MATHS HL

Overall grade boundaries

Higher level

Grade:	1	2	3	4	5	6	7
Mark range:	0-16	17-33	34-44	45-56	57-68	69-80	81-100

Higher level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-6	7-13	14-18	19-23	24-29	30-34	35-40

Many good portfolios were produced in this first session of the new syllabus. The changes to the assessment criteria were significant, yet expectations seemed to be well understood by most teachers and their students. The moderators have made a number of observations which are summarised below:

The tasks in the new syllabus:

The majority of the portfolio tasks were taken from the Teacher Support Material (TSM) for Mathematics HL. Unfortunately, where tasks were taken from the previous TSM, the new assessment criteria were not adequately met. Unless significant modifications are made, these older tasks should not be used. Some examples of tasks deficient in content included investigations (type I) that precluded the use of technology and modelling tasks (type II) in which the model was not created by the student as required in the new criteria, but given to students within the task.

The "Extended Closed Problem Solving" tasks (old type II) were deleted from the new internal assessment structure. Any such tasks taken from the previous TSM and submitted in future sessions may be subject to a non-compliance penalty, as well as a significant loss of marks in criteria C and D as the new achievement levels will not be met.

Criteria clarification and notes to moderators:

The following criteria notes were directed to moderators after the standardisation meeting in April and are presented here for all teachers to consider.

Criterion A: use of notation and terminology

Tasks will probably be set before students are aware of the notation and/or terminology to be used. Therefore the key idea behind this criterion is to assess how well the students' use of terminology describes the context.

Teachers should provide an appropriate level of background knowledge in the form of notes given to students at the time the task is set.

Correct mathematical notation is required, but it can be accompanied by calculator notation, particularly when students are substantiating their use of technology.

This criterion addresses appropriate use of mathematical symbols (for example, use of "$\approx$" instead of "$=$" and proper vector notation).

Word processing a document does not increase the level of achievement for this criterion or for criterion B.

Students should take care to write in appropriate mathematical symbols if the word processing software does not supply them. Calculator/computer notation should not be used. Notation such as *x* ^ 2 or ABS(*x*) should not be used and such use will be penalised. A single shortcoming would not preclude the awarding of level 2.

Terminology may depend on the task. In the case of Type I (Investigation) tasks, terminology may include terms devised by the candidate (e.g. "slide", "shift"), provided that such terms reasonably reflect the appropriate mathematical concept.

Criterion B: communication

This criterion also assesses how coherent the work is. The work can achieve a good mark if the reader does not need to refer to the wording used to set the task. In other words, the task can be marked independently.

Level 2 cannot be achieved if the student only writes down mathematical computations without explanation.

Graphs, tables and diagrams should accompany the work in the appropriate place and not be attached to the end of the document. Graphs must be correctly labelled and must be neatly drawn on graph paper. Graphs generated by a computer program or a calculator "screen dump" are acceptable providing that all items are correctly labelled, even if the labels are written in by hand. Colour keying the graphs can increase clarity of communication.

If, in reading a candidate's work, the teacher has to pause to clarify where a result came from or how it was achieved ("WHOA! Where did that come from?!"), then this generally indicates flawed communication.
Computer/calculator output may need clarification. Graphs generated by a calculator or computer should present the variables and labels appropriate to the task. Hand-written labels may need to be added to screen dumps or printouts if the software doesn't provide for custom labels.

A single shortcoming would not preclude the awarding of level 3.

Criterion C: mathematical process

Type I—mathematical investigation: searching for patterns

Students can only achieve a level 3 if the amount of data generated is sufficient to warrant an analysis.

This is the process of getting the statement. Student gets 4 if everything is ready for the statement. The correctness of the statement is assessed in D.

If student gives a proof of the correct statement, no further cases need be investigated to award a level 5.

Type II—mathematical modelling: developing a model

At achievement level 5, applying the model to other situations could include, for example, a change of parameter or more data.

Any form of definition of variables, parameters constraints (informal/implied) is acceptable (e.g. labelling a graph or table, noting domain and range).

Criterion D: results

Type I—mathematical investigation: generalization

A student who gives a correct formal proof of the general statement that does not take into account scope or limitations would achieve level 4.

