

## MATHS HL TZ1

### Overall grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 15	16 - 31	32 - 42	43 - 54	55 - 66	67 - 78	79 - 100

### Internal assessment

#### Component grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 6	7 - 13	14 - 18	19 - 23	24 - 29	30 - 34	35 - 40

Many portfolios provided strong evidence of excellent investigations in mathematics. Although the changes to the portfolio requirements and assessment criteria as of May 2006 were significant, most teachers and their students seemed to have understood the expectations. The moderators have made a number of observations that are summarised below:

#### The tasks in the new syllabus

The majority of all portfolio tasks were taken from the current Teacher Support Material (TSM) for Mathematics HL. Unfortunately, where tasks were taken from the previous TSM, even the best effort by the student resulted in work that did not fully satisfy the current assessment criteria. Unless significant modifications are made, these older tasks should not be used. Teachers are encouraged to design their own tasks, keeping in mind the need to satisfy all criteria fully. It should be noted that investigative tasks that preclude the use of technology, and modelling tasks in which the model is not created by the student, but given within the task, fall short of fulfilling the requirements of the portfolio.

Tasks taken from the TSM for Mathematics SL are not at a suitable level for Mathematics HL and should not be used.

#### Candidate performance against each criterion

Most candidates performed well against criterion A. The use of computer notation seemed to be very limited. Correct terminology should include the use of correct mathematical vocabulary.

Where a student's work began with an introduction to the task, and comments, annotations, and conclusions accompanied the steps and results, the work was easy to read and follow, and earned high marks in criterion B. On the other hand, those who have merely shown the

steps to the solutions of problems, or left graphs unlabelled, and tables disconnected by their relegation to an appendix, demonstrated poor skills in presentation.

Jointly, criteria C and D are meant to assess the mathematical content and comprise half the marks awarded to each piece of work. Generally, students have produced good work, and the assessments by their teachers have been more appropriate this session. However, in some type I tasks, insufficient exploration and patterning rendered the quick formulation of a conjecture questionable. Where several intermediate general statements were derived, the proof of *the* general statement needed to be evident to warrant full marks.

In type II tasks, variables should be explicitly defined. Some realisation of the significance of the results obtained in terms of the model when compared to the actual situation should have been provided, and students should have reflected on their findings. The analyses of data must be quantified, and if a regression analysis were appropriate, the student must have provided reasons for a particular choice. The use of software that automatically determines the “best” regression model leaves little for the student to interpret.

The use of technology varied considerably from the truly resourceful to the merely perfunctory. Full marks were given much too generously for an *appropriate* but not necessarily a *resourceful* use of technology, for example, in the inclusion of a single graph produced on a calculator. As one moderator remarked, technology must be used to do more than merely “decorate” the work. Students should be discouraged from including GDC key sequences – they are unnecessary and unwarranted.

There were many, many good pieces of work; however, the awarding of full marks in criterion F requires more than completion and correctness, but the evidence of mathematical sophistication. (Please refer to the *Criteria notes* below.)

## Recommendations for the teaching of future candidates

Teachers should select tasks that provide students with a variety of mathematical activities suitable at higher level. Tasks taken directly from the Mathematics SL TSM do not meet HL requirements and are not considered acceptable.

The teacher who remains uninformed or chooses to ignore the changes to the portfolio assessment criteria is generally the reason for a significant loss of marks in moderation. This is completely unfair to the student and must be rectified.

Teachers are expected to write directly on their students’ work, not only to provide feedback to students, but information to moderators as well. Some samples contained very few teacher comments. Moderation was extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks.

Moderators find the background to each portfolio task very useful in determining the context in which the task was given when confirming the achievement levels awarded. This information must accompany each sample.

If a teacher-designed task is submitted, a solution key must accompany the portfolios in order that moderators can justify the accuracy of the work, and appreciate the level of sophistication demonstrated in the work.

## Criteria notes

In order to clarify the requirements of the assessment criteria, notes were first directed to moderators after the standardisation meeting in April 2006, and then presented to all teachers through the two Subject Reports in 2006. At the risk of being redundant, they are presented here one last time in its entirety in the hope that teachers may find them useful.

### **Criterion A: use of notation and terminology**

Tasks will probably be set before students are aware of the notation and/or terminology to be used. Therefore the key idea behind this criterion is to assess how well the students' use of terminology describes the context.

