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MATHS HL TZ1

(IB Latin America & IB North America)

Overall grade boundaries

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 26	27 - 36	37 - 48	49 - 60	61 - 72	73 - 100

Series and differential equations

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 13	14 - 27	28 - 37	38 - 49	50 - 62	63 - 74	75 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 36	37 - 48	49 - 61	62 - 73	74 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 26	27 - 35	36 - 47	48 - 60	61 - 72	73 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2009 examination session the IB has produced time zone variants of the Mathematics HL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 18	19 - 23	24 - 29	30 - 34	35 - 40

The portfolios in this session were generally well presented. Teachers and students appear to have understood the assessment expectations. Generally, the work was clearly marked, and the requisite forms have been completed correctly. Observations made by the moderating team are summarised below:

The tasks:

Most portfolio tasks were taken from the current publication, "Mathematics HL – The portfolio – Tasks for use in 2009 and 2010". There were also a few good tasks submitted by a number of schools. Teachers are encouraged to design their own tasks, keeping in mind the need to satisfy all criteria fully.

There were three issues of concern this session:

1. Some teachers continued to use old tasks taken from a previous TSM. As explained in past Subject Reports and through the Coordinator's Notes, those tasks are no longer eligible for use; hence, a number of candidates lost a significant number of marks through no fault of their own! This is completely inexcusable and must be rectified.

2. Tasks taken from the document for Mathematics SL are not at a suitable level for Mathematics HL and should not have been used.

3. Gauging from the similarity of some student work, it would appear that some teachers are providing too much guidance or direction to students. To avoid the danger of malpractice, such guidance should not prescribe how students should proceed with any task assigned.

Candidates' performance

Most candidates performed well against criterion A. The use of computer notation seemed to be very limited; however, the inappropriate use of "^", "E09", and the like, continue to mar some student work. The careless misuse of some terminology (e.g. "equation" instead of "expression") must also be avoided.

Good communication skills were evident in some samples. Where a student's work began with an introduction to the task, and comments, annotations, and conclusions accompanied the steps and results, the work was easy to read and follow, and earned high marks in criterion B. However, there were many students whose work did not stand on its own, particularly when there was no introduction to a task or when a question-and-answer format to

a task was adopted. Unlabelled graphs and the relegation of tables to the appendix rate poorly in terms of an effective presentation and should have been penalised.

Criteria C and D are meant to assess the mathematical content and jointly comprise half of the total marks awarded to each piece of work. Generally, students have produced good work, and the assessments by their teachers have been appropriate. However, in some type I tasks, insufficient exploration and patterning rendered the quick formulation of a conjecture questionable. In some instances, results were quoted from internet sources and there was little individual work in exploration and investigation, the key to the type I task.

In type II tasks, variables should be explicitly defined. Some realisation of the significance of the results obtained in terms of the model when compared to the actual situation should have been provided, and students should have reflected on their findings. The analyses of data must be quantified, and if a regression analysis were appropriate, the student must have provided reasons for a particular choice. The use of software that automatically determines the "best" regression model leaves little for the candidate to interpret by himself and should be avoided.

The use of technology varied considerably. Full marks were given much too generously for an appropriate but not necessarily a resourceful use of technology, for example, in the mere inclusion of a graph of data. For full marks, the use of technology should contribute significantly to the development of each task. Students should be discouraged from including GDC key sequences – they are quite unnecessary.

There were many good pieces of work; however, the awarding of full marks in criterion F requires more than completion and correctness, but the evidence of mathematical sophistication.

Suggestions to teachers

Tasks from the TSM must not be used as of this examination session - they carry a 10-mark penalty for their use. Please refer to the document, "Mathematics HL – The portfolio – Tasks for use in 2009 and 2010" for suggested tasks. Teachers are encouraged to design their own.

Teachers should select tasks that provide students with a variety of mathematical activities suitable at higher level. Tasks taken from the Mathematics SL publication do not meet HL requirements. Please ensure that candidates do not lose marks due to inappropriate choices made by the teacher.

The teacher who is uninformed of the changes to the portfolio assessment criteria is generally the reason for a significant loss of marks in moderation. This is not only disastrous to the student, but also completely unfair, and should not happen.

