MATHEMATICS HL TZ1

(IB Latin America & IB North America)

Overall grade boundaries

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 12	13 – 26	27 – 38	39 – 49	50 – 61	62 – 72	73 – 100

Series and differential equations

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 12	13 – 25	26 – 35	36 – 46	47 – 57	58 – 68	69 – 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 12	13 – 26	27 – 37	38 – 48	49 – 60	61 – 72	73 – 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 12	13 – 25	26 – 36	37 – 47	48 – 59	60 – 70	71 – 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2011 examination session the IB has produced time zone variants of the Mathematics HL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 6	7 – 13	14 – 18	19 – 23	24 – 29	30 – 34	35 – 40

The majority of the portfolios produced this session represented a substantial amount of work on the part of the candidates. The assessment criteria were generally well understood by both the teachers and the candidates. However, a large number of teachers have not indicated their students' background information, or provided solutions keys which would have given moderators a much better understanding of the teachers' assessment of their students' work.

The tasks

Nearly all portfolio tasks were taken from the current publication, *"Mathematics HL – The portfolio – Tasks for use in 2011 and 2012".* Each of the four tasks from the document was very popular. There were also a few good teacher-designed tasks submitted by a number of schools; however, a disappointing number of teachers used very old tasks from an outdated TSM without significant modification. The resulting tasks fell short of meeting the current assessment criteria. Teachers are encouraged to design their own tasks, while keeping in mind the need to satisfy all criteria fully.

Candidates' performance

The majority of candidates performed well against criterion A. Unfortunately, the use of computer notation such as "^" and "E12" were still in evidence, and careless notation was often overlooked by teachers. The careless misuse of some colloquial terminology (e.g. "*plug in", "sub*") must be avoided.

Good communication skills were evident in many samples. Many candidates produced work that was a pleasure to read, starting with a personal introduction to the task, and providing comments, explanations, and conclusions to accompany their steps and results. Such work earned high marks in criterion B. On the other hand, there were some candidates whose work did not flow, particularly when there was no introduction to a task or when a question-and-answer format to a task was adopted in spite of the instructions given at the beginning of each task. Unlabelled graphs and the relegation of tables to the appendix rate poorly in terms of an effective presentation and should have been penalised by the teacher.

Candidates generally produced good and complete work, and the assessments against criteria C and D by their teachers have been appropriate. However, in some type I tasks, candidates formed conjectures much too soon from "patterns" observed from an inadequate number of observations. In some instances, generalisations were quoted from internet sources, leaving very little room for exploration and investigation, the essential components of the type I task.

In type II tasks, variables must be explicitly defined. Some realisation of the significance of the results obtained with the created model when compared to the actual situation should have been provided, and candidates should have reflected on their findings. The analysis of data must be quantified, and if a regression analysis were appropriate, the candidates should have explained the significance of the measure of the goodness of fit.

The use of technology varied considerably. The tasks from the current publication contained many opportunities for candidates to extend their work with the use of technology, ranging from 3-D graphs and dynamic spreadsheets to interactive graphs with sliders. Full marks were given much too generously for an *appropriat*e but not necessarily a *resourceful* use of technology. The use of technology should contribute to the *development* of each task, not merely its illustration.

There were many good pieces of work; nevertheless, the highest achievement level in criterion F requires more than completion and correctness, but the evidence of mathematical sophistication beyond what was expected in the statement of task.

Suggestions to teachers

Teachers are encouraged to design their own tasks; however, modifying an old task is ill advised due to the availability of model solutions to the originals readily available online.

The teacher must be fully informed of the portfolio assessment criteria to avoid a significant loss of marks in moderation. Where there is more than one teacher involved in Maths HL, a collegial process of “internal moderation” may help keep the criteria well defined.

To reiterate a recurring concern: the portfolios selected to comprise the school sample are expected to be originals with the teacher’s marks, not unmarked photocopies. Teachers are expected to write directly on their students’ work, not only to provide feedback to the students, but information to the moderators as well. Some samples contained very few comments, making moderation extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks.

