MATHS HL TZ2

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 32	33 - 44	45 - 55	56 - 68	69 - 79	80 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 6	7 - 13	14 - 18	19 - 23	24 - 29	30 - 34	35 - 40

Many portfolios provided strong evidence of excellent investigations in mathematics. Although the changes to the portfolio requirements and assessment criteria as of May 2006 were significant, most teachers and their students seemed to have understood the expectations. The moderators have made a number of observations that are summarised below:

The tasks in the new syllabus

The majority of all portfolio tasks were taken from the current Teacher Support Material (TSM) for Mathematics HL. Unfortunately, where tasks were taken from the previous TSM, even the best effort by the student resulted in work that did not fully satisfy the current assessment criteria. Unless significant modifications are made, these older tasks should not be used. Teachers are encouraged to design their own tasks, keeping in mind the need to satisfy all criteria fully. It should be noted that investigative tasks that preclude the use of technology, and modelling tasks in which the model is not created by the student, but given within the task, fall short of fulfilling the requirements of the portfolio.

Tasks taken from the TSM for Mathematics SL are not at a suitable level for Mathematics HL and should not be used.

Candidate performance against each criterion

Most candidates performed well against criterion A. The use of computer notation seemed to be very limited. Correct terminology should include the use of correct mathematical vocabulary.

Where a student’s work began with an introduction to the task, and comments, annotations, and conclusions accompanied the steps and results, the work was easy to read and follow, and earned high marks in criterion B. On the other hand, those who have merely shown the steps to the solutions of problems, or left graphs unlabelled, and tables disconnected by their relegation to an appendix, demonstrated poor skills in presentation.

Jointly, criteria C and D are meant to assess the mathematical content and comprise half the marks awarded to each piece of work. Generally, students have produced good work, and the assessments by their teachers have been more appropriate this session. However, in some type I tasks, insufficient exploration and patterning rendered the quick formulation of a conjecture questionable. Where several intermediate general statements were derived, the proof of *the* general statement needed to be evident to warrant full marks.

In type II tasks, variables should be explicitly defined. Some realisation of the significance of the results obtained in terms of the model when compared to the actual situation should have been provided, and students should have reflected on their findings. The analyses of data must be quantified, and if a regression analysis were appropriate, the student must have provided reasons for a particular choice. The use of software that automatically determines the "best" regression model leaves little for the student to interpret.

The use of technology varied considerably from the truly resourceful to the merely perfunctory. Full marks were given much too generously for an *appropriate* but not necessarily a *resourceful* use of technology, for example, in the inclusion of a single graph produced on a calculator. As one moderator remarked, technology must be used to do more than merely "decorate" the work. Students should be discouraged from including GDC key sequences – they are unnecessary and unwarranted.

There were many, many good pieces of work; however, the awarding of full marks in criterion F requires more than completion and correctness, but the evidence of mathematical sophistication. (Please refer to the *Criteria notes* below.)

Recommendations for the teaching of future candidates

Teachers should select tasks that provide students with a variety of mathematical activities suitable at higher level. Tasks taken directly from the Mathematics SL TSM do not meet HL requirements and are not considered acceptable.

The teacher who remains uninformed or chooses to ignore the changes to the portfolio assessment criteria is generally the reason for a significant loss of marks in moderation. This is completely unfair to the student and must be rectified.

Teachers are expected to write directly on their students' work, not only to provide feedback to students, but information to moderators as well. Some samples contained very few teacher comments. Moderation was extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks.

Moderators find the background to each portfolio task very useful in determining the context in which the task was given when confirming the achievement levels awarded. This information must accompany each sample.

If a teacher-designed task is submitted, a solution key must accompany the portfolios in order that moderators can justify the accuracy of the work, and appreciate the level of sophistication demonstrated in the work.

Criteria notes

In order to clarify the requirements of the assessment criteria, notes were first directed to moderators after the standardisation meeting in April 2006, and then presented to all teachers through the two Subject Reports in 2006. At the risk of being redundant, they are presented here one last time in its entirety in the hope that teachers may find them useful.