Teachers are expected to write directly on their students' work, not only to provide feedback to students, but information to moderators as well. Some samples contained very few teacher comments. Moderation was extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks.

Moderators find the background to each portfolio task very useful in determining the context in which the task was given when confirming the achievement levels awarded. This information must accompany each sample, either on Form A or through anecdotal comments.

A solution key for tasks from the current publication, as well as for those designed by teachers, must accompany the portfolios in order that moderators can justify the accuracy of the work, and appreciate the level of sophistication demonstrated in the work.

The tasks contained in the current document have now been in use with students completing their diplomas in 2009. They can only be reused with students finishing their diploma program in 2010. Students starting their first year this fall should not be assigned these tasks.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 24	25 - 34	35 - 47	48 - 61	62 - 74	75 - 120

The areas of the programme and examination that appeared difficult for candidates

On this paper candidates found difficulty with inverse trigonometric functions, logarithmic functions, complex numbers and absolute value. Generally most questions which involved formal, structured mathematical reasoning or algebraic simplifications were poorly done. Many candidates had difficulties in working with functions and showed poor understanding of the concepts of domain, range and inverse. Hand-drawn sketches were generally poorly done, with many candidates not labelling axes, asymptotes, or line of symmetry. A number of candidates seemed to have spent too much time on section A and hence had problems in answering all questions in part B. There are indications that some candidates were not prepared to answer questions on all aspects of the syllabus.

The areas of the programme and examination in which candidates appeared well prepared

In general candidates appeared to have been reasonably well prepared for routine questions on aspect of differentiation, integration, matrices, vectors and statistics.

The strengths and weaknesses of the candidates in the treatment of individual questions

Part A

Question 1

Most candidates made good attempts to answer this question. Weaker candidates did not get full marks due to difficulties recognizing the notation and working with modulus of a complex number.

Question 2

This was the most successfully answered question in the paper. Part (a) was done well by most candidates. In part (b), a small number of candidates used knowledge about transformations of functions to identify the coordinates of B. Most candidates used differentiation.

Question 3

There were many candidates showing difficulties in manipulating logarithms and the absolute value to solve the equation.

Question 4

In most cases candidates knew what a singular matrix was and made a good attempt to answer this question. However, many candidates found difficulties in solving the equation.

Question 5

Most candidates had difficulties with this question due to a number of misconceptions, including $\arctan x = \tan^{-1} x = \frac{\cos x}{\sin x}$ and $\arctan x = \frac{\arcsin x}{\arccos x}$, showing that, although candidates were familiar with the notation, they did not understand its meaning. Part (a) was done well among candidates who recognized $\arctan$ as the inverse of the tangent function but just a few were able to identify the relationship between parts (a) and (b). Very few candidates attempted a geometrical approach to this question.

Question 6

This question was successfully answered by most candidates using a variety of correct approaches. A few candidates, however, did not use a parameter for the angle, but instead substituted an angle directly, e.g., $\frac{\pi}{2}$ or $\frac{\pi}{4}$.

Question 7

Most candidates answered part (a) correctly although some candidates showed difficulty solving the equation using valid methods. Part (b) was less successful with many candidates failing to apply chain rule to obtain the derivative of the exponential function.

Question 8

This question was answered fairly well by most candidates using a diversity of approaches.

Question 9

Most candidates attempted this question but very often produced sketches lacking labels on axes and intercepts or ignored the domain of the function. For part (b) many candidates attempted to use integration to find the areas but seldom considered the absolute value. A small number of candidates used geometrical methods to determine the areas, showing good understanding of the problem.

Question 10

A fair amount of candidates had difficulties with this question. In part (a) many candidates were able to write down an expression for the volume in terms of *a*, but thereafter were largely unsuccessful. There is evidence that many candidates have lack of algebraic skills to manipulate the expression and obtain the volume in terms of x. In part (b) some candidates started with what they were trying to show to be true.

Part B

Question 11

Parts of this question were answered quite well by many candidates. A few candidates had difficulties with domain of $\arctan$ in part (a) and in justifying their reasoning in parts (b) and (c). In part (d) although most candidates were successful in finding the expressions of the derivatives and their values at $x = 0$, many were unable to use the results to find the nature of the curve at the origin. Very few candidates were successful in answering parts (e) and (f).

