MATHS HL TZ2

Overall grade boundaries

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 42	43 - 54	55 - 67	68 - 79	80 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 28	29 - 40	41 - 52	53 - 65	66 - 77	78 - 100

Series and differential equations

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 28	29 - 40	41 - 52	53 - 66	67 - 78	79 - 100

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 28	29 - 40	41 - 52	53 - 65	66 - 77	78 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2008 examination session the IB has produced time zone variants of the Mathematics HL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 18	19 - 23	24 - 29	30 - 34	35 - 40

Many excellent portfolios were in evidence this session. Both teachers and students appear to have understood the assessment expectations well. Observations made by the moderators are summarised below:

The tasks:

Most portfolio tasks were taken from the current Teacher Support Material (TSM) for Mathematics HL, and those chosen were undoubtedly familiar to many teachers. There were also several excellent new tasks submitted by a number of schools. Teachers are encouraged to design their own tasks, keeping in mind the need to satisfy all criteria fully.

There were three concerns that arose with portfolio tasks:

1. Risking dire consequences for their candidates, some teachers continue to use old tasks taken from the previous TSM. Those tasks do not fully satisfy the current assessment criteria; hence, a number of candidates lost a significant number of marks through no fault of their own. Unless appropriate modifications were made, these older tasks should not have been used.

2. The use of the new tasks for 2009 and 2010 is not only premature, but results in a 10-mark penalty for inclusion in this session. Though isolated, such instances were sad to note. It is imperative that teachers be aware of the ramifications of assigning tasks haphazardly to their students, and in particular, the consequence to the student who must bear the penalty.

3. Tasks taken from the TSM for Mathematics SL are not at a suitable level for Mathematics HL and should not be used.

Candidates' performance

Most candidates performed well against criterion A. The use of computer notation seemed to be very limited; however, the inappropriate use of "^", "E09", and the like, continue to be missed by some teachers.

Many samples contained work that was well written. Where a student's work began with an introduction to the task, and comments, annotations, and conclusions accompanied the steps and results, the work was easy to read and follow, and earned high marks in criterion B. However, there were a few whose work seemed disjointed, providing nothing more than a question and answer format to the task. Unlabelled graphs and the relegation of tables to the appendix rate poorly in terms of an effective presentation.

Criteria C and D are meant to assess the mathematical content and jointly comprise half of the total marks awarded to each piece of work. Generally, students have produced good work, and the assessments by their teachers have been appropriate. However, in some type I tasks, insufficient exploration and patterning rendered the quick formulation of a conjecture questionable. Where several intermediate general statements were derived, the proof of “the general statement”, as opposed to “a general statement”, needed to be evident to warrant full marks.

In type II tasks, variables should be explicitly defined. Some realisation of the significance of the results obtained in terms of the model when compared to the actual situation should have been provided, and students should have reflected on their findings. The analyses of data must be quantified, and if a regression analysis were appropriate, the student must have provided reasons for a particular choice. The use of software that automatically determines the “best” regression model often leaves little for the candidate to interpret by himself; consequently, little credit can be awarded.

The use of technology varied considerably. Full marks were given much too generously for an appropriate but not necessarily a resourceful use of technology, for example, in the inclusion of a scatter plot produced on a calculator. As one moderator remarked some time ago, technology must be used to do more than merely “decorate” the work. Students should be discouraged from including GDC key sequences – they are quite unnecessary.

There were many good pieces of work; however, the awarding of full marks in criterion F requires more than completion and correctness, but the evidence of mathematical sophistication.

Suggestions to teachers

Please be advised that tasks from the present TSM must not be used as of the May 2009 examination session; consequently, they must not be used with candidates who started their diploma program in September 2007. The use of any tasks from the current or older TSM will carry a 10-mark penalty as of the May 2009 session. Please refer to the document, “Mathematics HL – The portfolio – Tasks for use in 2009 and 2010” for suggested tasks.

Teachers should select tasks that provide students with a variety of mathematical activities suitable at higher level. Tasks taken directly from the Mathematics SL TSM do not meet HL requirements. Please ensure that candidates do not lose marks due to inappropriate choices made by the teacher.

The teacher who is uninformed of the changes to the portfolio assessment criteria is generally the reason for a significant loss of marks in moderation. This is not only disastrous to the student, but also completely unfair, and must be rectified.

