MATHEMATICS HL (IBNA & IBLA)

To improve the security of IB examinations, a selection of examination papers now have regional variants, including mathematics HL papers 1 and 2. The following report is for *mathematics HL* taken by candidates in the IB regions of North America and Latin America.

This was the last May session of the current courses. Teachers are reminded that the new courses will be examined for the first time in May 2006, and they should make sure they are preparing their candidates from the new guides. For this session, while it was pleasing to note the slight improvement in the portfolio results, many script examiners commented on some poor work seen. The examining team felt that there was a drop in the standard of the work produced, with many careless errors, and poor algebraic skills. While it was agreed that the papers were slightly more difficult than those of May 2004, the team did not feel that this explained some of the basic errors in understanding. Mathematics HL candidates should have a good background in mathematics and be competent in a range of analytical and technical skills. In general, they have a strong interest in mathematics and enjoy meeting its challenges and engaging with its problems. Students embarking on this course should expect to develop insight into mathematical form and structure, and should be intellectually equipped to appreciate the links between concepts in different topic areas. The course is a demanding one, requiring students to study a broad range of mathematical topics through a number of different approaches and to varying degrees of depth.

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-16	17-30	31-41	42-53	54-65	66-77	78-100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-4	5-6	7-8	9-11	12-13	14-16	17-20

There were many good portfolios produced this session. The expectations seem to be well understood by most teachers and their students. In their Examiner's Reports, moderators have made a number of observations in three areas – the tasks, suggestions to teachers, and the candidates' performance against the criteria. Their observations are summarised below:

The tasks:

Most tasks were taken from the Teacher Support Material (TSM) for Mathematics HL, but a few promising teacher-designed tasks were also noted. With some modification, these new tasks could be suitable for the revised assessment criteria in the new syllabus.

Task types appear to be better differentiated by teachers, as there were fewer tasks that were incorrectly categorised.

It is quite unfortunate that some teachers have taken tasks published in the TSM, and without any modification (except perhaps to the title), reclassified them for assessment as a different task type or against other unintended criteria. There were significant losses of marks in such instances.

Some teacher-designed type I tasks involved mathematical proof, but were lacking in the essential component that should have directed students to *generate and observe patterns* before formulating a conjecture, then producing an inductive generalisation to satisfy criterion E. Also, it would be appropriate for modelling tasks to direct students to look for the limitations of a model, suggest alternatives and make comparisons.

Suggestions to teachers:

Teachers should select tasks that provide students with a variety of mathematical activities suitable at higher level. Tasks taken from Mathematical Methods SL do not meet HL requirements and will not score high marks.

Teachers must provide more feedback to students on their work. Some samples contained very few teacher comments to students. As well, original student work must be sent in the sample, as teacher comments on photocopies are often illegible. Moderation was extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks.

Moderators find the background to each portfolio task very useful in determining the context in which the task was given when confirming the achievement levels awarded.

If a teacher-designed task is submitted, a solution key must accompany the portfolios in order that moderators can justify the accuracy of the work, and appreciate the level of sophistication demonstrated in the work.

Reminders for the new syllabus:

Teachers are reminded that in the new syllabus, the familiar but oversubscribed tasks in the TSM will not be appropriate to include in the portfolio. Teachers are encouraged to develop their own tasks. Please refer to the new TSM, published in September 2005.

The "Extended Closed Problem Solving" task (type II) will be deleted from the new internal assessment structure. The "Investigation" and "Modelling" tasks will remain and be known as "type I" and "type II" respectively.

Students' performance:

Students generally performed well against criterion A (Use of Notation and Terminology). The use of computer notation, such as "x^2 – 3x +1", "13*2", or "4.3E9", seemed to be very limited. Correct terminology should include the use of correct mathematical vocabulary, such as "substitute" instead of "plug in".

Criterion B (Communication) was often assessed with little attention being paid to the expected care and detail in the presentation of the work. It does not seem that all students were given sufficient direction in meeting the expectations under this criterion, as some samples contained very poorly presented work. Students must be directed to acquire some skills in technical writing. Some of them have merely shown the steps to the solutions of problems and their work was found to be severely lacking in explanation and links within and between parts of the tasks.

Some students were generously rewarded by their teacher with the highest level of achievement under criterion C (Mathematical Content) or criterion D (Results and Conclusions) for adequate work which, although complete, did not manifest much insight or sophistication.