The background information to each portfolio task is required to accompany each sample, either on Form A or through anecdotal comments. Moderators find them very useful in determining the context in which each task was given when confirming the achievement levels awarded; unfortunately, such information was often missing.

A solution key for tasks from the current publication, as well as for those designed by teachers, must accompany the portfolios in order that moderators can justify the accuracy of the work, and appreciate the level of sophistication demonstrated by the candidates.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 13	14 – 27	28 – 40	41 – 53	54 – 65	66 – 78	79 – 120

The areas of the programme and examination that appeared difficult for candidates

On this paper candidates found difficulty with vectors, complex numbers, conditional probability, transformation of functions and graphs. Many candidates also had difficulties in providing coherent and concise explanations, in using consistent and appropriate notation and setting their work in a logical manner. Many candidates also had some difficulties in applying knowledge to unfamiliar contexts.

There were also indications that a number of the candidates were not familiar with all topics of the Mathematics HL course.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to be reasonably well prepared for routine questions on certain techniques like simple differentiation and integration, arithmetic and geometric sequences and series, simple probability and use of trigonometric identities. Many candidates were also aware of the need to show working. It was pleasing to see evidence of good teaching at some schools whose candidates presented their work clearly, using appropriate notation and terminology and showing all the necessary steps in a logical manner.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates attempted this question and answered it well. A few misconceptions were identified (eg $\mathrm{P}(A \cup B) = \mathrm{P}(A)\mathrm{P}(B)$). Many candidates were unsure about the meaning of independent events.

Question 2

A number of different methods were adopted in this question with some candidates working through their method to a correct answer. However many other candidates either stopped with z still expressed as a quotient of two complex numbers or made algebraic mistakes.

Question 3

Part (a) was well done by most candidates. However (b) caused difficulty to most candidates. Although a number of different approaches were seen, just a small number of candidates obtained full marks for this question.

Question 4

This question was poorly done with most candidates having difficulties in using appropriate notation which made unclear the distinction between scalars and vectors. A few candidates scored at least one of the marks in (a) but most candidates had problems in setting up the proof required in (b) with many using a circular argument which resulted in a very poor performance in this part.

Question 5

The performance in this question was generally good with most candidates answering (a) well; (b) caused more difficulties, in particular the rationalization of the denominator. A number of misconceptions were identified, for example $\cot\frac{\pi}{8} = \tan\frac{8}{\pi}$.

Question 6

There was a mixed performance in this question with some candidates showing good understanding of probability and scoring well and many others showing no understanding of conditional probability and difficulties in working with decimals. Very few candidates were able to provide a valid argument to justify their answer to part (b).

Question 7

Many candidates were able to write down the correct expression for the required area, although in some cases with incorrect integration limits. However, very few managed to achieve any further marks due to a number of misconceptions, in particular $\arctan x = \cot x = \frac{\cos x}{\sin x}$. Candidates who realised they should use integration by parts were in general very successful in answering this question. It was pleasing to see a few alternative correct approaches to this question.

Question 8

Part (a) was in general well answered and part (b) well attempted. Some candidates had difficulties with the order of composition and in using correct notation to represent the domains of the functions.

Question 9

Implicit differentiation was attempted by many candidates, some of whom obtained the correct value for the gradient of the tangent. However, very few noticed the need to go further and prove that both points were on the same line.

Question 10

A significant number of candidates did not answer this question. Among the candidates who attempted it there were many who had difficulties in connecting vertical asymptotes and the domain of the function and dealing with transformations of graphs. In a few cases candidates managed to answer (a) but provided an answer to (b) which was inconsistent with the domain found.

