs to be developed in good practice in schools. Students should be further prepared in longer multi-stage questions which sometimes involve explanations and justifications. Students should be made aware of the importance of working accurately, and where this is numerical, good calculator use is essential.

Paper three - Discrete mathematics

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 17	18 - 27	28 - 33	34 - 40	41 - 46	47 - 60

The areas of the programme and examination which appeared difficult for the candidates

The candidates were less happy when they had to think and to create proofs for themselves e.g. Q.5, than when they were doing known algorithms. They did not seem to have practiced working in different bases much as shown by Q3.

The areas of the programme and examination in which candidates appeared well prepared

The candidates generally knew how to apply algorithms. The overall standard of the candidates was pleasing.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Part (a) was well answered and (b) fairly well answered. There were problems with negative signs due to the fact that there was a negative in the question, so candidates had to think a little, rather than just remembering formulae by rote. The lay-out of the algorithm that keeps track of the linear combinations of the first 2 variables is recommended to teachers. Part (c) was reasonably well done, not all candidates used the hint given and some did not read carefully enough that the smallest value was required.

Question 2

In part (a) the criteria for Eulerian circuits and trails were generally well known and most candidates realised that they must start/finish at G/E. Candidates who could not do (a) generally struggled on the paper. For part (b) the layout varied greatly from candidate to candidate. Not all candidates made their method clear and some did not show the temporary labels. It is recommended that teachers look at the tabular method with its backtracking system as it is an efficient way of tackling this type of problem and has a very clear layout.

Question 3

Part (a) was a good indicator of overall ability. Many candidates did not write both sides of the equation in terms of n and thus had an impossible equation, which should have made them realise that they had a mistake. Part (b) was not well answered and of those candidates that did, some only gave one side of the equation in base 7. The answers that were given in (a) and (b) could have been checked, so that the candidate knew they had done it correctly.

Question 4

Most candidates knew the method for part (a), a few did not. Part (b) was reasonably well answered with a variety of methods. Part (c) had generally good answers, some candidates in (ii) thought that the degree sequence being the same was sufficient for isomorphism. Part (d) was a good indicator of the overall ability of the candidate with good candidates giving a logical proof and poor candidates giving a single example.

Question 5

Only the top candidates were able to produce logically, well thought-out proofs. Too many candidates struggled with the summation notation and were not able to apply Fermat's little theorem. There was poor logic i.e. looking at a particular example and poor algebra. I was amused with the attempts at a proof by induction-starting with the prime $p=1$ and moving from prime k to prime $k+1$!

Recommendations and guidance for the teaching of future candidates

Although this option involves graphs and trees there was no need for candidates to use graph paper for some of their answers! It made it more difficult to read the answers of candidates that did this with the papers being scanned. If a candidate introduces a variable that is not given in the question then they need to say what it stands for so that the examiner can follow their working. Candidates need to be prepared for proofs as well as algorithms and know that “waffly” words rarely gain many marks. Looking at the structure of proofs on the mark-schemes of previous exams will help. For example, you cannot start with what you are trying to prove and examples are not proofs. With many of the points mentioned above, careful corrective marking of a trial exam should have assisted the candidates, if they were prepared to learn. It is important that the whole syllabus is covered in the teaching.

Paper three - Series and differential equations

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 11	12 - 22	23 - 30	31 - 37	38 - 45	46 - 52	53 - 60

The areas of the programme and examination which appeared difficult for the candidates

On this paper a significant number of candidates found difficulty in correctly applying Taylors Theorem. Candidates also found it difficult to sum an alternating series correctly to a given degree of accuracy. Similarly candidates were not confident in summing a telescoping series and managing the algebra to find a sum of such a series.

The areas of the programme and examination in which candidates appeared well prepared

A significant majority of candidates showed good knowledge of the conditions and technique required to find a limit using L’Hopital’s rule. Most candidates were also confident in solving differential equations using Euler’s methods and many candidates were able to solve a differential equation using an integration factor. Many candidates were able to find a radius of convergence and to verify the associated interval of convergence. Most candidates were able to make progress on a problem involving Riemann’s Sum.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Many candidates achieved full marks on this question but there were still a large minority of candidates who did not seem familiar with the application of Taylor series. Whilst all candidates who responded to this question were aware of the need to use derivatives many did not correctly use factorials to find the required coefficients. It should be noted that the formula for Taylor series appears in the Information Booklet.

Question 2

Most candidates knew Euler's method and were able to apply it to the differential equation to answer part (a). Some candidates who knew Euler's method completed one iteration too many to arrive at an incorrect answer. Surprisingly few candidates were able to efficiently use their GDCs to answer this question and this led to many final answers that were incorrect due to rounding errors.

