May 2012 subject reports

MATHEMATICS HL TZ2

(IB Africa, Europe & Middle East & IB Asia-Pacific)

Overall grade boundaries

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0-13	14-28	29-40	41-52	53-64	65-76	77-100

Series and differential equations

Grade:	1	2	3	4	5	6	7
Mark range:	0-14	15-29	30-41	42-53	54-66	67-79	80-100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0-13	14-27	28-39	40-51	52-63	64-75	76-100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0-13	14-27	28-38	39-50	51-63	64-75	76-100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2012 examination session the IB has produced time zone variants of the Mathematics HL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-6	7-13	14-18	19-23	24-29	30-34	35-40

The range and suitability of the work submitted

The portfolios produced this session were of a very good quality. Generally, the writing was clear and concise; unfortunately, some candidates produced massive documents, containing seemingly endless pages of tabular output or using a cut-and-paste approach.

The assessment criteria generally appeared to be well understood by the teachers and candidates with a few exceptions. Observations made by the moderating team are summarised below:

The tasks:

The majority of the tasks were taken from the older of the two publications in use, “Mathematics HL – The portfolio – Tasks for use in 2011 and 2012” with the most popular tasks still being “Patterns within Systems of Linear Equations” and “Modelling a Functional Building”. There were some tasks taken from the newer publication, “Tasks for use in 2012 and 2013”, and a very small number of teacher-designed tasks submitted. One task, “Patterns from Complex Numbers”, from this newer publication proved to be quite problematic, with many students (and some teachers) misreading the instruction and producing a familiar and trivial consequence of De Moivre’s theorem: that consecutive nth roots of unity when connected form a regular polygon. However, the instruction in this task, stated for cube roots and later applied to n-th roots, specifically required the student to “Choose a root and draw line segments from this root to the other two roots”, resulting in a tree by graph theory definition or, if viewed non-mathematically, a diagram resembling the frame of an oriental fan.

Candidate performance against each criterion

Overall, the candidates performed well against criterion A. The use of calculator notation and the absence of “dx” after the integrand appear to be declining steadily, but such careless use or absence of notation was often overlooked by teachers.

Some candidates failed to make a distinction between the terms, “equation”, “function”, and “expression”, which is a recurring shortcoming.

Communication skills have improved notably over the past few years. However, in spite of the instruction directed to students to avoid a “Q&A” approach given at the top of each statement of task, there were some candidates who continued to use such a format. Missing introductions and unlabelled graphs were still in evidence. Some student work, though correct, was far from concise, with the most extensive piece of work this session being in excess of 100 pages! The emphasis needs to return to quality over quantity.

Overall, the candidates produced good work, and the assessments against criteria C and D by their teachers have been appropriate. However, in some type I work, the investigation was extremely limited, with no evidence of a pattern to warrant a conjecture, let alone a generalisation, particularly in the task, "Patterns within Systems of Linear Equations". Unfortunately, in the apparent haste to reach a generalisation, many students did not qualify their conclusion to consider limitations involving dependent systems of equations.

In type II tasks, the choice of an appropriate model was seldom justified by the student, for example in the "Functional Building" task. However, improvement was noted in the candidates' careful definitions of variables and parameters. Many students did not pay enough attention in their type II task to the requirements in criterion D, generating results to an inappropriate degree of accuracy, such as a millionth of a second or billionth of a metre. Of concern was the failure to consider the reasonableness of parameter values, such as the choice of a "jogging speed" of 10 m/s in the task, "Running with Angie and Buddy".

The use of technology varied considerably. It should be noted that the mere inclusion of graphs is not sufficient to demonstrate a resourceful use of technology that enhances the work. Full marks were often given much too generously for the use of an overwhelming collection of similar graphs. Some graphs were carelessly included that did not support the work, as when a semi-circle was drawn to model a parabolic roof. Should the task require the domain to be limited to positive values, the graphs should have been so adjusted.

On the other hand, the use of sliders to adjust parameters provided an excellent investigative approach. The use of conditional statements in spreadsheets also worked very well in the modelling task, "Running with Angie and Buddy".