Criterion A: use of notation and terminology

Tasks will probably be set before students are aware of the notation and/or terminology to be used. Therefore the key idea behind this criterion is to assess how well the students' use of terminology describes the context.

Teachers should provide an appropriate level of background knowledge in the form of notes given to students at the time the task is set.

Correct mathematical notation is required, but it can be accompanied by calculator notation, particularly when students are substantiating their use of technology.

This criterion addresses appropriate use of mathematical symbols (for example, use of "$\approx$" instead of "$=$" and proper vector notation).

Word processing a document does not increase the level of achievement for this criterion or for criterion B.

Students should take care to write in appropriate mathematical symbols if the word processing software does not supply them. Calculator/computer notation should not be used. Notation such as *x* ^ 2 or ABS(*x*) should not be used and such use will be penalised. A single shortcoming would not preclude the awarding of level 2.

Terminology may depend on the task. In the case of Type I (Investigation) tasks, terminology may include terms devised by the candidate (e.g. "slide", "shift"), provided that such terms reasonably reflect the appropriate mathematical concept.

Criterion B: communication

This criterion also assesses how coherent the work is. The work can achieve a good mark if the reader does not need to refer to the wording used to set the task. In other words, the task can be marked independently.

Level 2 cannot be achieved if the student only writes down mathematical computations without explanation.

Graphs, tables and diagrams should accompany the work in the appropriate place and not be attached to the end of the document. Graphs must be correctly labelled and must be neatly drawn on graph paper. Graphs generated by a computer program or a calculator "screen dump" are acceptable providing that all items are correctly labelled, even if the labels are written in by hand. Colour keying the graphs can increase clarity of communication. If, in reading a candidate's work, the teacher has to pause to clarify where a result came from or how it was achieved ("WHOA! Where did that come from?!"), this generally indicates flawed communication.

Computer/calculator output may need clarification. Graphs generated by a calculator or computer should present the variables and labels appropriate to the task. Hand-written labels may need to be added to screen dumps or printouts if the software doesn't provide for custom labels.

A single shortcoming would not preclude the awarding of level 3.

Criterion C: mathematical process

Type I—mathematical investigation: searching for patterns

Students can only achieve a level 3 if the amount of data generated is sufficient to warrant an analysis.

This is the process of getting the statement. Student gets 4 if everything is ready for the statement. The correctness of the statement is assessed in D.

If student gives a proof of the correct statement, no further cases need be investigated to award a level 5.

Type II—mathematical modelling: developing a model

At achievement level 5, applying the model to other situations could include, for example, a change of parameter or more data.

Any form of definition of variables, parameters constraints (informal/implied) is acceptable (e.g. labelling a graph or table, noting domain and range).

Criterion D: results

Type I—mathematical investigation: generalization

A student who gives a correct formal proof of the general statement that does not take into account scope or limitations would achieve level 4.

It is important to note the difference between "a (i.e. any) general statement" in level 2 and "the general statement" in level 3.

Type II—mathematical modelling: interpretation

"Appropriate degree of accuracy" means appropriate in the context of the task.

Criterion E: use of technology

The emphasis in this criterion is on the contribution of the technology to the mathematical development of the task rather than to the presentation and/or communication.

The level of calculator or computer technology varies from school to school. Therefore teachers should state the level of the technology that is available to their students. While printed output is not required, some statement confirming appropriate use of technology (from the teacher or student) is necessary.

Using a computer and/or a GDC to generate graphs or tables may not significantly contribute to the development of the task, and therefore may not merit more than a level 1.

Criterion F: quality of work

Students who satisfy all the requirements correctly achieve level 1. For a student to achieve level 2, work must show precision, insight and a sophisticated level of mathematical understanding.

Award level 2 only if the work presented is beyond ordinary expectations. The teacher will take pause to admire the quality of such work ("Wow! Now, that's impressive!").