Teachers should provide an appropriate level of background knowledge in the form of notes given to students at the time the task is set.

Correct mathematical notation is required, but it can be accompanied by calculator notation, particularly when students are substantiating their use of technology.

This criterion addresses appropriate use of mathematical symbols (for example, use of " $\approx$ " instead of " $=$ " and proper vector notation).

Word processing a document does not increase the level of achievement for this criterion or for criterion B.

Students should take care to write in appropriate mathematical symbols if the word processing software does not supply them. Calculator/computer notation should not be used. Notation such as  $x^2$  or  $ABS(x)$  should not be used and such use will be penalised. A single shortcoming would not preclude the awarding of level 2.

Terminology may depend on the task. In the case of Type I (Investigation) tasks, terminology may include terms devised by the candidate (e.g. "slide", "shift"), provided that such terms reasonably reflect the appropriate mathematical concept.

### **Criterion B: communication**

This criterion also assesses how coherent the work is. The work can achieve a good mark if the reader does not need to refer to the wording used to set the task. In other words, the task can be marked independently.

Level 2 cannot be achieved if the student only writes down mathematical computations without explanation.

Graphs, tables and diagrams should accompany the work in the appropriate place and not be attached to the end of the document. Graphs must be correctly labelled and must be neatly drawn on graph paper. Graphs generated by a computer program or a calculator "screen dump" are acceptable providing that all items are correctly labelled, even if the labels are written in by hand. Colour keying the graphs can increase clarity of communication. If, in reading a candidate's work, the teacher has to pause to clarify where a result came from or how it was achieved ("WHOA! Where did that come from?!"), this generally indicates flawed communication.

Computer/calculator output may need clarification. Graphs generated by a calculator or computer should present the variables and labels appropriate to the task. Hand-written labels

may need to be added to screen dumps or printouts if the software doesn't provide for custom labels.

A single shortcoming would not preclude the awarding of level 3.

### **Criterion C: mathematical process**

Type I—mathematical investigation: searching for patterns

Students can only achieve a level 3 if the amount of data generated is sufficient to warrant an analysis.

This is the process of getting the statement. Student gets 4 if everything is ready for the statement. The correctness of the statement is assessed in D.

If student gives a proof of the correct statement, no further cases need be investigated to award a level 5.

Type II—mathematical modelling: developing a model

At achievement level 5, applying the model to other situations could include, for example, a change of parameter or more data.

Any form of definition of variables, parameters constraints (informal/implied) is acceptable (e.g. labelling a graph or table, noting domain and range).

### **Criterion D: results**

Type I—mathematical investigation: generalization

A student who gives a correct formal proof of the general statement that does not take into account scope or limitations would achieve level 4.

It is important to note the difference between “a (i.e. any) general statement” in level 2 and “the general statement” in level 3.

Type II—mathematical modelling: interpretation

“Appropriate degree of accuracy” means appropriate in the context of the task.

### **Criterion E: use of technology**

The emphasis in this criterion is on the contribution of the technology to the mathematical development of the task rather than to the presentation and/or communication.

The level of calculator or computer technology varies from school to school. Therefore teachers should state the level of the technology that is available to their students. While printed output is not required, some statement confirming appropriate use of technology (from the teacher or student) is necessary.

Using a computer and/or a GDC to generate graphs or tables may not significantly contribute to the development of the task, and therefore may not merit more than a level 1.

**Criterion F: quality of work**

Students who satisfy all the requirements correctly achieve level 1. For a student to achieve level 2, work must show precision, insight and a sophisticated level of mathematical understanding.

Award level 2 only if the work presented is beyond ordinary expectations. The teacher will take pause to admire the quality of such work (“Wow! Now, that’s impressive!”).

Only a totally inadequate response would receive 0.

## Paper one

**Component grade boundaries**

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 18	19 - 37	38 - 48	49 - 61	62 - 75	76 - 88	89 - 120

### The areas of the programme and examination that appeared difficult for the candidates

On this paper candidates found difficulty with complex numbers, probability and statistics, matrices, infinite geometric series, some aspects of differentiation and logarithms. Many candidates suffered an accuracy penalty with some candidates incorrectly rounding throughout the paper.

### The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been reasonably well prepared in that topics appeared to have been taught and that students had been exposed to the relevant concepts. A majority of candidates used the GDC well and showed the necessary working. On the whole candidates had success with the sine rule, inverse functions, and integration by parts.