There were many pieces of good and complete work; however, the awarding of full marks in criterion F requires more than completion and correctness, but the evidence of mathematical sophistication that extends beyond the requirements of the task. Work that is considered "very good" but is not characterised as exemplary should be awarded one mark against criterion F.

Recommendations for the teaching of future candidates

The portfolios in this session could have comprised tasks taken from the two documents mentioned above, but not taken from earlier publications. Please note that a significant penalty is applied in moderation for the use of "expired" tasks. Teachers may certainly choose to use a task proposed by a colleague or offered at a workshop, but be mindful that some such tasks may have been reused year after year and not always successfully by other teachers.

The portfolios in the sample are expected to contain originals with the teacher's marks, not unmarked photocopies. Teachers are expected to write directly on their candidates' work, not only to provide feedback to the candidates, but information to the moderators as well. The use of Form B would allow the teacher to indicate more relevant and descriptive comments.

There has been a noticeable improvement in the provision of background information to each portfolio task that is required to accompany each sample, particularly with the use of Form A or through anecdotal comments. Moderators find them very useful in determining the context in which each task was given when confirming the achievement levels awarded.

A solution key for each task in the sample must accompany the portfolios in order that moderators can justify the accuracy of the work and understand the teacher's expectations.

For candidates completing the diploma in 2013, the tasks contained in the document, "Mathematics HL – The portfolio – Tasks for use in 2012 and 2013", are the only published tasks eligible for use.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-16	17-33	34-45	46-60	61-75	76-90	91-120

The areas of the programme and examination that appeared difficult for the candidates

Some candidates seem to be unfamiliar with matrix algebra with matrices appearing in the denominator of an expression instead of multiplication by an inverse matrix.

Proof by induction is not understood by many candidates who fail to realise that the proof depends on **assuming** that the result is true for $n = k$ and showing that this leads to it being true for $n = k + 1$.

Graph plotting of functions and derivatives of a given graph causes problems for many candidates.

The areas of the programme and examination in which candidates appeared well prepared

On the evidence of this examination, candidates are generally competent in the solution of problems involving tree diagrams and trigonometrical identities.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Candidates who used the remainder theorem usually went on to find the two possible values of k. Some candidates, however, attempted to find the remainders using long division. While this is a valid method, the algebra involved proved to be too difficult for most of these candidates.

Question 2

Most candidates realised that the scalar product should be used to solve this problem and many obtained the equation $4\sin x\cos x = 1$. Candidates who failed to see that this could be written as $\sin 2x = 0.5$ usually made no further progress. The majority of those candidates who used this double angle formula carried on to obtain the solution $\frac{\pi}{12}$ but few candidates realised that $\frac{5\pi}{12}$ was also a solution.

Question 3

This question was well answered in general.

Question 4

It was disappointing to see many candidates expanding $\left(x-\frac{2}{x}\right)^4$ by first expanding $\left(x-\frac{2}{x}\right)^2$ and then either squaring the result or multiplying twice by $\left(x-\frac{2}{x}\right)$, processes which often resulted in arithmetic errors being made. Candidates at this level are expected to be sufficiently familiar with Pascal's Triangle to use it in this kind of problem. In (b), some candidates appeared not to understand the phrase 'constant term'.

Question 5

Some candidates seemed to be unfamiliar with matrix algebra. It was not uncommon to see solutions such as $\boldsymbol{X} = \frac{4\boldsymbol{A}\text{-}\boldsymbol{B}}{5\boldsymbol{B}}$ and some candidates seemed to be unaware that $\boldsymbol{AB}^{-1}$ is not in general equal to $\boldsymbol{B}^{-1}\boldsymbol{A}$.

Question 6

Part (a) was generally well answered. In (b), however, some candidates put $m = a + \mathrm{i}b$ and $n = c + \mathrm{i}d$ which gave four equations for two unknowns so that no further progress could be made.