Only a totally inadequate response would receive 0.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 19	20 - 39	40 - 53	54 - 66	67 - 80	81 - 93	94 - 120

General comments

There was general agreement among the examiners that this paper had proved accessible to the majority of candidates, who were able to show good performance levels on most subject areas. There was evidence that candidates had also managed their time well and been able to complete the paper in the time available.

The areas of the programme and examination that appeared difficult for the candidates

Generally speaking most candidates seem to have made relatively good attempts to most questions. The questions on integration, as well as the application of trigonometric identities, tended perhaps to be more poorly answered than others. Also, identifying and describing geometric transformations applied to functions caused difficulties for many.

The areas of the programme and examination in which candidates appeared well prepared

Good use of the GDC on the whole, with most candidates showing clear understanding of when its use was required. A high proportion of students showed a satisfactory degree of competence in all the subject areas; there were relatively few scripts with more than perhaps one or two questions unattempted.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates answered this question correctly. Accuracy penalties were common here.

Question 2

(a) Surprisingly few correct answers here; common mistakes were to give the 25th value as the median, or to write down the mode. (b) a very high proportion of correct answers, mostly applying the corresponding formula – relatively few answers found using a GDC.

Question 3

Most candidates were successful in obtaining the required cosine. Many went on to give the value of θ.

Question 4

Most candidates recognised this as a quadratic equation in $ln\ x$ and solved it correctly. Hardly any student failed to give the exact form of the solutions, as required. Some (not many) took $(ln\ x)^2$ to be equivalent to $2ln\ x$ and made the equation linear.

Question 5

Most candidates separated variables efficiently and were able to integrate successfully and obtain the correct solution, giving evidence that the overall strategy needed to solve this type of differential equation seemed to be mastered by most. A few omitted the constant of integration.

Question 6

Although many students found the required area correctly, a surprising number worked with the position vectors of two of the vertices of the triangle instead of the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$, for instance.

Question 7

(a). Most candidates identified the distribution as binomial and set out the necessary calculation correctly. Quite a few read their answer from the calculator. (b). Also well answered although a relatively frequent mistake was to look for $1 - P(X > 4)$. This was another question where many candidates received an accuracy penalty.

Question 8

(a). Many correct answers. (b). Most solved the equation $det\ A = 0$, but not all were then able to provide the reasoning behind deciding which values of k resulted in a system with a unique solution.

Question 9

Many candidates obtained full marks for this question. A disappointing number lost one mark for not showing their working explicitly enough. Many candidates included sketches in their answers, supporting the solution of the equation $s(t)=0$.

Question 10

(a) A surprising number of incorrect answers for the mode – some gave the value of $f(0)$, others looked for the value of the median and many set out long and very involved integrations leading nowhere, when the use of "state" in the question should have led candidates to realize that not much working was required. Poor GDC use was evident in answers such as 6.37 x 10^{-12}. (b) There were quite a few correct answers to this part of the question, with the vast majority knowing how to set out to find the answer, some eventually making mistakes in the integration by substitution required. A few ignored the instruction to give the exact answer.

Question 11

Although this question was conscientiously attempted by most candidates, not many were successful in reaching the correct required values for *m* and *n*. Careless algebraic errors were the main cause of difficulty, especially for those applying the factor theorem. Some candidates applied this theorem, but instead of simply equating $P(-1 - i)$ to 0, tried to work out the division of $P(z)$ by $(z+1+i)$ and equate the remainder to 0; very few of these were completely successful. Other candidates failed to equate real and imaginary parts. Few candidates took full advantage of their GDC here to work out, for instance, $(-1 - i)^3$.

Question 12

Most candidates gave the starting condition $\Delta > 0$ and found an expression for $b^2 - 4ac$ in terms of k successfully. Those who did not obtain the correct final answer made algebraic mistakes in manipulating the resulting quadratic inequality.

Question 13

This question proved very accessible to most candidates – there seemed to be no difficulty in identifying the strategy required and many students obtained full marks for their answers.