As noted earlier, some students were not given a suitable task that directed them to engage in generating a pattern, formulate a conjecture or to present an inductive generalisation with formal arguments in order to meet the requirements under criterion E. Also, as in the above remarks regarding criterion B, a number of students did not correctly state the important final conclusions after attempting an inductive proof.

Success in meeting criterion F (Use of Technology) varied considerably. The full capabilities of a graphic display calculator (GDC) or computer software were often not realised in the limited scope of some of the tasks set. Full marks were given much too generously for an *appropriate* but not necessarily a *resourceful* use of technology, for example, in the inclusion of a sole graph produced on a calculator.

Paper 1

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-19	20-39	40-52	53-66	67-81	82-95	96-120

Summary of the G2 forms

- Comparison with last year's paper:

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
0	1	10	10	2

- Suitability of question paper:

	Too easy	Appropriate	Too Difficult
Level of difficulty	0	28	4
	Poor	Satisfactory	Good
Syllabus coverage	1	17	14
Clarity of wording	0	13	17
Presentation of paper	0	9	21

Areas of Difficulty

Areas that proved difficult for most candidates were Probability and Statistics, Calculus and Trigonometry.

Centres need to ensure that their school scheme of work covers the syllabus; there seems to be evidence that some candidates are placed at a disadvantage through not having been taught certain topics thoroughly enough.

General comments about the strengths and weaknesses of the candidates

Although the use of GDCs was on the whole satisfactory, many candidates need more practice in discerning when it is a good strategy to use their calculators. Furthermore, many candidates incurred an accuracy penalty (AP) for failing to give an answer correct to three significant figures.

Some candidates had clearly learned methods well, and had learned how to apply them. Understanding, however, was not so good. In the questions that asked candidates to use techniques in a more creative way, there was evidence of uncertainty.

It has been disappointing to see the amount of marks lost by candidates due to careless work including miscopying information from the question and through algebraic errors one would not expect from students at this level, even under the pressure of examination conditions.

Performance on individual questions

Question 1

This question was quite well answered although it was not uncommon to see careless arithmetic errors. Some candidates confused the test for perpendicularity by attempting a cross product rather than a scalar product.

Question 2

This question was a standard use of the binomial theorem. Some candidates omitted the combinatorial term and a common error was $3^3 = 9$. Attempts to expand usually ended in numerous mistakes.

Question 3

Other than careless errors this was done well by the majority of candidates.

Question 4

In this question there were problems in calculation especially involving the 3×3 matrix. A number of candidates were unable to solve the quadratic equation correctly and there were many basic errors in arithmetic.

Question 5

Too many candidates failed to work in radians. Many of them made this question more difficult than it should be by trying to split up the area unnecessarily.

Question 6

Many candidates confused the standard deviation with the variance. Too many candidates seemed to find it difficult to correctly interpret the modulus sign in order to find the correct interval.

Question 7

Part (a) was correctly answered by most candidates. Of those who attempted part (b) a few forgot the binomial coefficient.

Question 8

Many of the candidates could find the measure of the central angle and the area of the sector but were unable to find the area of the triangle. Too few seemed to know the formula $\frac{1}{2}ab\sin C$ for area and some attempted to use the cosine rule.

Question 9

There were many errors in finding the first derivative. Many candidates did not acknowledge the square root or the natural logarithm, or both. However, the use of the quotient rule for part (b) was done well.

Question 10

There were few attempts to use a Venn diagram and a number of candidates tried to use a tree diagram. Many seemed to think that the two events were independent. Also although they could quote a formula for conditional probability they had difficulty applying it in this question.

Question 11

The majority of candidates knew what should be done with this question but their work was plagued with careless errors. Some changed z to $a+b\mathrm{i}$ and then completely confused themselves with the algebraic manipulations. Very few candidates used the GDC which was probably the best way to do the question.

Question 12

This question was not done very well. Surprisingly many candidates were able to use implicit differentiation well but many failed to realise the importance of the interval for x when finding the y value.

Question 13

Generally the majority of candidates did not do this question well. Some just put down an answer without any demonstration of the work behind the answer. Others just wrote down formulae but were unable to use them. Most candidates appreciated the need to use frequencies and extended the table. Fewer noted the need to use the mid-interval values. This question caused several candidates to have the AP applied.