Question 7

Solutions to this question were generally disappointing. In (a), the shape of the graph was often incorrect and many candidates failed to give the equations of the asymptotes and the coordinates of the image points. In (b), many candidates produced incorrect graphs although the coordinates of the image points were often given correctly.

Question 8

Candidates who are comfortable using implicit differentiation found this to be a fairly straightforward question and were able to answer it in just a few lines. Many candidates, however, were unable to differentiate $x^3 y$ with respect to *x* and were therefore unable to proceed. Candidates whose first step was to write $y = \frac{a \sin nx}{x^3}$ were given no credit since the question required the use of implicit differentiation.

Question 9

Solutions to this question were good in general with many candidates realising that multiplying the numerator and denominator by ($\cos A + \sin A$) might be helpful.

Question 10

Many candidates stated the two zeros of f correctly but the graph of f was often incorrectly drawn. In (c), many candidates failed to realise that integration by parts had to be used twice here and even those who did that often made algebraic errors, usually due to the frequent changes of sign.

Question 11

In (a), the ranges were often given incorrectly, particularly the range of *g* where the modulus signs appeared to cause difficulty. In (b), it was disappointing to see so many candidates making algebraic errors in attempting to determine the expression for $f \circ g$ (*x*). Many candidates were unable to solve (d) correctly with arithmetic errors and incorrect reasoning often seen. Since the solution to (e) depended upon a correct choice of function in (d), few correct solutions were seen with some candidates even attempting to use integration, inappropriately, to find the mean of *X*.

Question 12 – Part A

Since (a) was a ‘show that’ question, it was essential for candidates to give a convincing explanation of how the quoted results were obtained. Many candidates just wrote

$$(x + \mathrm{i}y)^2 = x^2 - y^2 + 2\mathrm{i}xy = -5 + 12\mathrm{i}$$

$$\text{Therefore } x^2 - y^2 = -5 \text{ and } xy = 6$$

This was not given full credit since it simply repeated what was given in the question. Candidates were expected to make it clear that they were equating real and imaginary parts. In (b), candidates who attempted to use de Moivre’s Theorem to find the square roots were given no credit since the question stated ‘hence’.

Question 12 – Part B

In (a), the explanations were often unconvincing. Candidates were expected to make it clear that the two intersections with the *x*-axis gave two real roots and, since the polynomial was a quartic and therefore had four zeros, the other two roots must be complex. Candidates who made vague statements such as ‘the graph shows two real roots’ were not given full credit. In (b), most candidates stated the values of *a* and *b* correctly but algebraic errors often led to incorrect values for the other parameters. Candidates who failed to solve (b) correctly were unable to solve (c), (d) and (e) correctly although follow through was used where possible.

Question 13

Even though the definition of the derivative was given in the question, solutions to (a) were often disappointing with algebraic errors fairly common, usually due to brackets being omitted or manipulated incorrectly. Solutions to the proof by induction in (b) were often poor. Many candidates fail to understand that they have to assume that the result is true for $n = k$ and then show that this leads to it being true for $n = k + 1$. Many candidates just write 'Let $n = k$' which is of course meaningless. The conclusion is often of the form 'True for $n = 1$, $n = k$ and $n = k + 1$' therefore true by induction'. Credit is only given for a conclusion which includes a statement such as 'True for $n = k \Rightarrow$ true for $n = k + 1$'.

Recommendations and guidance for the teaching of future candidates

Proof by induction continues to cause problems for many candidates and this important topic should perhaps be emphasized.

Algebraic manipulation is often poor with brackets often omitted or multiplied out incorrectly. Candidates need as much practice as possible in this area.

Higher level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-15	16-31	32-45	46-60	61-76	77-91	92-120

The areas of the programme and examination that appeared difficult for the candidates

On this paper candidates found difficulty with showing an understanding of the ambiguous case for the sine rule, sketching curves, working with a probability density function of a continuous random variable and applications of calculus.