Question 11

Most candidates attempted this question and scored at least a few marks in (a) and (b). Part (c) was more challenging to many candidates who were unsure how to find the required distance. Part (d) was attempted by many candidates some of whom benefited from follow through marks due to errors in previous parts. However, many candidates failed to give the correct answer to this question due to the use of the simplified vector found in (b) showing little understanding of the role of the magnitude of this vector. Part (e) was poorly answered.

Overall, this question was not answered to the expected level, showing that many candidates have difficulties with vectors and are unable to answer even standard questions on this topic.

Question 12

Most candidates attempted parts (a), (b) and (c) and scored well, although many did not gain the reasoning marks for the justification of the existence of local maximum and inflexion point. The graph sketching was poorly done. A wide selection of range shapes were seen, in some cases showing little understanding of the relation between the derivatives of the function and its graph and difficulties with transformation of graphs. In some cases candidates sketched graphs consistent with their previous calculations but failed to label them properly.

Question 13

This question proved to be very difficult for most candidates. Many had difficulties in following the instructions and attempted to use addition formulae rather than binomial expansions. A small number of candidates used the results given and made a good attempt to part (d) but very few answered part (e).

Recommendation and guidance for the teaching of future candidates

Besides covering the entire syllabus and providing extensive practice using past exam questions, teachers are strongly recommended to

- Ensure that their students have good basic skills and are able to manipulate algebraic expressions easily without wasting too much time in this routine tasks.
- Provide a wide variety of examples of proofs and ensure that students know how to set their work in a logical manner using appropriate notation.
- Provide more problem solving practice to ensure that their students are able to apply their knowledge in a wide variety of contexts.
- Ensure that candidates know how to present their work neatly and are aware of the need to use pen when they write answers in examinations.
- Ensure candidates are familiar with the command terms used in IB examinations.
- Ensure that the candidates are familiar with the information booklet used during examinations.
- Ensure that candidates understand the difference between scalars and vectors and know the properties of their operations.
- Ensure that candidates are familiar with transformation of functions and are able to provide clear and well labelled graphs.

It is also important that teachers provide more guidance about the suitability of this level of Mathematics in relation to candidates' ability and knowledge.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 13	14 – 26	27 – 37	38 – 51	52 – 66	67 – 80	81 – 120

The areas of the programme and examination that appeared difficult for candidates

On this paper candidates found difficulty with polynomial functions, piecewise functions, the binomial theorem, vectors in two dimensions, systems of three equations in three unknowns (intersection of planes) and volume of revolution about the y – axis. There are indications that a number of candidates were not prepared for questions on all aspects of the syllabus. A number of candidates spent too much time on section A and hence ran out of time on section B. A number of candidates had difficulty in providing complete and clear demonstrations when asked to “show that” in a question. There were a significant number of candidates who approximated prematurely in questions, leading to incorrect answers. A number of candidates seemed unaware of the degree of approximation required in the examination.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been reasonably well prepared for questions on most aspects of coordinate geometry involving differential calculus, kinematics involving calculus, logarithms and the basic principles behind a proof by induction. Some candidates were well prepared for questions involving probability distributions although some candidates seemed to have little knowledge of this topic. There was evidence of a good degree of competence in the use of the graphing calculator in questions where this was required although a number of candidates seemed unable to store answers to avoid premature approximation during a question.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

On a very straightforward question there were many correct answers. However, there was evidence that some candidates had not previously encountered cumulative frequency graphs and hence scored low marks on the question.

Question 2

There were a significant number of completely correct answers to this question. Many candidates demonstrated a good understanding of basic differential calculus in the context of coordinate geometry whilst other used technology to find the turning points. There were many correct demonstrations of the "show that" in (b).

Question 3

This was an accessible question to most candidates although care was required when calculating the angles. Candidates who did not annotate the diagram or did not take care with the notation for the angles and sides often had difficulty recognizing when an angle was acute or obtuse. This prevented the candidate from obtaining a correct solution. There were many examples of candidates rounding answers prematurely and thus arriving at a final answer that was to the correct degree of accuracy but incorrect.