It is important to note the difference between "a (i.e. any) general statement" in level 2 and "the general statement" in level 3.

Type II—mathematical modelling: interpretation

"Appropriate degree of accuracy" means appropriate in the context of the task.

Criterion E: use of technology

The emphasis in this criterion is on the contribution of the technology to the mathematical development of the task rather than to the presentation and/or communication.

The level of calculator or computer technology varies from school to school. Therefore teachers should state the level of the technology that is available to their students. While printed output is not required, some statement confirming appropriate use of technology (from the teacher or student) is necessary.

Using a computer and/or a GDC to generate graphs or tables may not significantly contribute to the development of the task, and therefore may not merit more than a level 1.

Criterion F: quality of work

Students who satisfy all the requirements correctly achieve level 1. For a student to achieve level 2, work must show precision, insight and a sophisticated level of mathematical understanding.

Award level 2 only if the work presented is beyond ordinary expectations. The teacher will take pause to admire the quality of such work ("Wow! Now, that's impressive!").

Only a totally inadequate response would receive 0.

Candidate performance against each criterion

Candidates generally performed well against criterion A. The use of computer notation seemed to be very limited. Correct terminology should include the use of correct mathematical vocabulary, such as "substitute" instead of "plug in".

Some students produced excellent pieces of technical writing. On the other hand, others have merely shown the steps to the solutions of problems and their work was found to be severely lacking in explanation and links within and between parts of the tasks. To meet the expectations in criterion B, students should be given explicit instructions to structure and present their work with more polish.

In criteria C and D, students have fared well, but the assessments by their teachers have been notably lenient. In type I tasks, sufficient data have often not been generated to justify the formulation of a conjecture. Where several intermediate general statements were derived, the proof of *the* general statement was not always evident to warrant full marks. In type II tasks, the statement of variables, perhaps in a "let statement", has often been implied, but should be explicit. Some realisation of the significance of the results obtained in terms of the model when compared to the actual situation needs to be given, but few students endeavoured to analyse their findings.

Success in meeting criterion E varied considerably. The full capabilities of a GDC or computer software were often not realised in the limited scope of some of the tasks set. Full marks were given much too generously for an *appropriate* but not necessarily a *resourceful* use of technology, for example, in the inclusion of a sole graph produced on a calculator. Students should be discouraged from including GDC key sequences – they are unnecessary and unwarranted.

Many good pieces of work were noted; however, the awarding of full marks in criterion F requires more than completion and correctness as noted in the clarifications above.

Recommendations for the teaching of future candidates

Teachers should select tasks that provide students with a variety of mathematical activities suitable at higher level. Tasks taken from the Mathematics SL TSM do not meet HL requirements as they do not generally offer access to the full range of criterion marks for HL.

Teachers are expected to write directly on their students' work not only to provide feedback to students but information to moderators as well. Some samples contained very few teacher comments.

Original student work must be sent in the sample, as teacher comments on photocopies are often illegible. Moderation was extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks.

Moderators find the background to each portfolio task very useful in determining the context in which the task was given when confirming the achievement levels awarded. This information must accompany each sample.

If a teacher-designed task is submitted, a solution key must accompany the portfolios in order that moderators can justify the accuracy of the work, and appreciate the level of sophistication demonstrated in the work.

Higher level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-20	21-40	41-52	53-67	68-82	83-97	98-120

The areas of the programme and examination that appeared difficult for the candidates

The ambiguous case of the sine rule was ignored even though the question stated in bold that the obtuse angle was required. Most candidates had no idea of how to handle a translation of a function. The use of the absolute value in an equation was not well understood. The chain rule was poorly demonstrated and the exact solution of trigonometric equations was poor. The algebra required by question 17, if the candidates chose Method 2 (which most did), was too much for most of them to handle. The work was filled with numerous algebraic errors. Permutations were not understood by the majority of candidates, this being the question that gained the fewest marks

The areas of the programme and examination in which candidates appeared well prepared

On the whole there was good use of the GDC for this exam, with evidence of good decision-making as to when its use was the best strategy. Graphing skills were demonstrated in this paper in question15 and question 20. However, there were a number of candidates who seemed unable to work with the indefinite graphs in question 20. Many candidates handled implicit differentiation and Poisson distributions well, even though they did not seem to be able to work with a conditional probability.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This question was correctly answered by most candidates. A few made careless algebraic mistakes and a handful seemed to have used a trial and error approach, since no working was shown, other than the list of terms in the sequence and the required value of d .