Question 14

Most of the candidates who tackled this question were successful in obtaining some marks, but few were awarded full marks. The most common error was to state the vertical translation *before* at least one of the two one-way stretches involved. Some candidates described the effect of each parameter on the shape of the graph without ever mentioning an actual geometric transfomation: it was common to find answers of the type "the amplitude is doubled" or "the period is shortened" instead of full descriptions of the stretches which cause these effects. There was evidence that some schools do not seem to have covered this part of the programme.

Question 15

Practically all the candidates knew they needed to apply $V = \pi \int_0^{\frac{\pi}{4}} \sin^2 \ 3x \ dx$ and managed therefore to obtain two marks for this question. Only a small minority realised they needed to use a trigonometric identity to transform the integrand and then evaluate the integral.

Question 16

This question was quite well answered on the whole. Most candidates obtained the correct answer in part (a). Using a diagram would have helped those giving 0.106 as the answer. Most candidates identified the need to use conditional probability in part (b).

Question 17

This was another question that most candidates found very accessible. The sketches in part (a) were satisfactorily drawn by most, although many did not label their sketches. (They were not penalised for this). In part (b), practically all candidates knew they needed to find the value of a definite integral and were successful in setting it out properly. A few used their GDC to find one of the x-coordinates of the points of intersection of the curves (1.227) but incorrectly assumed that the curves also intersected at $x = 0.5$.

Question 18

Practically all the candidates were able to differentiate f correctly. However, part (b) was on the whole not too well answered – only some candidates realised the connection between what they were asked here and their answer to (a). Very few actually mentioned the need for the function to be one-to-one and if they realised that, arguing for the least value of b was poor. Including a sketch to support the reasoning was a good strategy in this question, which a few candidates applied successfully.

Question 19

Most candidates recognised integration by substitution as the correct approach for this question (although some tried to integrate by parts). Of those who used the correct substitution, the majority obtained the correct answer; a few tried the substitution $u = e^{2x} + 9$ or $u = e^{2x}$.

Question 20

This question was relatively well answered -- a good number of students used the information given to find the value of z at the given point. Many also recognised the need to differentiate implicitly to find $\frac{dz}{dt}$, and then substitute values to obtain the required rate. Many candidates failed to note that z was decreasing and gave positive answers.

Recommendations and guidance for the teaching of future candidates

- Encourage students to pay attention to mathematical notation, accuracy, answering in degrees or in radians as appropriate, etc.
- Emphasize the importance of setting out their procedure neatly
- Make sure that students are fully aware of the statistics operations on their GDC.
- Discourage the use of calculator notation and encourage candidates to use correct mathematical language to communicate their reasoning and working.
- Refer often to the command terms provided in the subject guide, which frequently will be a good hint as to what type of answer is required. By the same token, the use of the work “exact” in the question should be clearly pointing students away from their GDC.
- Encourage the use of good sketches, which are very helpful both for the students and for the examiner.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 20	21 - 40	41 - 53	54 - 67	68 - 81	82 - 95	96 - 120

General comments

Several examiners have commented on the number of very good scripts they have marked, which is pleasing. Teachers must continue to emphasise the correct structure and language required in the setting out of mathematical induction proofs. A substantial number of candidates either did not seem to understand or ignored key phrases such as ‘hence’, ‘exact’ or ‘show that’. A significant proportion of candidates were awarded an accuracy penalty for not expressing numerical answers correct to three significant figures where required. Indeed, a substantial number of candidates can consider themselves fortunate that the accuracy penalty is capped to one mark only. Candidates need to understand the requirement that unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures where required and they also need to recognise when it is appropriate to use a GDC.

The areas of the programme and examination that appeared difficult for the candidates

The majority of candidates experienced a number of difficulties with the optimization problem that tested their ability to differentiate inverse trigonometric functions, determine the maximum value of a function and to justify this value as a maximum. Other areas of concern included difficulties locating the angle between a plane and a line, not recognising that matrix algebra was required to demonstrate or explain stated results and problems showing particular complex number results. In the probability question, a number of candidates did not apply the correct probability distribution i.e. demonstrating confusion of when to use either the Poisson, binomial or normal distributions. Generally, and also not surprisingly, question parts that required either more sophisticated mathematical reasoning or more demanding algebraic manipulation challenged the majority of candidates.