Question 4

Candidates found this question surprisingly challenging. The most straightforward approach was use of the Remainder Theorem but a significant number of candidates seemed unaware of this technique. This lack of knowledge led many candidates to attempt an algebraically laborious use of long division. In (b) a number of candidates did not seem to appreciate the significance of the word unique and hence found it difficult to provide sufficient detail to make a meaningful argument. However, most candidates did recognize that they needed a technological approach when attempting (b).

Question 5

Many candidates struggled to find an efficient approach to this problem by applying the Binomial Theorem. A disappointing number of candidates attempted the whole expansion which was clearly an unrealistic approach when it is noted that the expansion is to the 8^{th} power. The fact that some candidates wrote down Pascal's Triangle suggested that they had not studied the Binomial Theorem in enough depth or in a sufficient variety of contexts.

Question 6

Whilst most candidates were able to make the correct construction to solve the problem some candidates seemed unable to find the area of a segment. In a number of cases candidates used degrees in a formula that required radians. There were a number of candidates who followed a completely correct method but due to premature approximation were unable to obtain a correct solution.

Question 7

Many candidates recognised that integration was the appropriate technique to solve this question but the fact that the function was piecewise proved problematic for many. Good use of technology by some candidates was seen but few sketches of the function were made. A sketch would have been helpful to many candidates when attempting to solve (b (ii).

Question 8

This question was well answered by a large number of candidates and indicated a good understanding of calculus, kinematics and use of the graphing calculator. Some candidates worked in x and y rather than a, v and t but mostly obtained correct solutions. Although the majority of candidate used integration throughout the question some correct solutions were obtained by considering the areas in the diagram.

Question 9

This question was well answered by a significant number of candidates. There was evidence of good understanding of logarithms. The algebra required to solve the problem did not intimidate candidates and the vast majority noticed the necessity of technology to solve the final equation. Not all candidates recognized the extraneous solution and there were situations where a rounded value of x was used to calculate the value of y leading to an incorrect solution.

Question 10

Few candidates managed to make progress on this question. Many candidates did not attempt the problem and many that did make an attempt failed to draw a diagram that would have allowed them to make further progress. There were a variety of possible solution techniques but candidates seemed unable to interpret the equation of a straight line written in vector form or find a perpendicular direction. This meant that it was very difficult for meaningful progress to be made towards a solution.

Question 11

It was disappointing to see that a significant number of candidates did not appear to be well prepared for this question and made no progress at all. There were a number of schools where no candidate made any appreciable progress with the question. This was of concern since this is quite a standard problem in Mathematics HL exams. Parts (a) and (b) were intended to be answered by the use of determinants, but many candidates were not aware of this technique and used elimination. Whilst a valid method, elimination led to a long and cumbersome solution when a much more straightforward solution was available using determinants. Part (c) was also a standard question but more challenging. Very few candidates made progress on (c).

Question 12

Candidates who had been prepared to solve questions from this part of the syllabus did well on the question. As a general point, candidates did not always write down clearly which distribution was being used. There were many candidates who seemed unfamiliar with the concept of Normal distributions as well as the Poisson and Binomial distributions and did not attempt the question. Parts (a) – (c) of the question were a variation on similar problems seen on previous examinations and there were a disappointing number of candidates who seemed unable to start the question. The use of 830 and 850 rather than minutes after 8am was seen and this caused students to lose marks despite knowing the method required. In general technology was used well and this was seen in (d) when solving a problem that involved a Poisson distribution. A number of candidates were unable to identify the Binomial distribution in (e).

Question 13

This was the question that was answered best on section B. Although there were many good solutions there were few that were completely correct. In (a) candidates did not always take enough care when “showing that”, missing out steps and thus losing clarity in their arguments. In (b) the induction was reasonably well done with most candidates making a good attempt. In particular, the assumption that the proposition was true for $n = k$ was usually written explicitly and this has not always been the case in previous years. There were still a significant number of situations where candidates were not able to make a correct final statement in their induction proof. In (c), arguments were not always well organized and many candidates did not take enough care in showing that the inverse was the same as substituting $n = -1$.