Teachers are expected to write directly on their students’ work, not only to provide feedback to students, but information to moderators as well. Some samples contained very few teacher comments. Moderation was extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks.

Moderators find the background to each portfolio task very useful in determining the context in which the task was given when confirming the achievement levels awarded. This information must accompany each sample, either on Form A or through anecdotal comments.

If a teacher-designed task is submitted, a solution key must accompany the portfolios in order that moderators can justify the accuracy of the work, and appreciate the level of sophistication demonstrated in the work.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 18	19 - 36	37 - 54	55 - 68	69 - 83	84 - 97	98 - 120

The areas of the programme and examination that appeared difficult for the candidates

On this paper many candidates had difficulty with the interpretation of questions in context – Question 9 and to a lesser extent Question 13. Questions involving substantial reasoning and proof techniques continue to cause difficulties. Many candidates have a poor understanding of complex numbers.

The areas of the programme and examination in which candidates appeared well prepared

There was an encouraging overall response to the first non-calculator Paper 1, with most candidates demonstrating a good knowledge of the syllabus and an ability to process algebraic expressions and vectors.

The strengths and weaknesses of the candidates in the treatment of individual questions

Section A

Question 1

This question was generally well done, but a few candidates tried integration for part (b).

Question 2

Most candidates successfully answered this question. The majority used the factor theorem, but a few employed polynomial division or a method based on inspection to determine the third linear factor.

Question 3

Many candidates used a lot of space answering this question, but were generally successful. A few candidates incorrectly used the formula for the cosine of the difference of angles. An interesting alternative solution was noted, in which the side AB is reflected in AD and the required result follows from the use of the cosine rule.

Question 4

This question was generally well done, with very few candidates calculating $f \circ g$ rather than $g \circ f$.

Question 5

Part (a) was generally well answered, almost all candidates realising that implicit differentiation was involved. A few failed to differentiate the right hand side of the relationship. A surprising number of candidates made an error in part (b), even when they had scored full marks on the first part.

Question 6

This question was reasonably well done, with few candidates making the inappropriate choice of u and $\frac{dv}{dx}$. The main source of a loss of marks was in finding v by integration. A few candidates used the double angle formula for sine, with poor results.

Question 7

This question was generally well done, with a few candidates spotting an opportunity to use results for the *independent events A* and *B.*

Question 8

There was a disappointing response to this question from a fair number of candidates. The differentiation was generally correctly performed, but it was then often equated to $-2x+c$ rather than the correct numerical value. A few candidates either didn't simplify $\arctan(1)$ to $\frac{\pi}{4}$, or stated it to be 45 or $\frac{\pi}{2}$.

Question 9

This was a question in context which proved difficult for many candidates. Many appeared not to have fully comprehended the implications of the details of the diagram. A few candidates attempted integration, for no apparent reason.

Question 10

Those candidates who chose to use the trigonometric version of Pythagoras' Theorem were generally successful, although a minority were unconvincing in their reasoning. Some candidates adopted a full component approach, but often seemed to lose track of what they were trying to prove. A few candidates used 2-dimensional vectors or specific rather than general vectors.

Section B

Question 11

Most candidates scored reasonably well on this question. The most common errors were: Using ***OB*** rather than ***AB*** in (a); omitting the $r =$ in (b); failure to check that the values of the two parameters satisfied the third equation in (c); the use of an incorrect vector in (d). Even when (d) was correctly answered, there was usually little evidence of why a specific vector had been used.

Question 12

Part (a) was correctly answered by the majority of candidates, although a few found $r = -3$. Part (b) was often started off well, but a number of candidates failed to initiate the $n = k + 1$ step in a satisfactory way. A number of candidates omitted the ‘P(1) is true’ part of the concluding statement.

Question 13

Most candidates scored well on this question. The question tested their competence at algebraic manipulation and differentiation. A few candidates failed to extract from the context the correct relationship between velocity, distance and time.

Question 14

Parts (a) and (b) were generally well done, although very few stated that $w \neq 1$ in (b). Part (c), the last question on the paper was challenging. Those candidates who gained some credit correctly focussed on the real part of the identity and realise that different cosine were related.