### The strengths and weaknesses of the candidates in the treatment of individual questions

**Question 1**

Most candidates successfully answered this question but a significant number were penalised with an accuracy penalty. A number of candidates found a height of the triangle in part a), which often led to errors later on.

**Question 2**

Part a) was well done by the majority of candidates. In part b) most candidates knew how to show that A and B were independent, but there was some confusion. A significant number of

candidates attempted to use Bayes' theorem for part c), with the vast majority not doing so successfully.

### Question 3

On the whole this was well answered. Incorrect answers came from students who failed to identify the distribution as binomial or working with the standard deviation instead of the variance.

### Question 4

In part a) most candidates found  $f^{-1}(x)$  successfully, but a number did not write the answer in the form  $f^{-1}(x)=$ . A few candidates did not understand the notation and differentiated the function. Answers from part b) were mixed, with a number of candidates leaving the answer blank.

### Question 5

Most candidates were able to find the correct expressions for the area of the sector and the triangle, but a number had difficulty in forming the correct equation. Most students who got this far were able to successfully use the GDC to solve the equation, but a small number tried to find  $\theta$  analytically.

### Question 6

The vast majority of candidates recognised the use of the product rule in part a) and successfully found  $f'(x)$ . There were also many correct answers to part b) but a number of candidates did not use the answer they had found in part a).

### Question 7

There were few correct answers to this question. A relatively high number of candidates did not attempt the question and in part a) few thought of turning  $z$  into Cartesian form. Of the few candidates who gained the answer to part a), very few of these were able to complete part b), with a number trying to find  $z^3$ .

### Question 8

The vast majority of candidates found the correct answer to part a). On the whole part b) was well done, but there were a significant number of candidates who incorrectly rounded their values for the areas and then gained an incorrect value for  $k$ .

### Question 9

On the whole this question was attempted successfully by candidates, with most knowing the appropriate substitution. A few candidates did not know the formula for  $E(X)$ .

**Question 10**

Most candidates seemed to have a reasonable idea of what to do here, but were often let down by errors in finding the scalar product, including giving a non-scalar answer for this. Those who found the scalar product correctly then went on to work with the condition for perpendicular vectors and found the correct value for  $\lambda$ .

**Question 11**

Most candidates found the first derivative successfully and were able to apply the product or the quotient rule to begin finding the second derivative. However, the algebraic skills of many candidates were not strong enough to gain the final given answer.

**Question 12**

There was a mixed response to this question. Some candidates clearly did not understand the principles being tested here. However, a reasonable number of candidates knew the starting condition of  $b^2 - 4ac < 0$  and were able to find the correct inequality in terms of  $k$ . Those who did not obtain the correct final answer from this made algebraic mistakes in manipulating the inequality.

**Question 13**

This was a very poorly done question. Many candidates were able to find the two equations in  $x$  and  $y$ , but very few had any clear idea of how to proceed from here.

**Question 14**

Very few correct answers were seen for part a). Students did not recognise that  $4 - 3x$  was the ratio of the geometric series. Students had more success with part b) and a reasonable number of students gained the correct answer. However, it should be noted that relatively few gained the correct answer analytically, with many students just generating the terms.

**Question 15**

Most candidates recognised the method of implicit differentiation and knew the method to find a specific value of  $\frac{dy}{dx}$ . However, the actual process of differentiation was often poorly done, with the differentiation of  $e^{xy}$  causing many problems. Relatively few completely correct answers were seen.

**Question 16**

Candidates had some success with this question, provided they recognised the method of separating the variables. However  $\int \frac{x}{x^2 + 1} dx$  caused some difficulty and a number of candidates forgot to include a constant of integration.

**Question 17**

Candidates often started off quite well with this question, but there were very few fully correct answers. A few candidates thought that  $f \circ g(x)$  meant  $f \circ g(x) \times x$ . Many candidates tried an analytic approach involving cross multiplication or multiplying each term by a variable, which was usually incorrect. Those who tried to use the GDC were more successful, but were often let down by poor graphs.

**Question 18**

A small number of candidates gained full marks for this question, but the majority of candidates did not recognise that they needed to use the quotient rule in part a) and the chain rule in part b).

**Question 19**

This was the question that candidates found the most difficult. Very few knew the correct method, with the vast majority trying to take  $\log_2$  of each term.

