International Baccalaureate
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MATHS HL TZ1

Overall grade boundaries

Discrete mathematics

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 38	39 - 50	51 - 62	63 - 74	75 - 100

Series and differential equations

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 38	39 - 49	50 - 62	63 - 74	75 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 37	38 - 49	50 - 61	62 - 73	74 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 26	27 - 37	38 - 49	50 - 61	62 - 73	74 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2008 examination session the IB has produced time zone variants of the Mathematics HL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 18	19 - 23	24 - 29	30 - 34	35 - 40

Many excellent portfolios were in evidence this session. Both teachers and students appear to have understood the assessment expectations well. Observations made by the moderators are summarised below:

The tasks:

Most portfolio tasks were taken from the current Teacher Support Material (TSM) for Mathematics HL, and those chosen were undoubtedly familiar to many teachers. There were also several excellent new tasks submitted by a number of schools. Teachers are encouraged to design their own tasks, keeping in mind the need to satisfy all criteria fully.

There were three concerns that arose with portfolio tasks:

1. Risking dire consequences for their candidates, some teachers continue to use old tasks taken from the previous TSM. Those tasks do not fully satisfy the current assessment criteria; hence, a number of candidates lost a significant number of marks through no fault of their own. Unless appropriate modifications were made, these older tasks should not have been used.

2. The use of the new tasks for 2009 and 2010 is not only premature, but results in a 10-mark penalty for inclusion in this session. Though isolated, such instances were sad to note. It is imperative that teachers be aware of the ramifications of assigning tasks haphazardly to their students, and in particular, the consequence to the student who must bear the penalty.

3. Tasks taken from the TSM for Mathematics SL are not at a suitable level for Mathematics HL and should not be used.

Candidates' performance

Most candidates performed well against criterion A. The use of computer notation seemed to be very limited; however, the inappropriate use of "^", "E09", and the like, continue to be missed by some teachers.

Many samples contained work that was well written. Where a student's work began with an introduction to the task, and comments, annotations, and conclusions accompanied the steps and results, the work was easy to read and follow, and earned high marks in criterion B. However, there were a few whose work seemed disjointed, providing nothing more than a question and answer format to the task. Unlabelled graphs and the relegation of tables to the appendix rate poorly in terms of an effective presentation.

Criteria C and D are meant to assess the mathematical content and jointly comprise half of the total marks awarded to each piece of work. Generally, students have produced good work, and the assessments by their teachers have been appropriate. However, in some type I tasks, insufficient exploration and patterning rendered the quick formulation of a conjecture questionable. Where several intermediate general statements were derived, the proof of “the general statement”, as opposed to “a general statement”, needed to be evident to warrant full marks.

In type II tasks, variables should be explicitly defined. Some realisation of the significance of the results obtained in terms of the model when compared to the actual situation should have been provided, and students should have reflected on their findings. The analyses of data must be quantified, and if a regression analysis were appropriate, the student must have provided reasons for a particular choice. The use of software that automatically determines the “best” regression model often leaves little for the candidate to interpret by himself; consequently, little credit can be awarded.

The use of technology varied considerably. Full marks were given much too generously for an *appropriate* but not necessarily a *resourceful* use of technology, for example, in the inclusion of a scatter plot produced on a calculator. As one moderator remarked some time ago, technology must be used to do more than merely “decorate” the work. Students should be discouraged from including GDC key sequences – they are quite unnecessary.

There were many good pieces of work; however, the awarding of full marks in criterion F requires more than completion and correctness, but the evidence of mathematical sophistication.

Suggestions to teachers

Please be advised that tasks from the present TSM must not be used as of the May 2009 examination session; consequently, they must not be used with candidates who started their diploma program in September 2007. The use of any tasks from the current or older TSM will carry a 10-mark penalty as of the May 2009 session. Please refer to the document, “Mathematics HL – The portfolio – Tasks for use in 2009 and 2010” for suggested tasks.

Teachers should select tasks that provide students with a variety of mathematical activities suitable at higher level. Tasks taken directly from the Mathematics SL TSM do not meet HL requirements. Please ensure that candidates do not lose marks due to inappropriate choices made by the teacher.

The teacher who is uninformed of the changes to the portfolio assessment criteria is generally the reason for a significant loss of marks in moderation. This is not only disastrous to the student, but also completely unfair, and must be rectified.

Teachers are expected to write directly on their students’ work, not only to provide feedback to students, but information to moderators as well. Some samples contained very few teacher comments. Moderation was extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks.