Question 12

Part (a) was answered fairly successfully by most candidates. Many candidates found considerable difficulty in simplifying their expressions in part (b) (i).

Question 13

Part A was answered well by a fair amount of candidates, with some making mistakes in calculating the arguments of complex numbers, as well as careless mistakes in finding the products of complex numbers. Part B proved demanding for most candidates, particularly parts (c) and (d). A surprising number of candidates did not seem to know what was meant by the 'definition of derivative' in part (c) as they attempted to use quotient rule rather than first principles.

Recommendation and guidance for the teaching of future candidates

- Teacher should cover all of the syllabus, make students aware of appropriate terminology and enhance the teaching of basic definitions.
- Teacher should explore a wide range of problem solving techniques, provide unfamiliar problems about familiar concepts to allow students to understand mathematics better and give them the confidence to tackle questions which require more than just the use of a formula in a standard situation.
- The importance of writing the working clearly, showing all the steps, sketching graphs and label them clearly needs emphasis
- Candidates need more practice in questions that involve formal, structured mathematical reasoning.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 43	44 - 59	60 - 74	75 - 90	91 - 120

The areas of the programme and examination that appeared difficult for candidates

It was apparent that many students had been inadequately prepared in certain parts of the programme, in particular matrices, vectors and statistics. Given that a relatively easy long question on this paper was involving matrices, this resulted in many students who would normally be able to achieve a passing grade were unable to reach that level.

In addition, the use of graphing calculators was not well developed by many students and much time was wasted trying to do questions analytically which should be easily completed by GDC. Many students still do not give answers to the correct degree of accuracy, thereby losing a mark.

In questions where mathematical reasoning was required, many students had difficulty expressing themselves clearly. In the proof by induction, few students correctly finished the proof with a correct statement.

The areas of the programme and examination in which candidates appeared well prepared

It was apparent that the degree of preparation for the examination was dependant on the school, and some students were very well prepared in all areas, in contrast to other centres where certain areas had been neglected. It appeared that most students had been prepared well for the functions and calculus questions.

Many students were very well prepared in the use of a GDC, contrasting sharply with those students mentioned above.

In general the level of algebraic skills was reasonably good.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

A surprising number of students lacked the basic knowledge of the normal distribution and were unable to answer the first part of this question. Those students who showed a knowledge of the topic tended to answer the question well. In part (b) many students either had a misunderstanding of the difference between variance and standard deviation, or did not read the question properly.

Question 2

A surprising number of candidates solved the question by dividing the expression by $1-i$ rather than substituting l into the expression. Many students were not aware that complex roots occur in conjugate pairs, and many did not appreciate the difference between a factor and a root. Generally the question was well done.

Question 3

Most students were able to find the derived function correctly, although attempts to solve the inequality algebraically were often unsuccessful. This was a question where students prepared in good use of GDC were able to easily obtain good marks.

Question 4

It was clear that many students had not been taught the topic and were consequently unable to make an attempt at the question. Of those students who were able to start, common errors were in a misunderstanding of the language. Many had difficulties in part (c) and “at least” in part (a) was sometimes misinterpreted.

Question 5

Most students knew how to find the angle between two vectors, although many could not find the correct two direction vectors.

Question 6

Generally well answered, although many students did not include the constant of integration.

Question 7

Few students were able to do this question efficiently. Many students were able to do part (a) by manipulating equations, whereas calculator methods would yield the solution quickly and easily. Part (b) was poorly attempted and it was apparent that many students used a lot of time manipulating equations without real understanding of what they were looking for.

Question 8

This was a more difficult question and it was apparent that students did find it so. For those that managed to rearrange the equation to separate the variables, few could manage to successfully integrate both sides. The unfamiliarity of e^{e^y} seemed to disturb some students.

Question 9

Few students were able to complete this question successfully, although many did obtain partial marks. Many students failed to recognise the difference between differentiating with respect to t or with respect to y. Very few were able to give a satisfactory geometrical meaning in part (b).

Question 10

A surprising number of students were unable to make a start on this question. Clearly if students have not been taught matrices, this would have adversely affected their final grade. In part (a), candidates who could use their GDC well, generally managed to obtain good marks.

In part (b) marks were commonly lost with a failure to show that $A^1 = A$ and a failure to satisfactorily write the final statement.