Question 14

Most candidates were able to find the composite function and many of them were able to state the period. Finding the interval proved more difficult with some candidates only writing one of the values. There were few sketches showing the line intersecting the graph, with candidates preferring to try to solve algebraically. However, their algebra skills were not up to the task.

Question 15

This question proved more difficult with only a few good candidates recognising the conversion of the second part to $-\log_3(x+1)$. There was confusion over the application of the rules for logarithms. Most candidates knew how to change base and used that as a preferred method. Many of those were not able to proceed further. However, there were a good number of correct solutions with only a very few of those including $x=0$ and therefore not gaining full marks.

Question 16

There was a tendency to draw detailed axes for the sketch and a number of candidates had obviously spent time in drawing their response, which in fact was not time well spent. The sketches often did not include the x-intercept even though candidates stated it correctly in the latter part of the problem. Vertical asymptotes were found by most candidates but few found the horizontal asymptotes.

Question 17

This question was not well answered. Some candidates tried, most unsuccessfully, to integrate immediately, not realising that partial fractions had to be employed first. Some split the numerator $\frac{x^2}{x(x+2)^2}+\frac{3x}{x(x+2)^2}+\frac{12}{x(x+2)^2}$ and simplified but few could progress further with this method. For those candidates who knew partial fractions were required, a good number found the correct answer. Some used $\frac{\text{A}}{x}+\frac{\text{B}x+\text{C}}{(\text{x}+2)^2}$ but found the resulting integral difficult and were unable to complete the problem.

Question 18

There were a good number of fully correct answers to this question. However, there were also a number of errors not only in manipulation but in the construction of the equations. Many found a and b correctly but were unable to find c, apparently not realising that f (1)=7.

Question 19

The majority of candidates failed to make any progress on this question. They did not recognize the need to find AC. The main error was in assuming that the triangles ACD and ACB were similar and/or not using the fact that angle BAC is twice angle DAC.

Question 20

Few correct solutions were seen on this last question. The main source of error was in not establishing the correct relationship between r and h and therefore not writing the volume in terms of r.

Paper 2

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-14	15-29	30-40	41-51	52-63	64-74	75-100

Summary of the G2 forms

- **Comparison with last year's paper:**

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
	6	16	13	

- **Suitability of question paper:**

	Too easy	Appropriate	Too Difficult
Level of difficulty		39	4
	Poor	**Satisfactory**	**Good**
Syllabus coverage	1	22	20
Clarity of wording		18	25
Presentation of paper	1	17	25

Areas of difficulty

The following parts of the syllabus presented difficulties for many of the candidates.
3-D vector geometry, especially finding equations of planes; transformation geometry using matrices; Probability density functions; mathematical induction, especially the presentation of proofs.

Levels of Knowledge, Understanding and Skill

The use of calculators was generally satisfactory although candidates should be encouraged to support calculator work by explaining clearly what they are doing. Some candidates appear to be unfamiliar with the accuracy rules and marks are being lost through unnecessary accuracy penalties. Some candidates lost marks throughout the paper because of careless algebraic errors and miscopying previous answers.

Comments on individual questions

Question 1

Most candidates solved part (a)(i) correctly. In part (a)(ii), however, some candidates wrote, incorrectly, that $\overrightarrow{PQ} = \overrightarrow{OP} - \overrightarrow{OQ}$ instead of $\overrightarrow{OQ} - \overrightarrow{OP}$. Although most candidates showed that $\overrightarrow{PQ}$ is a multiple of $\boldsymbol{u}$, and therefore parallel to it, some used either scalar or vector products which are valid methods but unnecessarily long. The most common, and the most successful, method of finding the equation of the plane in part (b) was to find its normal vector as $\overrightarrow{PQ} \times \overrightarrow{TQ}$ (or $\overrightarrow{TP}$). Candidates who started with the plane either in the form $\boldsymbol{r} = \overrightarrow{OP} + \lambda \overrightarrow{PQ} + \mu \overrightarrow{TQ}$ or $Ax + By + Cz = D$ were generally less successful. In part (b)(ii), most candidates knew how to find the angle between two vectors although the correct vectors were not always chosen. Candidates should note that it is generally better to find such an angle using a scalar product leading to $\cos\theta$ rather than a vector product leading to $\sin\theta$ since this latter method can sometimes lead to an ambiguity.