Moderators find the background to each portfolio task very useful in determining the context in which the task was given when confirming the achievement levels awarded. This information must accompany each sample, either on Form A or through anecdotal comments.

If a teacher-designed task is submitted, a solution key must accompany the portfolios in order that moderators can justify the accuracy of the work, and appreciate the level of sophistication demonstrated in the work.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 30	31 - 44	45 - 59	60 - 73	74 - 88	89 - 120

The areas of the programme and examination that appeared difficult for candidates

On this paper candidates found difficulty with matrices, trigonometry, continuous probability distributions, some aspects of integration, determining between maximum points, minimum points and points of inflexion, vectors and differential equations. The quality of curve sketching was also weak. There are indications that a number of candidates were not prepared for questions on all aspects of the syllabus and that a number of candidates spent too much time on section A and hence ran out of time on section B.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been reasonably well prepared for questions on most aspects of differentiation, some aspects of integration, some aspects of series, functions and the basic principles behind a proof by induction. As this was the first time that a non-calculator paper had been set, there was no indication that candidates struggled with the arithmetic in any of the questions.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates made a meaningful attempt at this question using a variety of different, but correct methods. Weaker candidates sometimes made errors with the manipulation of the square roots, but there were many fully correct solutions.

Question 2

There were fewer correct solutions to this question than might be expected with a significant minority of candidates either not understanding the word singular or the condition for a matrix to be singular. In both these cases, candidates were unable to gain any of the marks. This highlights the need for teachers to teach students appropriate terminology.

Question 3

Stronger candidates had little problem with this question, but a significant minority of weaker candidates were unable to access the question or worked with area and very quickly became confused. Candidates who realised that the area of each sector was proportional to the angle usually gained the correct answer.

Question 4

There were many totally correct solutions to this question, but again a significant minority did not make much progress. The most common reasons for this were that candidates immediately assumed that because the question asked for the cosine of $\hat{\mathrm{C}}$ that they should use the cosine rule, or they did not draw a diagram and then confused which angles were opposite which sides.

Question 5

Most candidates were able to correctly differentiate the function and find the point where $f'(x) = 0$. They were less successful in determining the nature of the point.

Question 6

This was the question that gained the most correct responses. A few candidates struggled to find the limits of the integration or found a negative area.

Question 7

Part (a) was the first question that a significant majority of candidates struggled with. Only the best candidates were able to find the required set of values. However, it was pleasing to see that the majority of candidates made a meaningful start to part (b). Many candidates gained wholly correct answers to part (b).

Question 8

Part (a) was correctly done by the vast majority of candidates. In contrast, only the very best students gave the correct answer to part (b). Part (c) was correctly started by a majority of candidates, but many did not realise that they needed to use logarithms and were careless about the use of notation

Question 9

All but the best candidates struggled with part (a). The vast majority either did not attempt it or let $t = 1$. There was no indication from any of the scripts that candidates wasted an undue amount of time in trying to solve part (a). Many candidates attempted part (b), but few had a full understanding of the situation and hence were unable to give wholly correct answers.

Question 10

Only the best candidates were able to make significant progress with this question. It was disappointing to see that many candidates could not state that the formula for the required volume was $\pi \int_1^e \left(\frac{\ln x}{x}\right)^2 dx$. Of those who could, very few either attempted integration by parts or used an appropriate substitution.

Question 11

It was disappointing to see that a number of candidates did not appear to be well prepared for this question and made no progress at all. There were a number of schools where no candidate made any appreciable progress with the question. A good number of students, however, were successful with part (a) (i). A good number of candidates were also successful with part a (ii) but few realised that the shortest distance was the height of the triangle. Candidates used a variety of methods to answer (a) (iii) but again a reasonable number of correct answers were seen. Candidates also had a reasonable degree of success with part (b), with a respectable number of correct answers seen.

Question 12

This was the most accessible question in section B for these candidates. A majority of candidates produced partially correct answers to part (a), but a significant number struggled with demonstrating that the point is a minimum, despite the hint being given in the question. Part (b) started quite successfully but many students were unable to prove it is a point of inflexion or, more commonly, did not attempt to justify it. Correct answers were often seen for part (c). Part (d) was dependent on the successful completion of the first three parts. If candidates made errors in earlier parts, this often became obvious when they came to sketch the curve. However, few candidates realised that this part was a good way of checking that the above answers were at least consistent. The quality of curve sketching was rather weak overall, with candidates not marking points appropriately and not making features such as asymptotes clear. It is not possible to tell to what extent this was an effect of candidates not having a calculator, but it should be noted that asking students to sketch curves without a calculator will continue to appear on non-calculator papers. In part (e) the basic idea of proof by induction had clearly been taught with a significant majority of students understanding this. However, many candidates did not understand that they had to differentiate again to find the result for $(k+1)$.