The areas of the programme and examination in which candidates appeared well prepared

Overall students found this paper to be accessible with no single question producing significant problems. On the whole candidates appeared to have been reasonably well prepared for questions on arithmetic sequences, basic calculus, binomial distributions, Poisson distributions, and normal distributions.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This proved to be a good start to the paper for most candidates. The vast majority made a meaningful attempt at this question with many gaining the correct answers. Candidates who lost marks usually did so because of mistakes in the working. In part (b) the most efficient way of gaining the answer was to use the calculator once the initial inequality was set up. A small number of candidates spent valuable time unnecessarily manipulating the algebra before moving to the calculator.

Question 2

Again this proved to be an accessible question for students with many students gaining full marks. Most candidates used the calculator to find the answers to parts (b) and (c) which is what was intended, but candidates should be aware that there are often marks for recognising what needs to be found, even if the candidate does not obtain the final correct answer. It is suggested that in this style of question, candidates should indicate what they are trying to find as well as giving the final answer.

Question 3

Surprisingly few candidates were able to demonstrate diagrammatically the situation for the ambiguous case of the sine rule. More were successful in trying to apply it or to use the cosine rule. However, there were still a surprisingly large number of candidates who were only able to find one possible answer for AC.

Question 4

A good number of correct answers were seen to this question, but a significant number of candidates forgot to multiply by 2 in part (a) and in part (b) the most common error was to add the combinations rather than multiply them.

Question 5

Again this proved to be a successful question for many candidates with a good proportion of wholly correct answers seen. It was good to see students making good use of the calculator.

Question 6

Most candidates were able to make a meaningful start to this question, but many made errors along the way and hence only a relatively small number of candidates gained full marks for the question. Common errors included trying to use degrees, rather than radians, trying to use algebraic methods to find the gradient in part (b) and trying to find the equation of the tangent rather than the equation of the normal in part (c).

Question 7

Many candidates picked up some marks for this question, but only a few gained full marks. In part (a) many candidates did not appreciate the need for the calculator to find a value of *a*. Candidates had more success with part (b) with a number of candidates picking up follow through marks.

Question 8

The majority of candidates were able to start this question and gain some marks, but only better candidates gained full marks. In part (a) the common error was to assume the wrong number of bounces and in part (b) many candidates lost marks due to rounding the inequality in the wrong direction. Part (c) was found difficult with only a limited number recognising the need for the sum to infinity of a geometric sequence and many of those did not recognise how to link the sum to infinity to the total distance travelled.

Question 9

Given that this was the last question in section A it was pleasing to see a good number of candidates make a start on the question. As would be expected from a question at this stage of the paper, more limited numbers of candidates gained full marks. A number of candidates made the question very difficult by unnecessarily splitting the angles required to find the final answer into combinations of smaller angles, all of which required a lot of work and time.

Question 10

This was an accessible question for most students with many wholly correct answers seen. In part (b) a few candidates struggled to find the correct values from the calculator and in part (c) a small minority did not see the need to treat it as a binomial distribution.

Question 11

Many candidates were able to start this question, but only a few candidates gained full marks. Many candidates successfully used the augmented matrix in part (a) to find the correct answer. Part (b) was less successful with only a limited number of candidates using the calculator to its full effect here and with many candidates making arithmetic and algebraic errors. This was the hardest part of the question. Many candidates understood the vector techniques necessary to answer parts (c), (d) and (e) but a number made arithmetic and algebraic errors in the working.

Question 12

This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

Recommendations and guidance for the teaching of future candidates

- Students need to cover the entire syllabus.
- Students should be encouraged to pay attention to mathematical notation and accuracy.
- Teachers should emphasise the importance of students setting out their procedures in a logical fashion.
- Most of the questions in this paper used common problem solving strategies and this should be a focus for candidates.
- Students need to practice papers of a similar style in order that they understand the need to balance their time.
- Students need to be made aware of appropriate terminology.
- Students need to be aware of the capabilities of the calculator used and need to be aware of the full range of situations in which it can be helpful.
- Students need to remember that final answers should be fully accurate or given to 3 significant figures and that intermediate working needs to use more than 3 significant figures.