Question 2

The IB convention for transformations, is that points should be represented by column vectors. Thus, if the transformation $\boldsymbol{A}$ takes the point represented by the column vector $\boldsymbol{x}$ to the point represented by the column vector $\boldsymbol{y}$, then $\boldsymbol{y} = \boldsymbol{Ax}$. It would appear that some centres are using the convention that points are represented by row vectors. This is a valid approach if used consistently but it is not the IB approach and it should not be used. Candidates using this approach were given full credit in part (a) but only partial credit in part (b) where it led to a matrix $\boldsymbol{M}$ that could not easily be recognised. In part (i)(b)(ii), the word 'hence' implied that the area should be found using the fact that the determinant gives the area magnification and candidates using other methods were given no credit. Candidates who found $\boldsymbol{M}$ correctly usually recognised it as a reflection but were not always able to find the axis of reflection.

Question 3

In part (a), most candidates saw that it would be sensible to take u = arctan$x$ and dv = 2x. Those who then put $v = x^2$ found that $\int 2x\arctan x\,\mathrm{d}x = x^2\arctan x - \int \frac{x^2}{1+x^2}\mathrm{d}x$. Many candidates were unable to take this any further. Those few candidates who took $v = x^2 + 1$, whether by luck or good judgement, were able to proceed immediately to the required result. Many candidates failed to realise that $\mathrm{P}(X < a) \Rightarrow \int_0^a f(x)\mathrm{d}x = \frac{3}{4}$, although candidates who were unable to make any progress in part (b)(i) often solved part (b)(ii) correctly.

Question 4

In part (i), most candidates showed that the result is true for $n = 1$. Thereafter, the proof was often poorly presented. Many candidates do not appear to realise that you have to ASSUME that the result is true for $n = k$ and then show that this leads to its being true for $n = k + 1$. Many candidates were not given the final mark in the proof by induction. This is given for stating something like 'We have shown that if the result is true for $n = k$ then it is true for $n = k + 1$ and since it is true for $n = 1$, the result follows by induction'. It is not sufficient to follow the algebra with a statement such as 'Hence proved by induction' or as was sometimes seen 'QED'. Part (ii) caused problems for many candidates who often filled several pages of complicated algebra and sometimes (remarkably) obtained the correct answer. Candidates who realised initially that $ar - a = 2(ar^2 - ar)$ usually solved the problem fairly quickly.

Question 5

In part (a)(i), the sketches were often poor with the curve intersecting the y-axis and the scale not shown so that the domain could not be indicated. Most candidates attempted the differentiation of $g(x)$ and attempted to deduce the gradient of the normal although algebraic errors were fairly common. In part (b)(i), some candidates stated that the gradient of PQ is $\frac{y}{(x-1)}$ but failed to substitute for y in terms of x. Many candidates were unable to relate the result in part (b)(i) to the problem in part (b)(ii). Some candidates thought that the minimum distance could be found by joining Q to the minimum point on the graph of g. This, by coincidence, actually gave the correct answer but it was, of course, given no credit. The alternative solution of minimising the distance between Q and a general point on the curve was seen with the minimisation performed either using calculus or GDC.

Question 6

In part (i), many candidates found the confidence interval correctly although some omitted the $\sqrt{n}$ term in the standard error. Hardly any candidates were able to make reasonable statements about the Central Limit Theorem with many misconceptions evident, the most common being that the actual distribution, and not the sample mean, is approximately normal for large n. Part (i)(b) caused problems for many candidates with some failing to realise that $\bar{x}$ was no longer 72.04. In part (ii), most candidates wrote down correct hypotheses, although not always in mathematical notation. The sums were usually evaluated correctly. The calculation of the pooled estimate was often incorrect with candidates seemingly unconcerned by very large values of their estimate. In part (ii)(c), candidates who used their GDC were usually more successful than those who attempted to use the formula for t. In part (iii), almost all the candidates calculated the mean correctly but most candidates went on to make errors in the calculation of chi-squared, the most common errors being a failure to realise that values exceeding six had to be taken into consideration and the last two classes had to be combined.