Question 2

Although the vast majority of candidates gave the modulus-argument form correctly, relatively few were able to obtain the correct answer to part (b). Most incorrect answers involved working out the product of z_1 and z_2 , equating the product to 2 and then trying to work toward the value of r .

Question 3

A surprising number of candidates did not even attempt this question; very often not a single candidate from the school had answered it, indicating clearly that the topic had not been taught. Some candidates even wrote notes suggesting that it is not on the syllabus. Others knew to replace x but some replaced x by $x+2$ and/or added 1. Others correctly approached this problem by first completing the square and then shifting the vertex to find the new function.

Question 4

This question was poorly answered by most candidates. Most automatically associated 'tangent' with 'derivative', found $f'(x)$ but then were unable to go any further. Few used the more direct discriminant method.

Question 5

Most candidates were able to answer this question very satisfactorily. The few incorrect answers came from students who did not apply the Remainder Theorem and tried to work out the divisions of the polynomials, obtain algebraic expressions for the remainder and then equate them to 0 – the algebra involved led many to incorrect values.

Question 6

There were some problems, not many, with the frequencies. Some failed to show the use of mid-values in the calculation of the mean.

Question 7

The most common mistake here was to ignore the fact that the obtuse angle for B was required. Very few approached it through Method 2, using the cosine rule.

Question 8

Many candidates gained full marks for this question. The exception seemed to be those who lost the first A1. It was also one of the questions in which some candidates lost marks through not showing their working.

Question 9

There were many good answers to this question, although quite a few candidates gave only one of the two solutions through simply ignoring the absolute value completely. Others tired to square both sides of the equation but then incorrectly reduced it to $2\ln(x+3)=1$. There were fewer attempts than in equivalent questions in previous years' paper, to find the solutions from the GDC.

Question 10

Most found this problem quite easy. A few lost valuable marks through not indicating their method. Others entered the functions into the GDCincorrectly, using $y=2^{0.5}x$ and $y=3^{-0.5}x+\frac{5}{3}$ to produce two linear functions which intersected at 1.99. The reference to the y -axis in the question may have led a few students into trying to integrate with respect to the y -axis – these attempts were usually not successful, either because the limits of integration were not found correctly or simply because the functions in terms of y became so much more complicated.

Question 11

Part (a) was done without any apparent difficulty. In part (b) they were many correct approaches although some led to much more algebraic manipulations. Incorrect answers came from stating $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}=0$.

Question 12

Surprisingly, this question was done correctly by many candidates. They obviously had studied such problems and were not put off by it. There were a few, however, who knew it was integration by parts but could not even begin correctly or who, beginning it correctly, did not recognise the cyclic nature of the problem.

Question 13

Many candidates did this question without any problem. The most common mistakes made were to assume independence, using $P(A\cap B)=P(A)+P(B)$ to find $P(B)$ or $P(A\cap B)=P(A)\times P(B)$ to find $P(A\cap B)$. However, (and to some extent surprisingly, since many had assumed independence in answering part (b)), most candidates were successful in showing that A and B were not independent.

Question 14

This was fairly disappointing, as the candidates did not seem to be able to apply the chain rule correctly. A few candidates manipulated the function using trigonometric identities before differentiating. Certainly their skills in handling the resulting equations were almost non-existent. Those who did know how to find exact values often gave answers which were outside the given domain.

Question 15

The sketches in part (a) were usually well drawn, although the level of precision and neatness of the answers varied enormously. Some candidates were obviously working with their calculators set in degrees instead of radians. Many failed to give the correct values for the range.

Question 16

Part (a) was fine although some did not understand the meaning of 'more than two'. Only a minority of candidates tackled the conditional probability in part (b) successfully.

Question 17

This problem was definitely one of the most poorly answered questions in the paper. Very few spotted that $\det(AB)=(\det B)^2$. They tried to use method 2 and could not sustain the algebra it required. However, the majority were able to get the last two marks.