The areas of the programme and examination in which candidates appeared well prepared

Successful candidates generally exhibited excellent algebraic skills and judicious GDC use. These attributes were demonstrated in performing probability calculations, matrix arithmetic and equation solving. Routine questions that asked candidates to set up and solve a pair of simultaneous linear equations or to solve a quadratic equation were generally answered very well. The questions testing knowledge of vectors and probability were reasonably well answered by the majority of candidates.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Part (a) was very well done with the occasional candidate making a careless arithmetic error. In part (b), some candidates made numerical errors when calculating the vector product while many candidates did not continue on to find a *unit* vector normal to both vectors. In part (c), it appeared that some candidates were confused between the plane normal to ***b*** and the vector found in part (b). Surprisingly, a number of candidates who successfully found the Cartesian equation of π_1 in part (c) then experienced difficulties finding where the plane intersected with the coordinate axes. In part (e), a large number of candidates found the angle between the normal and the line rather than the angle between the plane and the line.

Question 2

The GDC was generally used well by the majority of candidates throughout this question with a small number of candidates still using normal distribution tables. Throughout this question, a large number of candidates failed to express final answers correct to three significant figures.

Part A

Part (a) was generally well done. A small minority of candidates either expressed their answer to an incorrect number of significant figures or made a rounding error while others stated that $P(T < 40) = \frac{5}{7}$ i .e. demonstrating confusion between finding a z-value and a probability. In part (b), common errors included not stating/showing that $P(T<t) = 0.90$, not using correct mathematical notation, premature rounding in calculations or not using the inverse normal function (tables) i.e. stating that $\frac{t-35}{7} = 0.9$. A substantial number of candidates did not recognise that the situation in part (c) is described by the binomial distribution with a small number of candidates attempting to apply the normal distribution.

Part B

In part (a), a common error was to misinterpret the phrase ‘at least two accidents’. Many candidates calculated $P(X=2)$ only while a smaller number calculated $1 - P(X \leq 2)$ instead of $1 - P(X < 2)$. In part (b), a large number of candidates experienced difficulties finding and justifying the most likely number of accidents. Many thought that one was the most likely number of accidents because this was the closest integer to the mean (0.6) while others thought that the answer was indeed the mean. It was pleasing to note that a number of candidates did offer an excellent justification of the correct result. Part (c) was generally well done. The most common error was to calculate $\frac{1}{7}P(X=0)$.

Question 3

Some candidates produced excellent solutions to this question. A large number of candidates however experienced difficulties with the ‘show that’ question parts involving the use of matrix algebra. Often the phrases ‘hence’, ‘use the result’ and ‘show that’ either were not understood or were ignored by a large proportion of candidates. Parts (a) and (b) (i) were generally well done by the majority of candidates. In part (b) (ii), many candidates started with the result and then attempted to verify it by using both matrix $\boldsymbol{A}$ and matrix $\boldsymbol{I}$ rather than demonstrating the result using matrix algebra. Again in part (b) (iii), many ignored the instruction from the previous part and found $det\,\boldsymbol{A}$ numerically. Only the strongest candidates were able to explain clearly why matrix $\boldsymbol{A}$ was non-singular from part (b) (ii). In part (c), the instruction was ignored by many candidates who calculated $\boldsymbol{A}^4$ directly from $\boldsymbol{A}$. Many candidates did not seem to understand the concept of a singular matrix and its relation to the determinant while others confused the identity matrix with the inverse matrix.