Recommendations and guidance for the teaching of future candidates

- Students should be encouraged to pay attention to the importance of structuring their extended answers using correct logic. This is particularly relevant to *Proofs by Mathematical Induction.*
- Teachers should emphasise the importance of drawing diagrams for vector questions particularly where lines and planes are involved.
- Teachers should emphasise the importance of carefully reading the question, especially where questions are set in unfamiliar contexts.
- As questions are often set where *exact* answers are required, and expected to be simplified, teachers should ensure that students are confident in the manipulation of surds and the relationship between logarithms and exponentials and between trigonometric functions and their inverses.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 33	34 - 46	47 - 60	61 - 75	76 - 89	90 - 120

General Comments

This was the first examination session where candidates sat a GDC-free Paper 1 and a GDC-required Paper 2. In Paper 2, several examiners commented on the number of very good scripts they have marked, which is pleasing. Evidence from candidates' scripts suggests that a significant majority used their time wisely and were able to sensibly attempt most of the paper. However, it was apparent that a significant proportion of candidates found it difficult to reconcile when a GDC approach was appropriate and when an analytic solution approach should be adopted (see recommendations and guidance for the teaching of future candidates for further comments).

Questions 3, 5, 7(b), 10, 11 and 13(a) were deemed to be GDC-required questions. Questions 2, 4 and 6 were questions where a GDC increases the number of plausible methods of solution available to candidates. Questions 1, 8, 9, 12 and 13(b) were questions where a GDC was not required and/or inappropriate to use.

Future candidates will need to be reminded that solutions obtained from a GDC should be supported by suitable working, *e.g.* either by an appropriate numerical or graphical approach. It is not considered sufficient for a candidate to offer the generic phrase 'done by GDC' as suitable supporting working. It is also important when using a GDC that candidates are well-versed in setting appropriate windows to assist with locating the intersection point coordinates of two functions.

A significant proportion of candidates were awarded an accuracy penalty for not expressing numerical answers correct to three significant figures where required. Indeed, a substantial number of candidates can consider themselves fortunate that the accuracy penalty is capped to one mark only. Candidates need to understand the requirement that unless otherwise stated in the question, all numerical answers must be given exactly correct to three significant figures where required.

Teachers must continue to emphasise the correct structure and language required in the setting out of mathematical induction proofs. A substantial number of candidates either did not seem to understand or ignored key phrases such as 'hence', or 'show that'.
For future teaching and learning, it is important that both teachers and students realise that not all questions set in Paper 2 will require the use of a GDC.

The areas of the programme and examination that appeared difficult for the candidates

In question 13, the majority of candidates were seemingly unaware that the method of separation of variables was required to set up the differential equations that described the motion of a particle. A significant proportion of candidates were unable to evaluate a definite integral determined from using separation of variables. Most candidates did not seem to recognise alternative forms of acceleration such as $\frac{dv}{dt}$ or $v\frac{dv}{dx}$.

Other areas of concern included difficulties solving a trigonometric inequality correctly, recognising the ambiguous case of the sine rule, confusing the mean, median and mode of a continuous probability density function, locating and justifying the existence of a point of inflection of a function specified with a restricted domain, solving equations numerically (including the use of a table of values) and graphically, sketching and locating the key features of the graph of a reciprocal function from the graph of an unspecified function, using complex number properties and algebra to demonstrate a given result, using correct Poisson distribution inequalities, setting out mathematical induction proofs correctly and correctly manipulating differentials when using either the method of separating variables or inversion.

Generally, and also not surprisingly, question parts that required either more sophisticated mathematical reasoning or more demanding algebraic manipulation challenged the majority of candidates.

The areas of the programme and examination in which candidates appeared well prepared

Successful candidates generally exhibited excellent algebraic skills, judicious GDC use and substantive conceptual reasoning. These attributes were demonstrated in performing probability calculations, exploring some matrix properties and in setting up and solving equations (including differential equations) using a variety of numerical and analytic techniques. Questions that asked candidates to set up and solve a pair of simultaneous linear equations were generally answered very well.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This was an easy question that was well done by most candidates. Careless arithmetic errors caused some candidates not to earn full marks. Only a few candidates realised that part (b) could be answered correctly by directly subtracting 3 from their answer to part (a). Most successful responses were obtained by redoing the calculation from part (a).