Question 18

Most candidates recognised this as a situation requiring implicit differentiation. The main error was in the differentiation of 3^{x+y}. Some candidates made the question more difficult by taking ln of both sides before differentiating. The majority of candidates knew to separate the $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms from the others.

Question 19

Very few candidates gained any marks on this question, even if they attempted something. Most tried to consider combinations rather than permutations.

Question 20

In part (a) some candidates simply took the absolute value of the given function. Others just reflected across the y-axis. In part (b) some tried to graph the inverse of the function, rather than the reciprocal function.

Recommendations and guidance for the teaching of future candidates

- Be aware of the difference between translations of functions (which is on the syllabus) and matrix representation of transformations (which is not on the syllabus)
- Concentrate more on the understanding and application of the concepts of probability and statistics
- Read the questions carefully and answer the questions asked.
- Consider the presentation of mathematics in a more systematic way. Many students missed a number of marks for their lack of working. In some cases working was shown but very often it consisted of unconventional and poorly presented methods of finding solutions. This makes it difficult for the candidate to logically follow the processes they use, thus resulting in simple errors.
- Teachers should be more careful in selecting students for HL mathematics. Many of the candidates were clearly ill suited for study at this level, to the point that they had clearly wasted two years' worth of study. Schools allowing student to enrol in a course for two years only to have these students achieve a mark of 5 out of 120 might consider monitoring the progress of their students more closely.

Higher level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-19	20-39	40-52	53-66	67-81	82-95	96-120

General comments

Several examiners have commented on the number of extremely poor scripts they have marked, which seem to indicate that it is a pity that these candidates have been registered for the subject at higher level, when they clearly lack many of the skills and the ability to face the demands of the course.

The areas of the programme and examination that appeared difficult for the candidates

Questions which many candidates had difficulty in answering were those based on the contents of Trigonometry, specifically those relating to the manipulation and proof of identities. Algebraic manipulation tended to be rather weak, too.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates seemed able to attempt all questions, although the work done often demonstrated varying degrees of competence with the material. There is some evidence that some students are placed at a disadvantage by not being taught all the areas of the programme. The use of graphics calculator was not, overall, satisfactory: some scripts showed unnecessary amounts of working in situations where it was not required (eg, when asked for the expected value, all that is required is that candidates write down the definite integral and then use the calculator to evaluate it). This tendency almost inevitably results in problems with timing in the remaining part of the examination.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a) (i) Correctly answered by most candidates.

(ii) Practically all the candidates started this part correctly (by using the scalar product) and many were successful in showing that m = -1. However, a good number lost marks through substituting m = -1 into the expression obtained for the scalar product.

(b) Most students were able to find the required equation of the plane. A few used the parametric equation of the plane and were often unsuccessful in dealing with the algebra required to derive the Cartesian form.

(c) The area of the triangle was correctly determined by most candidates.

(d) (i) Although most candidates tackled this part of the question adequately, quite a few lost marks due to incorrect notation for the equation – an amazing number wrote L = … --, or for not giving the equation in the required form.

(ii) Well answered on the whole, the most common errors being arithmetic mistakes in the determination of $\overrightarrow{AD}$ and the use of the wrong formula for the volume of the pyramid. A few candidates wasted time trying to "re-determine" the point of intersection between the height of the pyramid and the plane.

Question 2

This was the question on which candidates scored fewest marks, and which many left unanswered, or only partially attempted.

(a) (i) A high number of correct answers here, although quite a few incorrect expressions were seen also, mainly through giving the third and fourth terms of the expansion as $3\cos\theta\sin^2\theta i$ and $\sin^3\theta i$ respectively.

(ii) although it was clear many candidates were familiar with this type of question and had no difficulty proving the required identities, there was an unexpected number of students who did not know how to associate the answer to (i) with this second part, and either did not answer the question, or attempted the proof through other methods.

(b) This part of the question was successfully answered by many students, although again, a few ignored the "hence" instruction in the question and proved the identity through other methods.

(c) This was not on the whole well answered, most candidates failing to make the connection between this part and the previous parts of the question. Of those who did, relatively few were able to arrive at the final correct exact value: many simply found a value for θ, and then looked for the value of $\tan 3\theta$ on their calculators. Other successful attempts used the $\tan 3\theta = \tan(2\theta + \theta)$ approach.