Question 13

Candidates found this question quite difficult, with only the better students making appreciable progress on part (a). Relatively few candidates recognised that part (b) was asking them to solve a differential equation. Many students tried methods involving direct proportion, which did not lead anywhere.

Recommendation and guidance for the teaching of future candidates

- Students need to cover the entire syllabus.

- Students should be encouraged to pay attention to mathematical notation and accuracy.
- Teachers should emphasise the importance of students setting out their procedures in a logical fashion.
- Most of the questions in this paper used common problem solving strategies and this should be a focus for candidates.
- Students need to practice papers of a similar style in order that they understand the need to balance their time.
- Students need to be made aware of appropriate terminology.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 40	41 - 53	54 - 67	68 - 80	81 - 120

The areas of the programme and examination that appeared difficult for candidates

This is the first session with the new assessment format, i.e., no calculator is allowed on paper 1, and paper 2 requires a graphics display calculator. Additionally, both papers 1 and 2 now have short and long questions. The comments from some teachers on the G2 forms regarding a longer paper and increased level of difficulty over last year's paper may in part be due to the new format.

The areas of the core syllabus that candidates had particular difficulty with were complex numbers, vectors, some aspects of geometry, trigonometry, and differential equations. Many candidates incurred an accuracy penalty with some candidates incorrectly rounding throughout the paper.

The opportunities for use of the GDC on some questions were not always taken. This increased the working out time for questions designed to be solved using the GDC.

The areas of the programme and examination in which candidates appeared well prepared

The areas of the core syllabus that candidates manifested most ability in were calculus, probability and statistics, and functions. Candidates generally showed a good level of algebraic manipulative skills.

In general, candidates were successful in using their graphics display calculators to graph functions and find key points on their graphs, calculate probabilities, and perform routine calculations.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

Although the majority of the candidates understood the question and attempted it, excessive time was spent on actually expanding the expression without consideration of the binomial theorem. A fair amount of students confused “ascending order”, giving the last three instead of the first three terms.

Question 2

Part (a) was well executed by the majority of candidates. Most candidates had the correct graph with the correct x and y intercepts. For part (b), some candidates had the straight line intersect the x-axis at 3 rather than at π, and hence did not observe that there were 5 points of intersection.

Question 3

On the whole this question was well answered. Some candidates failed to find the complementary angle when using the formula with cosine.

Question 4

A fair amount of students did not use their GDC directly, but used tables and more traditional methods to answer this question. Part (a) was answered correctly by most candidates using any method. A large number of candidates reversed the probabilities, i.e., failed to use a negative z value in parts (b) and (c), and hence did not obtain correct answers.

Question 5

A large number of candidates did not use their GDC in this question. Some candidates who attempted analytical solutions looked for a point solution although the question specifically states that the planes intersect in a line. Other candidates eliminated one variable and then had no clear strategy for proceeding with the solution.

Some candidates failed to write $'r='$, and others did not give the equation in vector form.

Question 6

A large number of candidates obtained full marks on this question. Some candidates missed π and/or $\frac{dy}{dx}$ when differentiating the trigonometric function. Some candidates attempted to rearrange before differentiating, and some made algebraic errors in rearranging.

Question 7

The majority of candidates obtained the first two marks. Candidates who used their GDC to solve this question did so successfully, although few candidates provided a sketch as the rubric requires. Attempts to use “solver” only gave one solution.

Some candidates did not give the solutions as coordinate pairs, but simply stated the x and y values.

Question 8

This question was well answered by the majority of candidates. Most candidates used either tree diagrams or expected value methods.

Question 9

Many candidates obtained the first three marks, but then attempted various methods unsuccessfully. Quite a few candidates attempted integration by parts rather than substitution. The candidates who successfully integrated the expression often failed to put the absolute value sign in the final answer.

Question 10

Very few candidates scored more than the first two marks in this question. Some candidates had difficulty manipulating trigonometric identities. Most candidates did not get as far as defining the argument of the complex expression.

Question 11

Many candidates showed familiarity with the Poisson Distribution. Parts (a), (b), and (c) were straightforward, as long as candidates multiplied 0.2 by 30 to get the mean. Part (e) was answered successfully by most candidates. Parts (d) and (f) were done very poorly. In part (d), most candidates calculated $P(X=1)$ rather than $P(X \leq 1)$. Although some candidates realized the need for the Binomial in part (e), some incorrectly used 0.8 and 0.2.