**Question 20**

The better candidates had some success with this question, being able to find  $\frac{dV}{dt}$  and the appropriate value for  $h$ . However, many candidates made errors in finding  $\frac{dV}{dt}$ .

## Recommendations and guidance for the teaching of future candidates

- Students should be encouraged to pay attention to mathematical notation and accuracy.
- Teachers should emphasise the importance of students setting out their procedures in a logical fashion.
- When the word “exact” is given, students should realise that using a GDC approach is usually inappropriate.
- Most of the questions in this paper used common problem solving strategies and this should be a focus for most students.



## Paper two

### Component grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 15	16 - 31	32 - 47	48 - 61	62 - 75	76 - 89	90 - 120

### General comments

Whereas many teachers found the paper to be fair, accessible and properly challenging, others felt it to be more difficult than in previous years, mostly due to the time allowed for the examination. Overall it was the opinion of the examiners that the candidates found it to be quite a demanding paper. On the other hand it was also the opinion of the examiners that too many candidates were placed at a disadvantage by not being taught all the areas of the programme. It seems unfair to have registered these candidates for the subject at higher level, when they clearly lack many of the skills and the ability to face the demands of the course.

### The areas of the programme and examination that appeared difficult for the candidates

There were a number of centres whose candidates seemed to have little understanding of vectors. Most candidates knew the structure of a proof using mathematical induction but they were not very good at producing a convincing argument. There was a noticeable inability of candidates to develop an argument both in a proof and as a question moved from one part to the next. Too many candidates failed to recognize the importance of following the instruction given in problems, ignoring such words as “using” or “hence”.

### The areas of the programme and examination in which candidates appeared well prepared

The candidates demonstrated a good knowledge of calculus, trigonometry, graphing, finding simple z-scores, and factoring a polynomial.

### The strengths and weaknesses of the candidates in the treatment of individual questions

#### Question 1

In part (b) the candidates often did not clearly state the reasoning but simply showed that the scalar product was zero. In part (d) they often did not realise that the direction of the line required was the same as the direction of the line in part (c). Many were unable to complete part (e) because they could not see that they needed to put the parametric form of the line into the plane to find point Q. Surprisingly many candidates who found the coordinates of Q

were then unable to find PQ, mistakenly thinking they were to find the vector  $\overrightarrow{PQ}$  rather than the magnitude of that vector. Also there was some confusion over the different forms for the equation of a line. Some candidates even gave an equation of a plane instead of an equation of a line, clearly demonstrating a lack of understanding of the material. The relationship between the normals of planes and lines that are parallel to those planes is not understood well by many candidates. Errors in algebra also prevented candidates from earning full marks.

### Question 2

Circular reasoning was common on part (a) of this question. Some did not use the formula given as their starting point and lost marks by applying double angle formulas to simplify the expressions. The integration in part (b) was done well if they followed the hint to part (a). However, some candidates erroneously tried to use integration by parts thus creating an even more complex integral. Some candidates failed to determine the correct coefficient of the  $\sin 2x$  term. Many candidates were successful in finding the points of intersection in part (c) but some candidates did not acknowledge that exact values were required for the answers. Problems arose in part (e). Even though the graphs were generally correctly sketched in part (d) the candidates failed to write the correct expression for the volume. Most candidates had an idea of the correct setup for the integral, but tended to integrate the difference of the functions squared instead of the difference of the square of the functions. Some did not recall the anti-derivative of  $\sec^2 x$ . The algebra required for the exact answer also proved to be quite challenging for most candidates.

### Question 3

Part (a) was generally done very well except for the fact that too many candidates rounded prematurely and hence did not have the required degree of accuracy in the final answer. In part (b) some failed to adjust the mean for two hours instead of one hour. In part (c) again premature rounding caused a problem. Many recognised that a conditional probability was required but some tried to compute the probability given either exactly 10 messages received or more than 10 messages received. Candidates who attempted part (d) did so with varying degrees of success with some generally recognizing that the mean was 1.5 and that they were now considering a binomial distribution. For others the combination of the Poisson distribution with binomial probability made this very challenging. In some cases, the binomial probability was identified but the probabilities from the Poisson distribution were erroneously calculated partly due to an inability to recognize that the mean was now 1.5. One again there was a problem with giving the correct answer to 3 significant figures.