Question 14

This question was found to be challenging by many candidates and there were very few completely correct solutions. Many candidates did not seem able to find the volume of revolution when taken about the y – axis in (a). Candidates did not always recognize that part (b) did not involve related rates. Those candidates who attempted the question made some progress by separating the variables and integrating in (c) but very few were able to identify successfully the values necessary to find the correct answer.

Recommendation and guidance for the teaching of future candidates

- Students need to cover the entire syllabus.
- The ability to communicate in mathematics is an important objective of the course and teachers should emphasise the importance of students presenting their procedures in a clear and well-organized fashion.
- Teachers should make sure that students are exposed to the sophistication of solution technique expected of the course.
- Students should be taught to reflect on problems before solving them so as to select the most appropriate solution strategy to a problem.
- Student should be given the opportunity to write extended solutions to problems throughout the Mathematics HL course.
- The use of diagrams and annotation of diagrams to solve problems should be emphasised by teachers to students.
- Students need to make sure that they provide clear and complete arguments when solving “show that” problems.
- Teachers should teach students to use their graphing calculator to store answers so as to avoid premature approximation in solutions.
- Teachers should make sure that students are fully aware and have practised the accuracy requirements of the exam.

Paper three – Discrete mathematics

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 9	10 – 19	20 – 28	29 – 34	35 – 41	42 – 47	48 – 60

The areas of the programme and examination that appeared difficult to candidates

Some candidates seemed to be unfamiliar with methods for determining whether or not a number is prime.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates are confident in the use of the Euclidean Algorithm and the solution of Diophantine equations.

Candidates are generally able to solve straightforward problems involving graphs.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

This question was generally well answered although some candidates were unable to proceed from a particular solution of the Diophantine equation to the general solution.

Question 2

This question was generally well answered although some candidates failed to realise the significance of the equality of the upper and lower bounds.

Question 3

Part (a) was generally well answered. In (b), many candidates tested the result for $n=1$ instead of $n=0$. It has been suggested that the reason for this was a misunderstanding of the symbol N with some candidates believing it to denote the positive integers. It is important for candidates to be familiar with IB notation in which N denotes the positive integers and zero. In some scripts the presentation of the proof by induction was poor.

Question 4

Parts (a) and (b) were well answered by many candidates. In (c), candidates who tried to prove the result by adding edges to a drawing of G were given no credit. Candidates should be aware that the use of the word 'Prove' indicates that a formal treatment is required Solutions to (d) were often disappointing although a graphical solution was allowed here.

Question 5

In (a), some candidates tried to use Fermat's little theorem to determine whether or not 1189 is prime but this method will not always work and in any case the amount of computation involved can be excessive. For this reason, it is strongly recommended that this method should not be used in examinations. In (b), it was clear from the scripts that candidates who had covered this material were generally successful and those who had not previously seen the result were usually unable to proceed.

Recommendations and guidance for the teaching of future candidates

Candidates should be familiar with the notation used for certain standard sets in IB examinations, e.g. N indicates the set of the positive integers and zero.

Paper three – Series and differential equations

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 7	8 – 15	16 – 20	21 – 25	26 – 29	30 – 34	35 – 60

The areas of the programme and examination that appeared difficult for candidates

Although it is a core topic, some candidates were unable to use integration by parts correctly, especially when repeated use is required.

More generally, candidates taking this option need to have a thorough grasp of core calculus.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates are able to use Euler's method for solving differential equations.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

In (a), candidates who found the series by successive differentiation were generally successful, the most common error being to state that the derivative of $\ln(1+e^x)$ is $(1+e^x)^{-1}$. Some candidates assumed the series for $\ln(1+x)$ and e^x and attempted to combine them. This was accepted as an alternative solution but candidates using this method were often unable to obtain the required series. In (b), candidates were equally split between using the series or using l'Hopital's rule to find the limit. Both methods were fairly successful, but a number of candidates forgot that if a series was used, there had to be a recognition that it was not a finite series.