Question 2

Not as well done as expected with most successful candidates using a graphical approach. Some candidates confused t and h and subsequently stated the values of t for which the water depth was either at a maximum and a minimum. Some candidates simply gave the maximum and minimum coordinates without stating the maximum and minimum depths.

In part (b), a large number of candidates left out $t = 24$ from their final answer. A number of candidates experienced difficulties solving the inequality via algebraic means. A number of candidates specified incorrect intervals or only one correct interval.

Question 3

This was generally well done. In part (a), most candidates were able to find $x = 1.28$ successfully. A significant number of candidates were awarded an accuracy penalty for expressing answers to an incorrect number of significant figures.

Part (b) was generally well done. A number of candidates unfortunately omitted the dx in the integral while some candidates omitted to write down the definite integral and instead offered detailed instructions on how they obtained the answer using their GDC.

Question 4

A significant number of candidates attempted to find the mode and the mean using calculus when it could be argued that these quantities could be found more efficiently with a GDC.

A significant proportion of candidates demonstrated a lack of understanding of the definitions governing the mean, mode and median of a continuous probability density function. A significant number of candidates attempted to calculate the median instead of either the mean or the mode. A number of candidates prematurely rounded their value for the mode i.e. subsequently using 0.7 for example rather than using the exact value of $\frac{2}{3}$. A few candidates offered negative probability values or probabilities greater than one.

Question 5

A large proportion of candidates did not identify the ambiguous case and hence they only obtained one correct value of AC. A number of candidates prematurely rounded intermediate results (angles) causing inaccurate final answers.

Question 6

Most candidates adopted an algebraic approach rather than a graphical approach. Most candidates found $f'(x)$ correctly, however when attempting to find $f''(x)$, a surprisingly large number either made algebraic errors using the product rule or seemingly used an incorrect form of the product rule. A large number ignored the domain restriction and either expressed $x = \pm\frac{1}{2}$ as the *x*-coordinates of the point of inflection or identified $x = \frac{1}{2}$ rather than $x = -\frac{1}{2}$. Most candidates were unsuccessful in their attempts to justify the existence of the point of inflection.

Question 7

Part (a) was generally well done. The most common error was to omit the binomial coefficient *i.e*. not identifying that the situation is described by a binomial distribution.

Finding the correct value of n in part (b) proved to be more elusive. A significant proportion of candidates attempted algebraic approaches and seemingly did not realise that the equation could only be solved numerically. Candidates who obtained $n = 10$ often accomplished this by firstly attempting to solve the equation algebraically before 'resorting' to a GDC approach. Some candidates did not specify their final answer as an integer while others stated $n = 1.76$ as their final answer.

Question 8

A large number of candidates had difficulty graphing the reciprocal function. Most candidates were able to locate the vertical asymptotes but experienced difficulties graphing the four constituent branches. A common error was to specify incorrect coordinates of the local maximum *i.e.* $(0,-1)$ or $(0,-2)$ instead of $\left(0,-\frac{1}{2}\right)$. A few candidates attempted to sketch the inverse while others had difficulty using the scaled grid.

Question 9

This was a difficult question that troubled most candidates. Most candidates were able to substitute $z = x + y\mathrm{i}$ into w but were then unable to make any further meaningful progress. Common errors included not expanding $(x + y\mathrm{i})^2$ correctly or not using a correct complex conjugate to make the denominator real. A small number of candidates produced correct solutions by using $w = \frac{1}{z + z^{-1}}$.

Question 10

This question was generally not well done. A number of candidates attempted an 'ill-fated' algebraic approach. Most candidates who used their GDC were able to correctly locate one inequality. The few successful candidates were able to employ a suitable window or suitable window(s) to correctly locate both inequalities.

Question 11

This question was generally well done despite a large proportion of candidates being awarded an accuracy penalty. Candidates found part (a) (i) to be quite straightforward and was generally done very well. In part (a) (ii), a number of candidates used $\frac{d-6}{1.5} = 1.0364...$ instead of $\frac{d-6}{1.5} = -1.0364...$. In part (b), a pleasingly high number of candidates were able to set up and solve a pair of simultaneous linear equations to correctly find the values of μ and σ. Some candidates prematurely rounded intermediate results. In part (c), a number of candidates were unable to express a correct Poisson inequality. Common errors included stating $\mathrm{P}(T \geq 3) = 1 - \mathrm{P}(T \leq 3)$ and using $\mu = 7$.