Most candidates were able to correctly derive the Integration Factor in part (b) but some lost marks due to not showing all the steps that would be expected in a "show that" question. The differential equation was solved correctly by a significant number of candidates but there were errors when candidates multiplied by $\sec x$ before the inclusion of the arbitrary constant.

Question 3

It was pleasing that most candidates were aware of the Radius of Convergence and Interval of Convergence required by parts (a) and (b) of this problem. Many candidates correctly handled the use of the Ratio Test for convergence and there was also the use of Cauchy's n^{th} root test by a small number of candidates to solve part (a). Candidates need to take care to justify correctly the divergence or convergence of series when finding the Interval of Convergence. The summation of the series in part (c) was poorly handled by a significant number of candidates, which was surprising on what was expected to be quite a straightforward problem. Again efficient use of the GDC seemed to be a problem. A number of candidates found the correct sum but not to the required accuracy.

Question 4

Nearly all candidates were able to correctly find the partial fractions required by part (a). In part (b) whilst a majority of candidates were able to recognise the telescoping series using partial fractions many were not able to manipulate them so as to correctly find the sum of the series. It was not clear whether this was a lack of algebraic fluency to a lack of familiarity with telescoping series. When the sum was found most candidates were able to find the limit as $n \to \infty$.

Question 5

Many candidates made progress with this problem. This was pleasing since whilst being relatively straightforward it was not a standard problem. There were still some candidates who did not use the definite integral correctly to find the area under the curve in part (a) and part (b). Also candidates should take care to show all the required working in a "show that" question, even when demonstrating familiar results. The ability to find upper and lower bounds was often well done in parts (a) (ii) and (b) (ii).

Recommendations and guidance for the teaching of future candidates

- Candidates need to use their GDC correctly to apply Euler's method and the summation of series.
- Candidates need to be aware of the degree of accuracy required by a problem.
- Candidates need to apply the necessary rigour and level of communication required by a problem asking a student to prove a statement or show that it is true.
- Candidates need to be aware of techniques such as using partial fractions to find a telescoping series and using algebraic techniques efficiently to find the sum of such series.

Paper three - Sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 11	12 - 23	24 - 31	32 - 37	38 - 43	44 - 49	50 - 60

The areas of the programme and examination which appeared difficult for the candidates

In this paper candidates found difficulty presenting well constructed proofs. There was generally a lack of rigour evident in both poor terminology and notation as well as proof techniques. Candidates often started with what they were trying to prove and then used circular reasoning. In referring to bijections candidates often just stated the definition of injection and bijection and did not follow the instructions which required them to refer to the graphed function.

The areas of the programme and examination in which candidates appeared well prepared

Many candidates demonstrated a good knowledge of the syllabus throughout the examination, making a reasonable attempt at all questions. The candidates had an excellent knowledge of group theory in terms of considering the properties of the binary operations, completing a Cayley table and finding order of the elements.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

For the commutative property some candidates began by setting $a*b=b*a$. For the identity element some candidates confused $e*a$ and ea stating $ea=a$. Others found an expression for an inverse element but then neglected to state that it did not belong to the set of natural numbers or that it was not unique.

Question 2

There were no problems with parts (a), (b) and (d) but in part (c) candidates often failed to state that the set was associative under the operation because multiplication is associative. Likewise they often failed to list the inverses of each element simply stating that the identity was present in each row and column of the Cayley table. The majority of candidates did not answer part (d) correctly and often simply listed all subsets of order 2 and 3 as subgroups.

Question 3

For the most part the piecewise function was correctly graphed. Even though the majority of candidates knew that it is required to establish that the function is an injection and a surjection in order to prove it is a bijection, many just quoted the definition of injection or surjection and did not relate their reason to the graph. The majority of candidates found the inverse of the first part of the piecewise function but some struggled with the algebra of the second part. In finding the inverse of the quadratic part of the function some candidates omitted the plus or minus sign in front of the square root. Others who had it often forgot to eliminate the negative sign and so did not gain the reasoning mark. Most did not state the correct domain for either part of the inverse function.

Question 4

Candidates knew the properties of equivalence relations but did not show sufficient working out in the transitive case. Others did not do the modular arithmetic correctly, still others omitted the mod(5) in part or throughout.

Question 5

This question presented the most difficulty for students. Overall the candidates showed a lack of ability to present a formal proof. Some gained points for the proof of the identity element in the intersection and the statement that the associative property carries over from the group. However, the vast majority gained no points for the proof of closure or the inverse axioms.

Recommendations and guidance for the teaching of future candidates

- This option necessitates some preliminary work on the nature of mathematical proof. Teachers would do well to spend some time on the different kinds of mathematical proofs, e.g. proof by contradiction and direct proof.
- The need to expose candidates to problems requiring sophisticated mathematics reasoning and communication continues to exist. Candidates need to practice clearly communicating their ideas and arguments in a logical and legible manner.
- In problems involving inverse functions candidates need to be reminded to find the domain of the inverse function.