Question 3

(a) Most candidates answered this part correctly, although a surprising number did not actually give the required *value* for the maximum velocity, even after having justified that there was a maximum at $t = 3$. Sign diagrams with no values, to justify the maximum, were not awarded marks.

(b) The acceleration was found correctly by most candidates.

(c) Quite a high proportion of correct answers here. However, many candidates lost the marks allotted to including and then finding the values of the constants of integration.

(d) (i) The sketches here varied widely in their precision and neatness; many did not show and label clearly the intersections with the y-axis, others were inaccurate in that the curve drawn did not show the function over the whole of the required domain. Many answers seemed to reflect poor use of the gdc viewing window, with the graphs showing the displacements for values of t between –6 and 6 only, resulting in only one solution being found for part (ii). A few sketched the curves for the velocity instead of the displacement.

(ii) Many candidates were able to complete the question satisfactorily. Common errors were to leave out one of the solutions, as previously indicated, or to try and solve the equation analytically.

Question 4

Part A

(a) (i) Well done by many, with most candidates knowing to use the correct formula to find the value of μ . Many students, however, lost marks through algebraic or arithmetic mistakes in the calculation of the integral, not to mention the time invested in finding an answer which could be obtained easily and quickly from the gdc.

(ii) This part proved a bit more complicated for most candidates. Common errors were to simply calculate $\int_4^{10} t^2 f(T)\,dt$ or again, algebraic errors in the calculation of the integral.

(b) Relatively few correct answers here, with many candidates finding the limits of the interval correctly, but then trying to find the probability corresponding to a normal distribution.

Part B

(a) Both the mean and the standard deviation were correctly found by most students, although a few gave the variance instead of the standard deviation.

(b) (i) Correctly answered by most candidates.

(ii) Well answered on the whole, although a common mistake was to calculate this probability as $P(X \le 8) - P(X \le 4)$.

(c) Almost all candidates found the correct answer to this part.

(d) This was the point which candidates found most difficult; few were able to translate the required condition into an inequality or even an equation.

Question 5

(a) Successfully solved by practically all the candidates.

(b) (i) A considerable number clearly did not know what was required here, and others embarked on long algebraic "deductions" leading nowhere.

(ii) There was a very wide variety in the quality of answers to this part. Quite often it was hard to determine whether the candidate was actually solving the system, or trying to show that it could be solved, in some cases candidates showed that the system had infinitely many solutions, but then in solving it came to a unique solution. Overall, this part of the question was not very well answered.

(c) This proof was not on the whole well done. As usual, the most common flaws were the lack of rigour in some of the steps (verifying that the proposition is true for $n = 1$, writing the final statement) and the lack of clarity in the mathematical process when trying to show that $P(k)$ true $\Rightarrow$ $P(k+1)$ true.

Recommendations and guidance for the teaching of future candidates

- Make full use of the advantages the use of the gdc offers in answering this paper; candidates tend to waste precious time working analytically when this is not strictly necessary.
- Emphasize the need to adapt the viewing window on the gdc to the requirements of the question.
- Expose students to show that and prove that questions, making clear that, for example, verifying that m = -1 satisfies an equation is not equivalent to showing that given certain conditions, m = -1.
- Make sure that candidates answer each question on a separate sheet. This not only makes its marking easier – it also helps candidates organize their work and their time during the exam better.

Higher level paper three

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-10	11-20	21-26	27-33	34-39	40-46	47-60

General comment

Many G2s were received from schools with comments referring to the accessibility of option C – series and differential equations. The examiner reports also noted that many candidates had underperformed on this option, mainly through losing many marks on question 5. In light of these two separate reports and comments from the senior examining team it was decided to run an analysis of a sample of candidates. The analysis of the sample compared candidate performance on the compulsory core papers with that on paper 3. The results confirmed that there was inequity in the options – option C being considered more inaccessible and option D more accessible. As a result the marks of candidates who had chosen these options were adjusted . We feel that the adjustments made during grade award mean that no candidates have been penalised by choosing different options.