Question 12

No comment.

Question 13

No comment.

Recommendations and guidance for the teaching of future candidates

- Provide students with a range of GDC-required revision questions from appropriate areas of study. Discuss various solution approaches (*e.g*. the use of tabular, graphical and numerical solving features) with students and how solutions should be communicated clearly to examiners. In particular, dissuade students from using the phrase 'done with GDC' as a way of supporting argumentation. Ensure that students understand that GDC inputs should be expressed with correct mathematical notation and not in terms of brand specific calculator syntax and/or instructions.

- Discuss with students when it is appropriate to use a GDC and when an analytic (algebraic) solution approach is required. This discussion should be situated within the context of the new examination structure that commenced in May 2008.

- Ensure that students are aware that future Paper 2 questions will also contain questions or question parts that will not require the use of a GDC.

- Continue to highlight the importance of constructing a correct concluding statement when undertaking induction proofs. For example, $P(k)$ is true implies that $P(k+1)$ is true, and as $P(1)$ true then $P(n)$ is true for all positive integers.

- Encourage students to use mathematical notation when stating probability expressions.

- Encourage students to question the reasonableness of results they obtain.

- Discuss with students what phrases such as 'hence', 'exact' or 'show that' mean in the context of answering examination questions.

Paper three

Component grade boundaries

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 16	17 - 23	24 - 31	32 - 38	39 - 46	47 - 60

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 16	17 - 23	24 - 30	31 - 36	37 - 43	44 - 60

Series and differential equations

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 18	19 - 26	27 - 35	36 - 43	44 - 60

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 14	15 - 19	20 - 26	27 - 33	34 - 40	41 - 60

General Comments

Examiners opined that the four options were evenly balanced in terms of length and difficulty. G2 comments reflected a general consensus that the questions were accessible, well graded and enabled discrimination between candidates.

The areas of the program which proved difficult for candidates

1. **Statistics:** Expectation algebra involving $E(Y^2)$; choosing an appropriate test; type I and II errors; test of differences of dependent variables.

2. **Sets:** subgroups; injection and surjection; proofs involving function algebra; proofs involving sets; equivalence relations and classes.

3. **Series:** limits using series; deriving one series from another; Maclaurin series; correct approach to improper integrals; tests of convergence; Euler's method.

4. **Discrete:** finer points of Euclid's algorithm; modular arithmetic; Eulerian trails; Fermat's theorem; proofs involving face, edge and vertices relationships.

The levels of knowledge, understanding and skill demonstrated

In all of the options there was a noticeable lack of appreciation of the niceties of solving problems and in correctly writing down solutions. The impression was that the main aim was to find an answer and not worry too much about how the solution was set out for the examiner. There was often a distinct lack of rigor.

1. Statistics

The following were handled competently: linear transformation of a single random variable; recognition of probability distributions; p-values; use of graphics calculator; chi-squared test. Many candidates showed confidence in attempting this option although question 5 proved difficult. In this question too many candidates wasted time considering confidence intervals.

2. Sets

Cayley tables were found easy. De Morgan's laws were applied correctly in the most part. The two properties symmetry and reflexivity were well done. Recognition of a Latin square was good and the application of Lagrange's theorem was well known.

3. Series

Finding limits did not prove too difficult and most candidates knew how and when to apply l'Hopital's rule. Partial fractions are known by most students and many students managed to do the improper integral. Most candidates managed to solve the differential equation. Finding successive derivatives of $\ln(\cos x)$ did not prove difficult. A limited knowledge of convergence tests was evident.

4. Discrete

Most students could handle the Euclid algorithm bearing in mind the point made above. The drawing of the planar graph was also well done. The book-proof asked for in question 3(a) did not trouble most students and the knowledge of bipartite graphs was good. A basic knowledge of Euler's relationship was in evidence.

The strength and weaknesses of candidates in the treatment of individual questions

1. Statistics

Question 1

$E(Y)$ was calculated correctly but many could not go further to find $Var(Y)$ and $E(Y^2)$ $Var(2)$ was often taken to be 2. V was often taken to be discrete leading to calculations such as $P(V > 5) = 1 - P(V \leq 5)$.