Question 2

Most candidates were familiar with Euler's method. The most common way of losing marks was either to round intermediate answers to insufficient accuracy or simply to make an arithmetic error. Many candidates were given an accuracy penalty for not rounding their answer to three significant figures. Few candidates were able to answer (b) correctly with most believing incorrectly that the step length was a relevant factor.

Question 3

Most candidates recognised this differential equation as one in which the substitution $y = vx$ would be helpful and many reached the stage of separating the variables. However, the integration of $\frac{1}{v^2 + 2v + 2}$ proved beyond many candidates who failed to realise that completing the square would lead to an arctan integral. This highlights the importance of students having a full understanding of the core calculus if they are studying this option.

Question 4

Part (a) is essentially core work requiring repeated integration by parts and many candidates realised that. However, some candidates left the modulus signs in I_0 which invalidated their work. In parts (b) and (c) it was clear that very few candidates had a complete understanding of the significance of the modulus sign and what conditions were necessary for it to be dropped. Overall, attempts at (b) and (c) were disappointing with few correct solutions seen.

Question 5

Solutions to (a) were generally good although some candidates failed to reach the correct conclusion from correct application of the ratio test. Solutions to (b) and (c), however, were generally disappointing with many candidates unable to make use of the signposting in the question. Candidates who were unable to solve (b) and (c) often picked up marks in (d).

Recommendations and guidance for the teaching of future candidates

This option is likely to include questions which require competence in algebraic manipulation and calculus and candidates need to be confident in these areas.

Candidates should be made aware of the accuracy rules which require answers to be given either exactly or to three significant figures. Many candidates taking this option are given accuracy penalties.

Paper three – Sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 9	10 – 18	19 – 26	27 – 33	34 – 39	40 – 46	47 – 60

The areas of the programme and examination that appeared difficult for candidates

Although candidates seem able, in general, to show that a relation is an equivalence relation, many find it difficult to determine the equivalence classes.

Isomorphism continues to be a difficult topic for many candidates.

Some candidates appeared to be unable to show that a function of two variables is a bijection.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates are confident in dealing with specific finite groups.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Parts (a) and (b) were well done in general. Some candidates, however, when considering closure and associativity simply wrote ‘closed’ and ‘associativity’ without justification. Here, candidates were expected to make reference to their Cayley table to justify closure and to state that multiplication is associative to justify associativity. In (c), some candidates tried to show the required result without actually identifying the elements of T. This approach was invariably unsuccessful.

Question 2

It was disappointing to find that many candidates wrote the elements of A and B incorrectly. The most common errors were the inclusion of 1 as a prime number and the exclusion of 3 in B. It has been suggested that some candidates use N to denote the positive integers. If this is the case, then it is important to emphasise that the IB notation is that N denotes the positive integers and zero and IB candidates should all be aware of that. Most candidates solved the remaining parts of the question correctly and follow through ensured that those candidates with incorrect A and/or B were not penalised any further.

Question 3

Many candidates solved (a) correctly but solutions to (b) were generally poor. Most candidates seemed to have a weak understanding of the concept of equivalence classes and were unaware of any systematic method for finding the equivalence classes. If all else fails, a trial and error approach can be used. Here, starting with 1, it is easily seen that 4, 6,… belong to the same class and the pattern can be established.

Question 4

Candidates who knew that they were required to give a rigorous demonstration that *f* was injective and surjective were generally successful, although the formality that is needed in this style of demonstration was often lacking. Some candidates, however, tried unsuccessfully to give a verbal explanation or even a 2-D version of the horizontal line test. In 2-D, the only reliable method for showing that a function *f* is injective is to show that $f(a,b) = f(c,d) \Rightarrow (a,b) = (c,d)$.