Paper three – Discrete mathematics

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-8	9-17	18-25	26-31	32-36	37-42	43-60

The areas of the programme and examination that appeared difficult for the candidates

Many candidates were not able to cope with the demands of the instruction ‘Prove that’. In particular, Question 4(b) was poorly answered. This was disappointing, as HL candidates should realise that reasoning is just as important in Mathematics as carrying out standard routines.

Some candidates failed to realise that in Question 2 it was necessary to demonstrate that they had really used Prim’s algorithm. The order of addition of edges is fundamental to the algorithm, so just giving the minimal weight spanning trees, without comment, gained few marks.

The areas of the programme and examination in which candidates appeared well prepared

Candidates performed well on questions involving the application of standard algorithms and procedures: Q1 (a); .Q2; Q3 (a) (c); Q4 (a); Q5 (a) (b). So the Euclidean algorithm, Prim's algorithm, drawing a graph from adjacency/weight data and the use of Fermat's Little Theorem, were secure areas of the syllabus for many candidates.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

The majority of candidates were successful in parts (a) and (b). In part (c), some candidates failed to understand the distinction between a particular solution and a general solution. Part (d) was a 1 mark question that defeated all but the few who noticed that the gcd of the numbers concerned was 3.

Question 2

This question was well answered by many candidates. A disappointing minority failed to realise that the instruction 'Use Prim's ..' means that they have to convince the marker that they have actually used the algorithm. It is not good enough to just draw the minimal weight spanning trees. It is the order in which edges are added that is the important part of the algorithm. A (very) few candidates started at A – this was minimally penalised.

Question 3

Parts (a) and (c) were generally correctly answered. In part (b), a minority of candidates failed to mention that the starting and end points had to coincide. A large number of candidates gave all walks (trails were asked for) – an unnecessary loss of marks.

Question 4

Part (a) was well done. The various parts of Parts (b) were often attempted, but with a disappointing feeling that the candidates did not have a confident understanding of what they were writing.

Question 5

Many candidates were able to complete part (a) and then went on to part (b). Some candidates raced through part (c). Others, who attempted part (c) using the alternative strategy of repeatedly solving linear congruencies, were sometimes successful.

Recommendations and guidance for the teaching of future candidates

Although this is an option with some topics involving the implementation of algorithms, we also expect candidates to be familiar with the basic notions of proof and that they may be expected to explain their work.

Examiners expect candidates to have covered the whole syllabus.

Paper three – Series and differential equations

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-9	10-19	20-29	30-36	37-42	43-49	50-60

The areas of the programme and examination that appeared difficult for the candidates

On this paper a significant number of candidates found difficulty in recognising the difference between a sequence and a series. It appeared that many candidates were applying knowledge that they had learnt without reflecting on whether the conditions were appropriate to apply this knowledge. Candidates also found it difficult to identify and confirm specific conditions for convergence tests to be applied. Finally, candidates often lacked the confidence to adequately prove or provide rigorous demonstrations when they were asked to “show that” in a problem.

The areas of the programme and examination in which candidates appeared well prepared

A significant majority of candidates showed good knowledge of the conditions and technique required to find a limit using L’Hôpitals rule. Most candidates were also confident in solving differential equations using Euler’s methods and there were also many candidates who were able to solve a differential equation using separation of variables. Candidates who recognised the need to use the Integration Factor techniques were usually successful in applying it.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

The vast majority of candidates were familiar with L’Hôpitals rule and were also able to apply the technique twice as required by the problem. The errors that occurred were mostly due to difficulty in applying the differentiation rules correctly or errors in algebra. A small minority of candidates tried to use the quotient rule but it seemed that most candidates had a good understanding of L’Hôpital’s rule and its application to finding a limit.