Some candidates had no idea how to approach the question in part B. Many did not know which formula for variance to use. A common error was to use the total number of marks as

20 instead of as 22. On the other hand this question seemed to pose no problem for a number of candidates. Normally they were able to earn full marks unless a careless algebra mistake was made along the way.

#### Question 4

On the surface the question in Part A appears to be routine. However, minor errors in elementary counting soon caused unsuspecting candidates to lose easy marks. In part (a) a common error was to use 600 as the total number of numbers. A variety of approaches were seen in part (b) including listing and counting. Many seemed unable to manipulate the facts and the formulae required to solve this question.

In Part B the proof by induction proved to be challenging. Many candidates were unprepared to tackle this type of proof. In particular, the induction hypothesis was not always clearly stated. The correct decomposition of the expression for  $n=k+1$  proved to be too difficult for many candidates. Typically, candidates either made errors in algebra or gave up on the proof. Also the final three reasoning marks were not awarded unless the candidates clearly demonstrated an understanding of the induction process.

#### Question 5

In part (a) the factoring was done without too much trouble but amazingly a few candidates were unable to use the quadratic formula to find exact solutions for  $t$ . Many candidates were able to do part (b) (i) and (ii) if they followed the instructions given. However, some tried to use sum and difference identities and for this approach they were only able to gain a minimum number of marks. Candidates were generally unsuccessful with parts (c) and (d) of this question. In (c) the first two marks for this proof were routine, but many candidates had difficulty devising a method to produce the given result. When identifying restriction on the identity, many omitted the answer that  $\theta \neq \frac{\pi}{2}$ . Many candidates did not attempt the final part of the question. Typically, candidates could identify the results from parts (a) and (c) which were needed to solve the problem. Even those who seemingly knew what to do were not good about stating the reasoning behind their work.

### Recommendations and guidance for the teaching of future candidates

Where there is a choice, perhaps more advice could be given to candidates on the appropriate level of mathematics suitable to individual students. Although most of the calculator use was appropriate, there were still problems with accuracy and candidates need to be aware of the error of premature rounding. Candidates should be given a variety of different types of questions for proof by mathematical induction. Teachers should continue to provide candidates with problem solving situations that involve multiple topics of the course.

Candidates had difficulty linking the parts of the problems together. They also need to clearly state their reasoning and above all follow the instructions given in the questions.

## Paper three

### Component grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 11	12 - 22	23 - 28	29 - 35	36 - 42	43 - 49	50 - 60

### General comments

The examiners were of the opinion that the questions in the four sections were broadly comparable.

### The areas of the programme and examination that appeared difficult for the candidates

#### Section A

Solutions to questions on probability distributions are often poor, particularly involving the relationships between the various distributions.

#### Section B

Questions requiring a proof are, in general, not well answered. Candidates should be aware that a proof usually requires a rigorous algebraic argument. The use of Venn diagrams and the consideration of special cases are insufficient.

#### Section C

Many candidates find problems involving convergence difficult, especially where no indication of an appropriate test is given in the question.

#### Section D

Many candidates find some aspects of number theory, especially the use of Fermat's little theorem, particularly difficult.

### The areas of the programme and examination in which candidates appeared well prepared

#### Section A

Solutions to questions involving hypothesis testing are generally well done although the wrong test is sometimes selected. It was, however, evident in this examination that some of

the terminology used in this area is unfamiliar to some candidates, e.g. critical region and Type I and Type II errors.

### **Section B**

The properties of groups and equivalence relations are generally well understood.

### **Section C**

Candidates are generally confident in questions involving Taylor series, including the use of l'Hôpital's Rule and also differential equations although the integration sometimes causes problems.

### **Section D**

Candidates are generally comfortable with the graph algorithms.

## **The strengths and weaknesses of the candidates in the treatment of individual questions**

### **Section A**

#### **Question 1**

Most candidates are reasonably happy with chi-squared tests and this question was well done in general. The correct hypotheses are that the Poisson law either does or does not provide a suitable model and not that the Poisson law with mean 2.16 either does or does not provide a suitable model. This latter pair of hypotheses was not penalised on this occasion although it might be in future papers. Common errors were to calculate the expected number of exactly 5 instead of at least 5 and/or to combine the last two rows because the expected frequency was less than 5.

#### **Question 2**

Although the question emphasised the use of a matched pairs test, some candidates used a 2-sample test which, although no longer on the syllabus, is readily available on their GDC.