Question 12

The majority of the candidates attempted part A of this question. Parts (a) and (b) were answered reasonably well. In part (c), many candidates scored the first two marks, but failed to recognize that the result was a quadratic equation, and hence did not progress further. Correct answers to part B were rarely seen. Although many candidates expressed RS correctly in two different ways, they failed to go on to use the cosine rule.

Question 13

Many candidates scored the full 6 marks for part (a). The main mistake evidenced was to treat k as a variable, and hence use the product rule to differentiate. Of the many candidates who attempted parts (b) and (c), few scored the R1 marks in either part, but did manage to get the equations of the straight lines.

Question 14

Part (a) of this question was answered fairly well by candidates who attempted this question. The main error was the sign of the argument of z_2. Few candidates attempted part (b), and of those who did, most scored the first two marks for equating the modulii. Only a very small number equated the arguments correctly using $2\pi k$.

Recommendation and guidance for the teaching of future candidates

- Teachers should ensure that students are familiar with explicit terminology stated in the syllabus, in particular, the argument of a complex number.
- Students need more guidance in giving clear and succinct mathematical explanations for questions involving "show that".
- Students need more training in knowing when to use, and not to use, the GDC. At times an analytical solution is not possible or accessible, and an early recognition of this would be more time effective under examination conditions.

Paper three – Discrete mathematics

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 17	18 - 25	26 - 32	33 - 38	39 - 45	46 - 60

The areas of the programme and examination that appeared difficult for candidates

Some candidates appear not to understand the significance of the term 'if and only if' and also not to appreciate fully the notion of a statement and its converse.

The areas of the programme and examination in which candidates appeared well prepared

Candidates are generally better prepared in the sections on graphs than the sections on number theory.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1

Most candidates were able to use the Euclidean Algorithm correctly to find the greatest common divisor. Candidates who used the GCD button on their calculators were given no credit. Some candidates seemed unaware of the criterion for the solvability of Diophantine equations.

Question 2

While most candidates gave a correct meaning to $x \equiv y(\bmod n)$, there were some incorrect statements, the most common being $x \equiv y(\bmod n)$ means that when *x* is divided by *n*, there is a remainder *y*. The true statement $8 \equiv 5(\bmod 3)$ shows that this statement is incorrect. Part (b) was solved successfully by many candidates but (c) caused problems for some candidates who thought that the result in (c) followed automatically from the result in (b).

Question 3

The response to this question was disappointing. Many candidates were successful in showing the 'if' parts of (a) and (b) but failed even to realise that they had to continue to prove the 'only if' parts.

Question 4

A fairly common error in (a) was to draw a non-planar version of *G*, for which no credit was given. In (b), most candidates realised that only one extra edge could be added but a convincing justification was often not provided. Most candidates were reasonably successful in (c) although in some cases not all possible Hamiltonian cycles were stated.

Question 5

In (a), many candidates derived the minimum spanning tree although in some cases the method was not clearly indicated as required and some candidates used an incorrect algorithm. Part (b) was reasonably answered by many candidates although some justifications were unsatisfactory. Part (c) caused problems for many candidates who found difficulty in writing down a rigorous proof of the required result.

Recommendation and guidance for the teaching of future candidates

- Candidates need to understand the meaning of 'if and only if'.

Paper three – Series and differential equations

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 17	18 - 23	24 - 30	31 - 37	38 - 44	45 - 60

The areas of the programme and examination that appeared difficult for candidates

Many candidates are unfamiliar with the remainder terms in Taylor series.

Many candidates are unable to use integration to estimate the sum of a series.

The areas of the programme and examination in which candidates appeared well prepared

L'Hopital's Rule and the Ratio Test are well understood by most candidates.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates used the Ratio Test successfully to establish convergence. Candidates who attempted to use Cauchy's (Root) Test were often less successful although this was a valid method.

Question 2

Most candidates made a reasonable attempt at (a). In (b), however, it was disappointing to note that some candidates were unable to use integration by parts to perform the integration. In (c), while many candidates obtained the correct value of the integral, proof of its convergence was often unconvincing.

Question 3

The response to this question was often disappointing. Many candidates were unable to find the integrating factor successfully.

Question 4

Many candidates failed to give a convincing argument to establish the inequality. In (b), few candidates progressed beyond simply evaluating the integral.