Question 5

Solutions to (a) were often poor with inadequate explanations often seen. It was not uncommon to see $pq = pr$

$$p^{-1}pq = p^{-1}pr$$
$$q = r$$

without any mention of associativity. Many candidates understood what was required in (b)(i), but solutions to (b)(ii) were often poor with the tables containing elements such as ab and bc without simplification. In (b)(iii), candidates were expected to determine the isomorphism by noting that the group defined by {1, – 1, i, – i} under multiplication is cyclic or that – 1 is the only self-inverse element apart from the identity, without necessarily writing down the Cayley table in full which many candidates did. Many candidates just stated that there was a bijection between the two groups without giving any justification for this.

Paper three – Statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 8	9 – 16	17 – 22	23 – 29	30 – 35	36 – 42	43 – 60

The areas of the programme and examination that appeared difficult for candidates

Many candidates do not appreciate the difference between nX and $\sum_{i=1}^{n} X_i$. The situation is not helped by the fact that some candidates write the former to indicate the latter.

Many candidates suffer an arithmetic penalty for not giving numerical answers correct to three significant figures.

Candidates should be aware that questions on this paper may involve mathematical topics from the core, including calculus.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates are extremely competent in the use of the calculator to solve problems involving statistical inference. The one exception in the case of many candidates is determining the value of an unbiased estimate of variance which is usually not given directly but has to be obtained by squaring an appropriate standard deviation.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

As might be expected, (a) was well answered by many candidates, although those who gave 0.6915 straight from tables were given an arithmetic penalty. Parts (b) and (c), however, were not so well answered with errors in calculating the variances being the most common source of incorrect solutions. In particular, some candidates are still uncertain about the difference between nX and $\sum_{i=1}^{n} X_i$.

Question 2

This question was generally well answered. Some candidates lost marks by excessive rounding of the expected frequencies, sometimes going to the nearest integer. Here, the candidates should have given all expected frequencies exactly.

Question 3

In (a), most candidates gave a correct estimate for the mean but the variance estimate was often incorrect. Some candidates who use their GDC seem to be unable to obtain the unbiased variance estimate from the numbers on the screen. The way to proceed, of course, is to realise that the larger of the two ‘standard deviations’ on offer is the square root of the unbiased estimate so that its square gives the required result. In (b), most candidates realised that the t-distribution should be used although many were awarded an arithmetic penalty for giving either t = 2.911 or the critical value = 2.821. Some candidates who used the p-value method to reach a conclusion lost a mark by omitting to give the critical value. Many candidates found part (c) difficult and although they were able to obtain t = 2.49…, they were then unable to continue to obtain the confidence interval.

Question 4

Solutions to this question were often disappointing with many candidates not knowing what had to be done. Even those candidates who knew what to do sometimes made errors in evaluating the probabilities, often by misinterpreting the inequality signs. Candidates who used the Central Limit Theorem to evaluate the probabilities were given only partial credit on the grounds that the answers obtained were approximate and not exact.

Question 5

In general, candidates were able to start this question, but very few wholly correct answers were seen. Most candidates were able to write down the probability function but the process of taking logs was often unconvincing. The vast majority of candidates gave an incorrect domain for *f*, the most common error being $x \geq 3$. Most candidates failed to realise that the solution to (b) was to be found by setting the right-hand side of the given equation equal to zero. Many of the candidates who obtained the correct answer, 6.195…, then rounded this to 6 without realising that both 6 and 7 should be checked to see which gave the larger probability.

Recommendations and guidance for the teaching of future candidates

- Candidates should be made aware of the accuracy rules which require answers to be given either exactly or to three significant figures. Many candidates taking this option are given accuracy penalties.
- Candidates should be aware that questions on this paper may involve the use of topics from the core including calculus.
- Candidates need to cover the entire syllabus and be prepared for questions on any of the distributions given in the syllabus.