Question 7

Part (i) was reasonably well done by many candidates who saw that closure, identity and inverse are satisfied so that it must be associativity that is not. Solutions to part (ii)(a) were often incomplete, for example to show symmetry, some candidates stated that $\boldsymbol{ARB} \Rightarrow \boldsymbol{A} = \boldsymbol{HBH}^{-1} \Rightarrow \boldsymbol{B} = \boldsymbol{H}^{-1}\boldsymbol{AH} \Rightarrow \boldsymbol{BRA}$, omitting the crucial expression $\boldsymbol{H}^{-1}\boldsymbol{A}(\boldsymbol{H}^{-1})^{-1}$. Some candidates chose $\boldsymbol{H}$ unwisely in part (ii)(c)(i) giving themselves unnecessary algebra and others failed to realise the relevance of part (ii)(b) to part (ii)(c)(ii). Most candidates made a reasonable attempt at showing in part (iii)(a) that f is a bijection although showing that $f \circ f$ is self-inverse proved too difficult for many. Although the result to be proved in part (iv) is a standard one, most candidates were unable to make much progress in proving it.

Question 8

Most candidates made a reasonable attempt at using Euclid's algorithm in part (i). Part (ii) was reasonably well done although the explanation as to why G has a Eulerian circuit was often unclear. In part (iii), many candidates failed to realise what was required, going straight to the auxiliary equation and then wondering what to do with the double root. In part (iv), many candidates just wrote down the shortest path, apparently by inspection, without using Djikstra's algorithm for which they were given no credit. Candidates should be aware that when a problem requires the use of a particular algorithm, they should use only that algorithm and their solution should make it clear that the particular algorithm has been used.

Question 9

In part (i)(a), most candidates realised that the ratio test should be used although this test was not always correctly applied. In part (i)(b), some candidates used their GDC to sum the series although this approach was not permitted on the grounds that although the sum to infinity appeared to be 1, this approach did not actually prove that. Candidates who showed that $\sum_{k=1}^{n} \frac{(k-1)}{k!} = 1 - \frac{1}{n!}$ were able to answer parts (a) and (b) simultaneously. Most candidates made a reasonable attempt at part (ii)(a)(i) although algebraic errors were not uncommon. Very few candidates, however, solved part (ii)(a)(ii), not realising that they simply had to differentiate n times and put $x = 0$. Some candidates attempted, unsuccessfully, to prove the result by induction. Most candidates attempting part (ii)(b) made algebraic errors along the way. In part (iii)(a), the sketches were often poor, with no labelling whatsoever and no explanation as to why the sketch showed that there was only one root. Most candidates made a reasonable attempt at differentiating $g(x)$ and $h(x)$ although the attempts at proving the inequalities in part (c)(i) were often unconvincing. Many candidates went on to use the convergent fixed-point iteration to solve the equation correctly although some were clearly using their GDC in degree mode which, of course, gave an incorrect answer. Candidates who simply wrote down the answer were given no credit on the grounds that they probably used SOLVER to find the root.

Question 10

In part (i), most candidates found the coordinates of P, Q, R and S correctly. However, very few went on to show the existence of a harmonic ratio, giving the impression that most candidates did not know the meaning of this term. Solutions to part (ii) were often disappointing with most candidates writing down the equation of l correctly but making little further progress. Few candidates made any progress at all in part (iii) – in fact one candidate wrote on his script that the result stated in part (a) could not possibly be correct because you cannot have negative lengths.

Guidance for future candidates

It is suggested that more time be spent on the topics mentioned as areas in which candidates found difficulty. These topics mentioned as presenting difficulties seem to appear most years. In particular, candidates should concentrate on presenting proofs by induction in a more rigorous way.

Care needs to be given to the presentation and setting out of work; as usual candidates lost credit because it was impossible to decipher their trains of thought. Some scripts are laid out poorly and some are almost illegible. For paper 2, as stated in the rubrics, each question should be started on a new page.

It is important to encourage students to use their GDC critically and effectively. At the same time, it is also important not to lose sight of the need to maintain the basic algebraic skills that are essential to the course.

Teachers should ensure that IB conventions and notation are used in their teaching. It was noticeable this year that some candidates were using transformation matrices in Question 2 on paper 2 in a different way from that given in the guide. Although this particular problem will not occur next year because of syllabus changes, there are other areas of the syllabus where care needs to be taken.