In (c) few candidates correctly used the identity $A^{n\ -1}A^n = I$ and consequently could not obtain full marks

Question 11

Generally this question was answered well by those students who attempted it. It was common to see confusion between coordinates and position vectors. Part (d) was most easily answered with the use of a GDC, but fewer candidates took advantage of this. In part (e) many students had difficulties expressing their reasoning well to obtain the marks.

Question 12

Generally there were many good attempts to this, more difficult, question. A number of students found b to be equal to 1, rather than 5. In the final part few students could successfully work through the entire integral successfully.

Recommendation and guidance for the teaching of future candidates

First and foremost, it is important to cover the whole syllabus if students are to enter the exam. Many students were clearly not prepared in large parts of the syllabus and consequently found the examination experience very negative and demoralizing.

Many students are in need of more practice in the use of graphic calculators, in particular in the choice of method. Many students are unable to select when a calculator method would be the most appropriate.

Paper three – Discrete mathematics

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 15	16 - 23	24 - 30	31 - 36	37 - 43	44 - 60

The areas of the programme and examination that appeared difficult for candidates

On this paper candidates found difficulty with using adjacency matrices, Chinese remainder theorem and using aspects of Fermat's little theorem.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been reasonably well prepared for questions on some aspects of graph theory and using the Euclidean algorithm.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

Most candidates were able to name an algorithm to find the lowest cost road system and then were able apply the algorithm. All but the weakest candidates were able to make a meaningful start to this question. In 1(b) some candidates lost marks by failing to indicate the order in which edges were added.

Question 2

Part (a) of this question was the most accessible on the paper and was completed correctly by the majority of candidates. Most candidates were able to start part (b), but a number made errors on the way and quite a number failed to give the general solution.

Question 3

Stronger candidates had little problem with this question, but a significant number of weaker candidates started by making errors in drawing the graph G, where the most common error was the omission of the loops and double edges. They also had problems working with the concepts of Eulerian circuits and Hamiltonian cycles. A majority of candidates were unable to complete part (d), with a significant number showing no indication that they understood what was required.

Question 4

There were a number of totally correct solutions to this question, but many students were unable to fully justify the result. Some candidates had learnt a formula to apply to the Chinese remainder theorem, but could not apply it well in this situation. Many worked with the conditions for divisibility but did not make much progress with the justification.

Question 5

There were very few fully correct answers. If Fermat's little theorem was known, it was not well applied.

Recommendation and guidance for the teaching of future candidates

- Students need to cover the entire syllabus.
- Students need to know the correct terminology.
- Students need to be aware that contextual questions can be asked.

Paper three – Series and differential equations

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 16	17 - 25	26 - 32	33 - 40	41 - 47	48 - 60

The areas of the programme and examination that appeared difficult for candidates

On this paper candidates found difficulty in deciding which was an appropriate series convergence test and in solving differential equations correctly.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been well prepared for questions on L'Hopital's rule and using Euler's method to solve a differential equation.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This question was accessible to the vast majority of candidates, who recognised that L'Hopital's rule was required. A few of the weaker candidates did not realise that it needed to be applied twice in part (b). Many fully correct solutions were seen.

Question 2

Part (a) of the question was set up in an unusual way, which caused a problem for a number of candidates as they tried to do part (b) first and then find the Maclaurin series by a standard method. Few were successful as they were usually weaker candidates and made errors in finding the solution $y = f\ x$. The majority of candidates knew how to start part (b) and recognised the need to use an integrating factor, but a number failed because they missed out the negative sign on the integrating factor, did not realise that $e^{\ln\cos x} = \cos x$ or were unable to integrate $\cos^2 x$. Having said this, a number of candidates succeeded in gaining full marks on this question.

Question 3

This question was found to be the hardest on the paper, with only the best candidates gaining full marks on it. Part (a) was very poorly done with a significant number of candidates unable to start the question. More students recognised part (b) as an integral test, but often could not progress beyond this. In many cases, students appeared to be guessing at what might constitute a valid test.

Question 4

Part (a) was well done by many candidates, but a number were penalised for not using a sufficient number of significant figures. Part (b) was started by the majority of candidates, but only the better candidates were able to reach the end. Many were unable to complete the

question correctly because they did not know what to do with the substitution $y = vx$ and because of arithmetic errors and algebraic errors.