The areas of the programme and examination that appeared difficult for the candidates

In Section A, the question on the exponential distribution was not well done in general. Also, questions which attempt to explore a candidate's deeper understanding, eg questions 2(c) and 3(c) are poorly done in general, indicating that although the statistical methods can be applied, the theory underlying them is not well understood.
In Section B, more care needs to be taken when establishing that a relation is an equivalence relation. Also, questions of a more theoretical nature, eg Ques 5, are very badly done in general with candidates seemingly not knowing even where to start.
In Section C, the convergence (or otherwise) of series continues to present problems with some candidates having only a superficial knowledge of this topic.
In Section D, although candidates are able in general to solve problems involving graphical algorithms, some find more theoretical problems, eg Question 3, beyond their capabilities.

The areas of the programme and examination in which candidates appeared well prepared

In Section A, candidates are generally well prepared for using a graphic display calculator to carry out statistical tests although some would be better advised to give more explanation so that method marks can be awarded if their answer is incorrect.
In Section B, candidates are generally able to solve problems on group theory except where these are more theoretical.
In Section C, problems on differential equations are well done in general although some candidates lost marks, especially in Question 1, by not showing all their working even though their final answer was correct.
In Section D, work involving graphical algorithms is generally good.

The strengths and weaknesses of the candidates in the treatment of individual questions

Section A

Question 1

This was well answered by most candidates. In (b), the most common approach was to use the asymptotic normality of a sample proportion either using the statistics menu on a calculator or calculating the z-value. Some candidates used the binomial distribution which gives the exact p-value but both methods were accepted.

Question 2

As one might expect, some candidates divided by 200 instead of 199 to estimate the variance. Others used a 2-stage process, dividing by 200 and then multiplying by 200/199. In some cases, this produced a rounding error. Strictly speaking, since the distribution was not given to be normal, the t-distribution should not be used to find the confidence interval. This method, for which the appropriate percentile is 1.97, was however accepted on this occasion. Answers to (c) rarely involved a correct mention of the role of the Central Limit Theorem.

Question 3

This was well answered by many candidates, usually with the help of the statistical function on the GDC. In (c), few candidates mentioned the fact that the t-test could be used because the data are normal.

Question 4

Most candidates realised in (a)(i) that the probability density function had to be integrated although some candidates simply carried out an indefinite integration and then forgot about the negative sign produced. Many candidates used the incorrect notation $\int_t^\infty \mathrm{e}^{-t}\mathrm{d}t$. Candidates should be aware that it is incorrect to use the same symbol to denote the variable of integration and one of the limits although this was condoned on this occasion. The conditional probability introduced in (a)(ii) caused problems for many candidates. Few candidates realised the connection between (a)(ii) and (b).

Question 5

This was well answered by many candidates although some calculated the expected frequency for '5 or more' as that of 'exactly 5', obtaining 3.125 instead of 6.25 and therefore having to combine classes.

Section B

Question 1

This was well answered by many candidates although in (a) some gave a decimal approximation despite being asked for the exact range. In (b), most candidates knew what an injection was but some seemed to be unfamiliar with the term surjection. In (c), most candidates found a correct expression for $g^{-1}(x)$ although some were unable to state its domain correctly.

Question 2

Most candidates showed that R is reflexive and symmetric. Proof of transitivity was often inadequate with many simply stating that $a^2 \equiv b^2$ and $b^2 \equiv c^2 \Rightarrow a^2 \equiv c^2$ without any justification. Candidates were expected to write

$$a^2 \equiv b^2 \Rightarrow a^2 - b^2 = 6m, b^2 \equiv c^2 \Rightarrow b^2 - c^2 = 6n \text{ so } a^2 - c^2 = 6(m+n) \Rightarrow a^2 \equiv c^2$$

Question 3

Most candidates proved closure correctly but the fairly straightforward algebra to prove non-associativity proved too difficult for some. Many candidates stated that 1 was the identity, not realising that an identity must be both a left-hand identity and a right-hand identity.

Question 4

In (a), it was disappointing to note that many candidates were unable to define a Latin square. Many candidates solved the equation in (b) correctly although some omitted some of the intermediate steps. Part (c) was well answered by many candidates although some failed to find both generators and others failed to show that 2 and 5 were the only generators.