Question 4

This was generally not well done with many algebraic and conceptual difficulties evident. Most candidates demonstrated the required result in part (a) although some attempted to use $\tan\theta = \tan\alpha - \tan\beta$ while others did not provide enough evidence to 'show' the given result. In part (b), many candidates ignored differential calculus techniques. These candidates typically produced a sketch graph and deduced that the maximum value occurred at $x = \sqrt{2}$ (1.41). Many overlooked the instruction 'find the **exact** value'. A large majority of candidates who did employ a calculus approach either failed to make a correct substitution or to use the chain rule correctly. Those that managed to find a correct derivative were not able to justify the maximum value, with many candidates either resorting to an ill-conceived graphical justification or ignoring the requirement to justify the maximum. Where a justification was attempted, often an unlabelled sketch graph was used rather than 1^{st} or 2^{nd} derivative analysis. In part (a) (iii), a number of candidates gave their maximum value of θ in degrees rather than in radians. In part (b), students were expected to use their GDC to solve the equation using either numerical or graphical means. It was surprising to note the number of candidates who found only one of the two solutions while a number of candidates tried unsuccessfully to solve the equation algebraically.

Question 5

Part (a) (i) was generally well done although a small minority multiplied the numerator and denominator by v. In part (a) (ii), a large number of candidates did not find u and v separately in modulus-argument form while a common error for those who did employ the correct approach was to neglect to show that the arguments needed to be subtracted. A common error in part (a) (iii) was to not use the instruction '**hence**'. A reasonable number of candidates produced well-structured mathematical induction proofs in part (b). Common structural errors in the induction proof included not showing that $P(1)$ is true, not stating the assumption that $P(k)$ is true, not showing that $P(k+1)$ is true or not supplying a correct concluding statement. The most common error here was to omit stating that $P(1)$ is true. In part (c), a substantial number of candidates were able to substitute correctly for u and v but were then unable to collect the real and imaginary parts together. A common error was to multiply the numerator and denominator by $\sqrt{2}v + u$. A number of candidates inappropriately resorted to a GDC in attempting to establish that $\mathrm{Re}\, z = 0$.

Recommendations and guidance for the teaching of future candidates

- Provide students with a range of extended answer revision questions that tests knowledge of a range of probability distributions in a variety of contexts.

- Discuss with students when it is appropriate to use a GDC and when an analytic (algebraic) solution approach is required. This discussion should be situated within the context of the new examination structure commencing in May 2008.
- Ensure that students understand that appropriate GDC use should be accompanied by appropriate documentation of solutions. GDC inputs should be expressed in correct mathematical notation and not in terms of brand specific calculator syntax.
- Continue to highlight the importance of constructing a correct concluding statement when undertaking induction proofs. For example, $P(k)$ is true implies that $P(k+1)$ is true, and as $P(1)$ true then $P(n)$ is true for all positive integers.
- Encourage students to question the reasonableness of results they obtain.
- Discuss with students what phrases such as ‘hence’, ‘exact’ or ‘show that’ mean in the context of answering examination questions.

Paper three

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 17	18 - 25	26 - 32	33 - 39	40 - 46	47 - 60

General comments

Options A, B and C remain the most popular with D coming in last but seemingly more schools chose this option than in previous years. There were more good scripts this year indicating that the paper might have been a little easier.

Not all candidates cooperated with the first request on page 2 to begin each question on a new page which makes the marking a little more difficult. Writing answers in pen is also not yet universal.

The requirement ‘all numerical answers must be given exactly or to three significant figures unless stated otherwise in the question’ is also too often overlooked.

Finding and equilibrium between writing enough and writing too much is still a problem for many candidates.

The areas of the programme and examination that appeared difficult for the candidates

Section A

This option was covered quite well. Expectation algebra and the relationship between the various kinds of distributions were not secure. The notion of proof in statistics is not something that candidates feel at ease with also understanding of type I and type II errors.

Section B

This option is perhaps more heavily weighted towards questions involving proof than the others and this is the weakest aspect of the scripts seen. In particular, proof by contradiction is not widely understood nor its uses appreciated. Knowledge of equivalence classes is not thorough. The use of De Morgan's laws needs attention.

Section C

Successive use of integration by parts led to errors and finding intervals of convergence proved difficult.

Section D

Generally accessible to most candidates with perhaps only the use of Fermat's little theorem being a source of difficulty.