Recommendation and guidance for the teaching of future candidates

- Students need to cover the entire syllabus.
- Students need to understand the conditions for the application of series convergence tests.
- Students need to have a solid background with skills and understanding in the core calculus portion of the HL programme to be successful with this option.

Paper three – Sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 16	17 - 24	25 - 31	32 - 39	40 - 46	47 - 60

The areas of the programme and examination that appeared difficult for candidates

On this paper candidates found difficulty with finding equivalence classes, showing that a function is a bijection, and finding that set difference is associative.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been reasonably well prepared for questions on most aspects of group theory.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates were aware of the group axioms and the properties of a group, but they were not always explained clearly. A number of candidates did not understand the term “Abelian”. Many candidates understood the conditions for a group to be cyclic. Many candidates did not realise that the answer to part (e) was actually found in part (d), hence the reason for this part

only being worth 1 mark. Overall, a number of fully correct solutions to this question were seen.

Question 2

Part (a) of this question was the most accessible on the paper and was completed correctly by the majority of candidates. Part (b) was completed by many candidates, but a significant number either did not understand what was meant by associative, confused associative with commutative, or were unable to complete the algebra.

Question 3

Stronger candidates had little problem with part (a) of this question, but proving an equivalence relation is still difficult for many. Equivalence classes still cause major problems and few fully correct answers were seen to this question.

Question 4

Many students were able to show that the expression was injective, but found more difficulty in showing it was subjective. As with question 1 part (e), a number of candidates did not realise that the answer to part (b) came directly from part (a), hence the reason for it being worth only one mark.

Question 5

This question was found difficult by a large number of candidates, but a number of correct solutions were seen. A number of candidates who understood what was required failed to gain the final reasoning mark. Many candidates seemed to be ill-prepared to deal with this style of question.

Recommendation and guidance for the teaching of future candidates

- Students need to cover the entire syllabus.
- Students need to know the correct terminology.
- Students need to understand that they will be penalised for poor explanation or layout of work.
- In this option questions involving proof will be asked and it is essential that students understand that a degree of rigour is needed in these proofs.

Paper three – Statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 15	16 - 21	22 - 28	29 - 34	35 - 41	42 - 60

The areas of the programme and examination that appeared difficult for candidates

On this paper candidates found difficulty with working with the exponential distribution and the geometric distribution.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been reasonably well prepared for questions on expectation algebra, t-distributions, Normal distributions and confidence intervals.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates were able to access this question, but weaker candidates did not always realise that parts (b) and (c) were testing different things. Part (b) proved the hardest with a number of candidates not understanding how to find the variance of the sum of variables.

Question 2

This question also proved accessible to a majority of candidates with many wholly correct or nearly wholly correct answers seen. A few candidates did not recognise that part (a) was a t-distribution and part (b) was a Normal distribution, but most recognised the difference. Many candidates received an accuracy penalty on this question for not giving the final answer to part (b) to 3 significant figures.

Question 3

Stronger candidates had little problem with this question, but a significant number of weaker candidates encountered a number of problems. Many did not realise that part (b) could be done using the answer to part (a) and the manipulation of logarithms in part (iii) was weak. Weaker candidates knew how to start part (c), but encountered problems by rounding the expected values and forgetting to combine equivalence classes. Some candidates seemed to

think that the criterion for combining classes is that the observed frequency rather than the expected frequency is less than 5.

Question 4

This question was found difficult by the majority of candidates and few fully correct answers were seen. Few candidates were able to find $P(X = x)$ in terms of n and x and many did not realise that the last part of the question required them to find the sum of a series. However, better candidates received over 75% of the marks because the answers could be followed through.

Recommendation and guidance for the teaching of future candidates

- Candidates need to be aware of the potential of the GDC in this paper. The majority of candidates were not using it to the full potential.
- Students need to cover the entire syllabus and be prepared for questions on any of the distributions given in the syllabus.
- In the statistics and probability option many students lose the accuracy penalty mark and other marks due to accuracy. Full accuracy should be used, except in the final answer, which should be given to 3 significant figures.
- Students need to have a solid background with skills and understanding in the core statistics and probability section of the HL programme to be successful with this option.