Question 5

In (a), some candidates appeared not to understand the term 'constant term'. In (b), many candidates found the differentiation beyond them with only a handful realising that the best way to proceed was to rewrite the function as $f(x) = -\ln(1-x)$. In (d), many candidates were unable to use the Lagrange formula for the upper bound so that (e) became inaccessible.

Recommendation and guidance for the teaching of future candidates

- Candidates need to be familiar with the forms of the remainder in Taylor series.

- Candidates need to understand the use of integration to estimate the sum of an infinite series.

Paper three – Sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 16	17 - 22	23 - 28	29 - 35	36 - 41	42 - 60

The areas of the programme and examination that appeared difficult for candidates

While many candidates are comfortable solving problems which deal with specific groups, theoretical problems on group theory are found difficult by many candidates. The notion of isomorphism is difficult for some candidates to grasp.

The areas of the programme and examination in which candidates appeared well prepared

Candidates are generally well prepared to solve problems involving binary operations and equivalence relations.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Almost all the candidates thought that the binary operation was associative, not realising that the non-closure prevented this from being the case. In the circumstances, however, partial credit was given to candidates who 'proved' associativity. Part (b) was well done by many candidates.

Question 2

Most candidates found the range of f correctly. Two algebraic methods were seen for solving (b), either showing that the derivative of f is everywhere positive or showing that $f(a) = f(b) \Rightarrow a = b$. Candidates who based their 'proof' on a graph produced on their graphical calculators were given only partial credit on the grounds that the whole domain could not be shown and, in any case, it was not clear from the graph that f was an injection.

Question 3

This question was reasonably well answered by many candidates, although in (b)(iii), some candidates were unable to give another group isomorphic to *G*.

Question 4

Part (a) was well answered by many candidates although some misunderstandings of the terminology were seen. Some candidates appeared to believe, incorrectly, that reflexivity was something to do with $(a,a)R(a,a)$ and some candidates confuse the terms 'reflexive' and 'symmetric'. Many candidates were unable to describe the equivalence classes geometrically.

Question 5

Few solutions were seen to this question with many candidates unable even to start.

Recommendation and guidance for the teaching of future candidates

- Candidates need to understand the notion of isomorphic groups.
- Candidates should be aware that it is not always possible to establish that a function is an injection using a graphical method.

Paper three – Statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 22	23 - 29	30 - 35	36 - 42	43 - 60

The areas of the programme and examination that appeared difficult for candidates

Candidates are often unable to distinguish between nX and $\sum_{i=1}^{n} X_i$, where X is a random variable.

In goodness-of-fit questions, candidates sometimes round their expected frequencies to the nearest integer which leads to inaccurate values of their chi-squared statistic.

The areas of the programme and examination in which candidates appeared well prepared

Candidates are generally well prepared in the use of their graphical calculators although they sometimes fail to explain fully exactly what they are doing.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This question was well answered in general with several correct methods seen. The most popular method was to use a GDC to carry out a proportion test which is equivalent to using a normal approximation. Relatively few candidates calculated an exact p-value using the binomial distribution. Candidates who found a 95% confidence interval for p, the probability of obtaining a head, and noted that this contained 0.5 were given full credit.

Question 2

Most candidates made a reasonable attempt at this question. Some candidates gave incorrect hypotheses, omitting any mention of the mean. It is important to realise that the hypotheses 'H_0 : Data can be modelled by an exponential distribution' and 'H_0 : Data can be modelled by an exponential distribution with mean 100 hours' are different and lead to different degrees of freedom. Some candidates rounded their expected frequencies to the nearest integer and lost marks for an inaccurate value of their chi-squared statistic.

Question 3

The response to this question was disappointing. Many candidates are unable to differentiate between quantities such as $3X$ and $X_1 + X_2 + X_3$. While this has no effect on the mean, there is a significant difference between the variances of these two random variables.

Question 4

Most candidates realised that the unbiased estimate of the mean was simply the central point of the confidence interval. Many candidates, however, failed to realise that, because the variance was unknown, the *t*-distribution was used to determine the confidence limits. In (b), although the p-value was asked for specifically, some candidates solved the problem correctly by comparing the value of their statistic with the appropriate critical values. This method was given full credit but, of course, marks were lost by their failure to give the p-value.

Question 5

Many candidates were unable even to start this question although those who did often made substantial progress.

Recommendation and guidance for the teaching of future candidates

- Candidates need to be more precise in stating the hypotheses being tested.
- Candidates should be aware of the importance difference between nX and $\sum_{i=1}^{n} X_i$.
- Candidates should understand that the *t*-distribution, and not the normal distribution, must be used when the variance is unknown.