The areas of the programme and examination in which candidates appeared well prepared

Section A

Application of the χ^2 test. Use of the calculator and p-values. Matched pairs.

Section B

Equivalence relations. Group properties. Permutation groups and composite functions.

Section C

Limits, Maclaurin series, convergence and partial fractions.

Section D

Algorithms for spanning trees, gcd and minimum walks.

The strengths and weaknesses of the candidates in the treatment of individual questions

Section A

Question 1

The mean was easily found though not always evident in the solution. Hypotheses were generally clear. Premature rounding caused some errors in final results when calculating χ^2 'by hand'. Occasionally rows were combined because of the presence of an **observed** frequency of 4. Some candidates did not realize there were two restrictions and had $\upsilon = 5$.

Question 2

This was quite well done partly (I suspect) because of the help given in the question. An encouragingly increasing number of students are using the p-value approach to statistical questions.

Question 3

Interpreting the text of the question posed some difficulties but when candidates realized they were dealing with a straight forward normal question the results were good.

Question 4

Quite a tricky question involving the sorting out of introduction and expressing each error in combinatorial terms. Good solutions were getting rarer as perhaps is expected as we near the end of the option.

Question 5

Realizing that a negative binomial distribution was involved escaped many candidates. Some managed the mean but then floundered on the remaining parts of the question.

Section B

Question 1

This was quite well done but candidates sometimes leave too much to the imagination of the examiner. Explicit statements to back-up R being an equivalence relation were missing.

Question 2

Again the same comments as in the previous question. The meaning of the expression ‘Show that’ is not at all clear to many candidates. Otherwise the question was straight forward.

Question 3

Part (a) was often done directly, i.e. working out p^4, p^5, p^6, rather than using the method shown in the mark scheme. The composite function in part (b) was often read backwards so that the two results were reversed. It was encouraging to see at least some students using the reversal rule in part (c).

Question 4

Parts (a) and (b) were found to be easy but many candidates treated the multiplication as if they were dealing with real numbers and ignored any caution concerning commutativity. So although the first M1A1A1 were gained the remaining marks were lost.

Question 5

Far too many candidates tried to answer this question using Venn diagrams. A ‘solution’ of this kind received no marks. It was refreshing to see the occasional good solution.

Section C

Question 1

The first two parts were well done, although some candidates got lost in differentiating twice in part (ii). Part (b) caused some difficulty as often happens when a question requires a candidate to show understanding rather than mere technique.

Question 2

This question required accuracy and patience. It was not inherently difficult but the unwillingness or inability of some candidates to set out their work clearly and logically caused them to lose a considerable number of marks.

Question 3

Part (a) was well done as was the first part of (b). Seeing a telescoping series was not difficult if the work had been well organized but many missed this.

Question 4

This was quite an easy question but accuracy was again involved and this caused problems. Some candidates worked ‘backwards’ from the given answer. Some missed the connection between parts (a) and (c).

Question 5

The most difficult question. Modulus signs were often missing although the inequality was found for the absolute value of x. Clear analysis of the results for $x = 1$ and $x = -1$ were not often seen. The logic of this last part was seen only very occasionally.

Section D

Question 1

Most candidates found this question to be entirely accessible and gained full marks.

Question 2

There were some strange but correct methods of setting out Euclid’s algorithm but $gcd(43,73) = 1$ was not difficult to find and the second part of (a) followed easily. (b)(i) was easy but too many candidates saw the word ‘minimizes’ and launched immediately into some sort of weird calculus.

Question 3

An easy question that most candidates did well on.

Question 4

This question caused great difficulty for many candidates since, again, a proof (logical argument) is required. Few good solutions were seen.

Question 5

The definition proved difficult ads did the second part. Yet again difficulty with how to do a proof.

Recommendations and guidance for the teaching of future candidates

There is an overwhelming need to develop the general ideas of proof and the specific nature of proof in a particular option. It is relatively easy to get students to learn results and to understand demonstrated arguments but getting candidates to provide clear solutions, logical arguments and complete proofs remains the challenge.