Paper three - Statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 16	17 - 24	25 - 31	32 - 38	39 - 45	46 - 60

The areas of the programme and examination which appeared difficult for the candidates

It was disturbing to note that so many candidates were unable to find an unbiased estimate of the variance in Question 1. Candidates at this level should know that division by $(n-1)$ and not n is required to find an unbiased estimate.

Many candidates appear to be unfamiliar with the cumulative distribution function. Even those who know the definition often write it as $F(x)=\int_a^x f(x)\mathrm{d}x$. The double use of *x* is, of course, an abuse of notation and candidates should be encouraged to use the definition $\int_a^x f(u)\mathrm{d}u$.

Many candidates are unfamiliar with the Central Limit Theorem. This is a major concern since it is widely regarded as the most important theorem in Statistics and candidates taking the Statistics Option should be familiar with it. The most common fallacy is that the sampled distribution instead of the sample mean tends to normality as the sample size increases.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates are extremely competent in the use of the calculator to solve problems involving statistical inference although in some cases, it would be advisable to explain more fully what is being done.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

In (a), most candidates estimated the mean correctly although many candidates failed to obtain a correct unbiased estimate for the variance. The most common error was to divide Σx^2 by 20 instead of 19. For some candidates, this was not a costly error since we followed through their variance into (b) and (c). In (b) and (c), however, since the variance was estimated, the confidence interval and test should have been carried out using the t-distribution. It was extremely disappointing to note that many candidates found a Z-interval and used a Z-test and no marks were awarded for doing this. Candidates should be aware that having to estimate the variance is a signpost pointing towards the t-distribution.

Question 2

The most common error in (a) was to define hypotheses including the sample mean 2.3. It is important for candidates to realise that testing for a Poisson goodness of fit is different from testing for a Poisson goodness of fit with specified mean. Indeed the two tests are carried out with different degrees of freedom. In (b), many candidates worked with a '5' cell instead of a '≥ 5' cell. Consequently, the sum of their expected frequencies was less than the sum of the observed frequencies which led to an invalid test. Some candidates failed to realise that, since the parameter was estimated, the degrees of freedom would be two less than the number of cells.

Question 3

This question was well answered by many candidates. The most common error was to attempt to use a normal approximation to find approximate probabilities instead of the Poisson distribution to find the exact probabilities. Some candidates appeared not to be familiar with the term 'Type II error probability' which made (b)(ii) inaccessible. Another fairly common error was to believe that the complement of $x \leq 25$ is $x \geq 25$.

Question 4

Solutions to (a)(i) were disappointing in general, suggesting that many candidates are unfamiliar with the concept of the cumulative distribution function. Many candidates knew that it was something to do with the integral of the probability density function but some thought it was $\int_1^2 f(x)\mathrm{d}x$ which they then evaluated as 1 while others thought it was just $\int f(x)\mathrm{d}x = \frac{(x^2 + x^3)}{10}$ which is not, in general, a valid method. However, most candidates solved (a)(ii) correctly, usually by integrating the probability density function from 1 to *m*. In (b)(i), the statement of the central limit theorem was often quite dreadful. The term 'sample mean' was often not mentioned and a common misconception appears to be that the actual distribution rather than the sample mean tends to normality as the sample size increases. Solutions to (b)(ii) often failed to go beyond finding the mean and variance of X. In calculating the variance, some candidates rounded the mean from 1.5916666.. to 1.59 which resulted in an incorrect value for the variance. It is important to note that calculating a variance usually involves a small difference of two large numbers so that full accuracy must be maintained.

Question 5

Part (a) was well answered in general although some candidates were unable to distinguish between the binomial and negative binomial distributions. In (b)(ii), most candidates knew what to do but algebraic errors were not uncommon. Candidates often used equal instead of inequality signs and this was accepted if it led to $x = 23.5$. The difficulty for these candidates was whether to choose 23 or 24 for the final answer and some made the wrong choice. Some candidates failed to see the relevance of the result in (b)(ii) to finding the most likely value of X and chose an 'otherwise' method, usually by creating a table of probabilities and selecting the largest.

Recommendations and guidance for the teaching of future candidates

As indicated earlier, many candidates were unfamiliar with what is regarded as fairly basic knowledge. With the new material on estimation in the programme, candidates should be more aware of the need to divide by (n – 1) rather than n to find an unbiased estimate of the variance. Candidates should be encouraged to explain more fully the methods used when they use a calculator to solve a problem. If their final answer is incorrect, there is always the possibility of method marks being awarded if a method is indicated.