Question 5

Theoretical questions like this often cause problems for candidates and this was no exception even though this particular result is mentioned specifically in the syllabus. Many candidates failed even to show that the identity was in *H*.

Section C

Question 1

This question was well answered by many candidates although some failed to give intermediate answers. Candidates should be aware that marks can be lost for not showing all working.

Question 2

It is important in 'Show that' questions that candidates show their work in full. In this case, candidates who wrote

$$\int \tan x \mathrm{d}x = \int \frac{\sin x}{\cos x} \mathrm{d}x = \ln \sec x$$

were not given full credit on the grounds that an intermediate result '$= -\ln \cos x$' was omitted. Candidates who proved the result by showing that the derivative of ln sec*x* is equal to tan*x* were given full credit on the grounds that integration is actually the reverse process to differentiation.
Most candidates solved (b) correctly. In (c), however, some failed to include an arbitrary constant which led to the loss of 4 marks since the initial condition could not be applied.

Question 3

Most candidates solved (a) correctly but (b) caused problems for many. Those who realised that the expression could be rewritten as $\ln x \div (1/x)$ were usually successful.

Question 4

Solutions to (a)(i) were often disappointing. Some candidates had a vague idea of what had to be done but solutions usually lacked rigour. In (a)(ii), many candidates just guessed the answer with only a few realising that $\sum n^{-p}$ with $p = 1$ and 2 provides a useful counter-example. In (b), many candidates realised that the integral $\int \frac{\mathrm{d}x}{x(\ln x)^p}$ had to be evaluated but many failed to spot that this integral could be simplified by putting $u = \ln x$.

Question 5

Most candidates failed to see that (a)(i) could be solved by putting $x = 0$ in the given equation and noting that $f^{(n)}(0) = n!a_n$. Some candidates who failed to solve (a)(i) nevertheless used the result in it in (a)(ii) to find an expression for a_n. Candidates who reached this point usually went on to solve (b) and (c).

Section D

Question 1

This question was well answered by many candidates, although some were unable to carry out the multiplication in base 6 required in (b).

Question 2

In (b), many candidates found a particular solution but the general solution was not always given correctly.

Question 3

Most candidates made a reasonable attempt at this question although some solutions were not sufficiently precise. In (b), some candidates tried to draw the appropriate graph with the intention of showing that this could not be done. Such attempts were usually unconvincing although partial credit was given.

Question 4

This was well answered by many candidates. It was pleasing to note that in (a), many candidates were familiar with the properties of powers of adjacency matrices, in (b), many candidates used a systematic method for investigating whether or not a graph is bipartite and in (c), many candidates were familiar with the conditions necessary for the existence of an Eulerian circuit.

Question 5

In (a), many candidates failed to realise that the simplest way to find an upper bound is to evaluate the length of any cycle. A common solution involved finding the weight of a minimum spanning tree and doubling this. Although this is a valid method, it takes much longer and does not give as good an upper bound. In (b)(i), some candidates simply wrote down the minimum spanning tree without explaining their method. In problems requiring a particular algorithm, it is necessary to show clearly that this algorithm is being used for full credit to be given. In (b)(ii), some candidates appeared not to be familiar with the relationship between the minimum spanning tree and the travelling salesman problem.

Recommendations and guidance for the teaching of future candidates

In Section A, be aware of the requirement for answers to be given to three significant figures. Many candidates suffered an accuracy penalty in this section. Ensure that in problems where candidates use a calculator, as much explanation as possible is provided to allow for the award of method marks if the answer is incorrect. Ensure that candidates are fully aware of all the theoretical distributions in the new syllabus.

In Section B, ensure that candidates are thoroughly familiar with all the extensions to function theory mentioned in Paragraph 9.3 of the syllabus. Since solutions to theoretical questions are often poor in general, try to ensure that candidates realise that clarity and rigour are important in such questions.

In Section C, ensure that candidates are aware that all steps of a solution must be given. Some candidates lost marks by omitting intermediate steps. Emphasise that the general solution to a differential equation contains an arbitrary constant whose value is determined using an extra condition.

In Section D, ensure that candidates realise that when a question stipulates that a certain algorithm should be used, the solution should clearly indicate that this algorithm has been used.