Question 2

Most candidates had a good knowledge of Euler's method and were confident in applying it to the differential equation in part (a). A few candidates who knew the Euler's method completed one iteration too many to arrive at an incorrect answer but this was rare. Nearly all candidates who applied the correct technique in part (a) correctly calculated the answer. Most candidates were able to attempt part (b) but some lost marks due to a lack of rigour by not clearly showing the implicit differentiation in the first line of working. Part (c) was reasonably well attempted by many candidates and many could solve the integrals although some did not find the arbitrary constant meaning that it was not possible to solve (ii) of the part (c).

Question 3

Perhaps a small number of candidates were put off by the unusual choice of variables but in most instances it seemed that candidates who recognised the need for an integration factor could make a good attempt at this problem. Candidates who were not able to simplify the integrating factor from $e^{2\ln t}$ to t^2 rarely gained full marks. A significant number of candidates did not gain the final mark due to a lack of an arbitrary constant or not dividing the constant by the integration factor.

Question 4

The "show that" in part (a) of this problem was not adequately dealt with by a significant minority of candidates and simply stating the limit and not demonstrating its existence lost marks. Part (b), whilst being possible without significant knowledge of limits, seemed to intimidate some candidates due to its unfamiliarity and the notation. Part (c) was somewhat disappointing as many candidates attempted to apply rules on the convergence of series to solve a problem that was dealing with the limits of sequences. The same confusion was seen on part (d) where also some errors in algebra prevented candidates from achieving full marks.

Question 5

A good number of candidates were able to find the integral in part (a) although the vast majority did not consider separately the integral when $k = 1$. Many candidates did not explicitly set a limit for the integral to let this limit go to infinity in the anti – derivative and it seemed that some candidates were "substituting for infinity". This did not always prevent candidates finding a correct final answer but the lack of good technique is a concern. In part (b) many candidates seemed to have some knowledge of the relevant test for convergence but this test was not always rigorously applied. In showing that the series was not absolutely convergent candidates were often not clear in showing that the function being tested had to meet a number of criteria and in so doing lost marks.

Recommendations and guidance for the teaching of future candidates

- Candidates need to be clear about the difference between a series and a sequence.

- Candidates need to be able to recognise the conditions when an integrating factor is required.
- Candidates need to apply the necessary rigour and level of communication required by a problem asking a student to prove a statement or show that it is true.
- Candidates need to clearly understand the criteria and conditions for applying a rule of convergence.

Paper three – Sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-7	8-15	16-22	23-28	29-33	34-39	40-60

The areas of the programme and examination that appeared difficult for the candidates

On this paper candidates found difficulty with problems that required the application of concepts in general and abstract situations although in a particular concrete situation they had more success. Candidates often had difficulty in applying their knowledge to prove results taking account of all the conditions of the problem being asked. Also candidates seemed often to lack the skills to clearly communicate their thinking. In particular candidates had difficulties when dealing with injective and surjective functions. Also, despite candidates knowing the conditions for an equivalence relation, they had difficulty showing it to be true.

The areas of the programme and examination in which candidates appeared well prepared

It was pleasing that so many candidates showed knowledge of the syllabus throughout the exam paper. This did not always mean that the candidate was able to correctly answer questions but it was rare to see a student who did not have a minimum of knowledge to attempt a problem. Students were particularly successful in this paper on answering simpler problems in group theory and also displayed a good knowledge of sets.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This was on the whole a well answered question and it was rare for a candidate not to obtain full marks on part (a). In part (b) the vast majority of candidates were able to show that the set satisfied the properties of a group apart from associativity which they were also familiar with. Virtually all candidates knew the difference between commutativity and associativity and were able to distinguish between the two. Candidates were familiar with Lagrange's Theorem and many were able to see how it did not apply in the case of this problem. Many candidates found a solution method to part (iii) of the problem and obtained full marks.

Question 2

This was also a well answered question with many candidates obtaining full marks on both parts of the problem. Some candidates attempted to use a factorial rather than a sum of combinations to solve part (b) (ii) and this led to incorrect answers.