#### **Question 3**

This question was well done in general with many candidates correctly taking into account the fact that one piece of information related to the **mean** daily rainfall.

#### **Question 4**

Candidates found this question difficult with some not knowing where to start. The term 'critical region' was unfamiliar to many although it is specifically mentioned in the Guide. Some students appeared to be unfamiliar with the terms Type I and Type II errors and even those familiar with these terms were often unable to evaluate the appropriate probabilities.

**Question 5**

Many candidates appear to be unaware of the relationship between the geometric and negative binomial distributions apart from the fact the geometric distribution is a special case of the negative binomial distribution. Some candidates realised that the formulae for the mean, variance and probability function for the negative binomial distribution are given in the Information Booklet but many, apparently, did not.

**Section B****Question 1**

Part (a) was well done in general although the proof of transitivity was sometimes unconvincing. It is not enough simply to state

$$'a^2 \equiv b^2 \pmod{3} \text{ and } b^2 \equiv c^2 \pmod{3} \Rightarrow a^2 \equiv c^2 \pmod{3}'$$

This statement requires justification. Some candidates found the determination of the equivalence classes beyond them.

**Question 2**

This question was well done by many candidates.

**Question 3**

Some candidates appeared not to be familiar with permutations with many not realising that the first three elements cycle with order 3 and the last two cycle with order 2 so that the overall order is 6. Many candidates dealt with the composite permutations the wrong way around. Part (c) was beyond the capabilities of many of the candidates.

**Question 4**

This question was well done by many candidates although the double solution in (c) was missed by some.

**Question 5**

Very few correct solutions were seen with either nothing written or an unrewarded Venn diagram 'proof' offered. Although Venn diagrams may be used in certain situations, they may not be used in a question where a proof is required.

**Section C****Question 1**

This question was well done by many candidates. It was pleasing to note that many candidates realised why the argument in (b) was incorrect.

**Question 2**

It was disappointing to see that so many candidates made arithmetic errors either in the differentiation of  $e^{\sin x}$  or in combining the series for  $e^x$  and  $\sin x$ .

**Question 3**

Many candidates successfully used a comparison test to demonstrate convergence in (a). In (b), the partial fractions were usually found correctly but the process of telescoping series appeared to be unfamiliar to some candidates.

**Question 4**

Part (a) proved to be too difficult for some candidates who seemed to be unable to find the two functions necessary for integration by parts to be used. Many candidates found the correct integrating factor in (b) but careless working in (c) often led to the wrong answer. In particular, the incorrect result  $y = -\sin x - 2$  was often seen due to careless manipulation involving the constant of integration..

**Question 5**

Most candidates made a reasonable attempt to find the radius of convergence using the ratio test although the modulus sign was sometimes omitted. Some candidates wasted time

evaluating the limit of  $\frac{\sin \pi / (n+1)}{\sin \pi / n}$  using l'Hôpital's rule although this limit, being a standard

result, could have been assumed. Many candidates failed to realise that the behaviour for  $x = \pm 1$  had to be considered.

**Section D****Question 1**

This question was well answered by many candidates although some failed to indicate the order in which the edges were introduced. This meant that no credit could be given for finding the minimum spanning tree since there was no indication that Prim's Algorithm had been used.

**Question 2**

Part (a) was well done by the majority of candidates. Most candidates were able to find a particular solution to the Diophantine equation although many were unable to derive the general solution. Part (b)(ii) caused problems for many candidates with some attempting to use calculus to perform the minimisation.

**Question 3**

Most candidates were familiar with both the route inspection algorithm and Hamiltonian cycles.

**Question 4**

Although this was a fairly straightforward application of Fermat's little theorem, and the question indicated that this theorem should be used, few candidates were able to make any progress with this question.

**Question 5**

Some candidates appeared to be unaware of the meaning of the complement which made the question inaccessible. Even those candidates who were familiar with the concept often found it extremely difficult to construct a logical argument to prove the required result in (b).

## Recommendations and guidance for the teaching of future candidates

Many candidates are unable to construct a proof and this applies in all four sections of the paper. Not only do many attempts lack rigour – the presentation is often poor.

Many candidates seem unaware of the accuracy rules and many suffer an accuracy penalty unnecessarily.

Some scripts are barely legible and candidates should be encouraged to present their work in a way that can be easily followed by the assistant examiner marking the script.