Question 2

Many candidates used a *t*-test on this question. This was possibly because the sample was large enough to approximate normality of a proportion. The need to use a one-tailed test was often missed. When using the *z*-test of proportions $\hat{p} = 0.04$ was often used instead of $\hat{p} = 0.02$. Not many candidates used the binomial distribution.

Question 3

Although this question was reasonably well done the hypotheses were often not stated precisely and the fact that the two data sets were dependent escaped many candidates.

Question 4

This question was well done although the word 'exact' in (a) was often ignored.

The problem of combining columns remains and the fact that the expected frequencies must add to 80 escaped many candidates. There is still confusion about the accuracy of intermediate calculations and how they affect the final answer.

Question 5

This question proved to be the most difficult. The range of solutions ranged from very good to very poor. Many students thought that $P(TypeI) = 1 - P(TypeII)$ when in fact $1 - P(TypeII)$ is the power of the test.

2. Sets

Question 1

The table was well done as was showing its group properties. The order of the elements in (b) was done well except for the order of 0 which was often not given. Finding the generators did not seem difficult but correctly stating the subgroups was not often done. The notion of a 'proper' subgroup is not well known.

Question 2

'Using features of the graph' should have been a fairly open hint but too many candidates contented themselves with describing what injective and surjective meant rather than explaining which graph had which properties. Candidates found considerable difficulty with presenting a convincing argument in part (b).

Question 3

Venn diagram 'proof' are not acceptable. Those who used de Morgan's laws usually were successful in this question.

Question 4

Not a difficult question although using the relation definition to fully show transitivity was not well done. It was good to see some students use an operation binary matrix to show transitivity. This was a nice way given that the set was finite. The proof in (b) proved difficult.

Question 5

Part (a) presented no problem but finding the order two subgroups (Lagrange's theorem was often quoted correctly) was beyond some candidates. Possibly presenting the set in non-alphabetical order was the problem.

3. Series

Question 1

Part (a) was well done but too often the instruction to use series in part (b) was ignored. When this hint was observed correct solutions followed.

Question 2

Not a difficult question but combination of the logarithms obtained by integration was often replaced by a spurious argument with infinities to get an answer. $\log(\infty+1)$ was often seen.

Question 3

Some incomplete tables spoiled what were often otherwise good solutions. Although the intermediate steps were asked to four decimal places the answer was not and the usual degree of IB accuracy was expected.

Some candidates surprisingly could not solve what was a fairly easy differential equation in part (b).

Question 4

Some candidates had difficulty organizing the derivatives but most were successful in getting the series. Using the series to find the approximation for $\ln 2$ in terms of π was another story and it was rare to see a good solution.

Question 5

Some corners were cut in applying the ratio test and some candidates tried to use the comparison test. With careful algebra finding the radius of convergence was not too difficult. Often the interval of convergence was given instead of the radius.

Part (b) was done only by the best candidates. A little algebraic manipulation together with an auxiliary series soon gave the answer.

4. Discrete

Question 1

This problem was not difficult but presenting a clear solution and doing part (b) alongside part (a) in two columns was. The simple answer to part (c) was often overlooked.

Question 2

Drawing the graph usually presented no difficulty. Distinguishing between Eulerian and semi-Eulerian needs attention but this part was usually done successfully.

A simple, clear argument for part (c) was often hidden in mini-essays on graph theory.

Question 3

Part (a) (i) was not found difficult but using it in part (a)(ii) resulted in two or three correct lines and then abandonment of the problem.

Question 4

Part (a) was usually done correctly but then clear argument for parts (b) and (c) were rare.

Question 5

Part (a)(i) was done successfully but many students did not read part(ii) carefully. It said '**adding an edge'** nothing else. Many candidates assumed it was necessary to add a vertex when this was not the case.

Part (b) was not found to be beyond many candidates if they used the inequality $e \leq 3v - 6$

The type of assistance and guidance the teachers should provide for future candidates

There were many good and sometimes even exceptional scripts.

My concern is that even with the time pressures in teaching the course not enough emphasis is given to situations that are more than just routine. Using unusual examples, possibly from the history of mathematics, helps engender interest and effort. Euler's work is a particularly good source for series work (the Basel problem for example) and graph theory. Contrasting the similarities and differences between probability distributions, between convergence tests, between numerical and analytical methods of solving differential equations helps encourage mathematical discretion in a student.