Question 3

Candidates were mostly aware of the conditions required to show an equivalence relation although many seemed unsure as to the degree of detail required to show that the different conditions are met for the example in this question. In part (b) many candidates found the correct set although a number were unable to write down the set correctly, including or excluding elements that were not part of the equivalence class. Part (c) saw candidate being less successful than (b) and relatively few candidates were able to prove the equivalence class in part (d) although there were a number of very good solutions.

Question 4

Nearly all candidates were aware of the conditions for an injection and a surjection in part (a). However, many missed the fact that the function in question was mapping from the set of integers to the set of integers. This led some to lose marks by applying graphical tests that were relevant for functions on the real numbers but not appropriate in this case. However, many candidates were able to give two integer counter examples to prove that the function was neither injective or surjective. In part (b) candidates seemed to lack the communication skills to adequately demonstrate what they intuitively understood to be true. It was usually not stated that the number of elements in the sets of the image and pre – image was equal. Part (c) was well done by many candidates although a significant minority used functions that mapped the positive integers to non – integer values and thus not appropriate for the conditions required of the function.

Question 5

This question was by far the problem to be found most challenging by the candidates. Many were able to show that ghg^{-1} had order one or two although hardly any candidates also showed that the order was not one thus losing a mark. Part a (ii) was answered correctly by a few candidates who noticed the equality of h and ghg^{-1}. However, many candidates went into algebraic manipulations that led them nowhere and did not justify any marks. Part (b) (i) was well answered by a small number of students who appreciated the nature of the identity and element h thus forcing the other two elements to have order four. However, (ii) was only occasionally answered correctly and even in these cases not systematically. It is possible that candidates lacked time to fully explore the problem. A small number of candidates "guessed" the correct answer.

Recommendations and guidance for the teaching of future candidates

- Candidates need to practise constructing mathematical arguments that clearly communicate their ideas and arguments.
- Candidates need to look carefully for the conditions given in a problem.
- In problems involving functions candidates need to take care with the sets being used for the domain and range.
- Candidates need to review carefully the demonstration of the conditions for equivalence relationships.
- Candidates need to be exposed to problems that require a sophisticated level of mathematical communication.

Paper three – Statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-7	8-14	15-20	21-26	27-33	34-39	40-60

The areas of the programme and examination that appeared difficult for the candidates

Surprisingly, the cumulative distribution function was an area of the programme that seemed difficult for many candidates. There was a feeling amongst examiners that many candidates were just carrying out routine procedures without any real understanding of how to tackle situations that were slightly different from those previously encountered.

Candidates did not seem to be aware of the shape of the Poisson distribution.

The areas of the programme and examination in which candidates appeared well prepared

Candidates were generally competent at hypothesis testing – both the t-test and the chi-squared test. There was a good use of the GDC.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

A successful question for many candidates. A few candidates did not read the question and adopted a 2-tailed test.

Question 2

In part (a) some candidates thought that the geometric distribution was continuous, so attempted to integrate the pdf! Others, less seriously, got the end points of the summation wrong.

In part (b) It was very disappointing that may candidates, who got an incorrect answer to part (a), persisted with their incorrect answer into this part.

Question 3

Many candidates picked up good marks for this question, but lost marks because of inattention to detail. The mean of the data was usually given correctly, but sometimes the variance was wrong. It may seem a small point, but the correct hypotheses should not mention the value of the estimated mean. Some candidates did not notice that some columns needed to be combined.

Question 4

There was a curious issue about the lower quartile in part (a): The LQ coincides with a quarter of the range of the distribution $\frac{2}{4}=0.5$. Sadly this is wrong reasoning – the correct reasoning involves a consideration of areas.

In part (b) many candidates used hand calculation rather than their GDC.

The random variable Y was not well understood, and that followed into incorrect calculations involving Y – 2X.

Question 5

Most candidates were able to complete part (a). The remainder of the question involved some understanding of the shape of the distribution and some facility with algebraic manipulation.

Recommendations and guidance for the teaching of future candidates

Although this is an option with some topics involving the implementation of standard techniques, it is important that students obtain some understanding of the real applications and limitations of statistics.

Examiners expect candidates to have covered the whole syllabus.