MATHEMATICS HL (IBAP & IBAEM)

To improve the security of IB examinations, a selection of examination papers now have regional variants, including mathematics HL papers 1 and 2. The following report is for *mathematics HL* taken by candidates in the IB regions of Africa, Europe, the Middle East and Asia/Pacific

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 16	17 – 31	32 – 42	43 – 55	56 – 66	67 – 78	79 – 100

Portfolio

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 4	5 – 6	7 – 8	9 – 11	12 – 13	14 – 16	17 – 20

Many candidates produced excellent portfolios, and exemplary pieces of work were noted this session. Whereas there were still a number of portfolio tasks not adequate for HL in terms of difficulty or length, there were also a number of pieces of work that were very extensive and more demanding than required. In their Examiner's Reports, moderators have made a number of observations in three areas – the tasks, suggestions to teachers, and the candidates' performance against the criteria.

The tasks:

The majority of the tasks were taken from the Teacher Support Material (TSM) for Mathematics HL. A few excellent teacher-designed tasks were noted, as were poorly designed tasks of questionable content and standard. It is worth noting that with the introduction of the new syllabus, none of the familiar but oversubscribed tasks presently in the TSM will be permitted for portfolio use. Teachers are encouraged to develop their own tasks with shorter "shelf lives".

Some teacher-designed investigation tasks involved mathematical proof, but did not fully meet the requirements of criterion E, as the opportunity to formulate a conjecture was missing. For full marks, there must be suitable components to direct candidates to generate and observe patterns, formulate a conjecture, then produce an inductive generalisation with proof.

Suggestions to teachers:

It is critical that teachers provide more feedback to candidates on their work. Very few samples contained actual teacher comments to candidates. As well, originals must be sent in the sample, as teacher comments on photocopies are often illegible. Moderation was extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks.

Only a few teachers explained the background to the portfolio tasks which moderators need and appreciate.

If a teacher-designed task is submitted, it is recommended that the solution key accompany the portfolios for moderators to justify the accuracy and appropriateness of the work.

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Candidates' performance:

Candidates generally performed well against criterion A (Use of Notation and Terminology). The use of computer notation, such as "x^3 – 5x +2" or "2.3E6", seemed only sporadic. Correct terminology should include the use of correct mathematical vocabulary, such as "substitute" instead of "plug in".

Criterion B (Communication) was often assessed with little attention being paid to the expected care and detail in the presentation of the work. It does not seem that all candidates were given sufficient direction in meeting the expectations under this criterion.

Some samples contained very poorly presented work. Candidates must be directed to acquire some skills in technical writing. Many of them have merely shown the steps to the solutions of problems and their work was found to be severely lacking in introduction, explanation, annotation, or justification.

Some candidates misinterpret a "conclusion" to be a comprehensive summary or a statement of the personal value of the work undertaken.

Some candidates were generously rewarded by their teacher with the highest level of achievement under criterion C (Mathematical Content) or criterion D (Results and Conclusions) for adequate work, which, although complete, did not manifest much insight or sophistication.

Criterion E (Making Conjectures) was treated inconsistently by teachers and candidates. Some candidates were not given the task of engaging in formulating a conjecture or in presenting an inductive generalisation with formal argument, as noted above.

Success in meeting criterion F (Use of Technology) varied considerably. The full capabilities of a GDC were often not realised in the limited scope of some of the tasks set. Full marks were given incorrectly for an *appropriate* but not necessarily a *resourceful* use of technology. The inclusion of a single graph produced on a calculator should merit even less – just one mark out of three.

Paper 1

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 18	19 – 37	38 – 52	53 – 66	67 – 79	80 – 93	94 – 120

Comparison with last year's paper:

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
1	12	47	13	1

Suitability of question paper:

	Too easy	Appropriate	Too Difficult
Level of difficulty	7	88	4
	Poor	Satisfactory	Good
Syllabus coverage	3	41	52
Clarity of wording	0	34	62
Presentation of paper	0	26	70

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Areas of Difficulty

Candidates were, in general, least comfortable in the areas of complex numbers, probability distributions (where some candidates were unable to distinguish between discrete and continuous distributions), vector projections, related rates and inequalities.

Levels of Knowledge, Understanding and Skill

Apart from the topics listed above, the overall standard of scripts was satisfactory.
However, although the use of calculators was generally satisfactory, many candidates use their calculators in degree mode when radian mode is required. Furthermore, many candidates incurred an accuracy penalty for failing to give an answer correct to three significant figures.

Performance on Individual Questions

Question 1

Answer: $a = 1$

Most candidates spotted that $x^2 - 4x + 3 = (x-1)(x-3)$ and most then substituted 1 or 3 into the cubic expression. The most common errors thereafter were algebraic ones in solving for a. Candidates who attempted to use long division were generally less successful.

Question 2

Answer: $y = \mathrm{e}^x - x^2 + 2$

A fairly common error was the omission of the constant of integration. Candidates who did this could only gain a maximum of 2 marks for this question.

Question 3

Answer: $x = -2.67, y = 1.22$
$x = -0.827, y = 0.609$
$x = 0.439, y = 0.187$

The most common error was to use the calculator in degree mode instead of radian mode. Candidates should be advised to check that they are in radian mode whenever a question involving calculus is being done.

Question 4

Answer: $a = -2, b = 4$

Candidates who went straight to the equations $a + b = 2$ and $b = a^2$ were generally more successful than those who introduced the common difference d and common ratio r.

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Question 5

Answer: Rotation (about the origin)
90° (anticlockwise)

This question was well answered by most candidates.

Question 6

Answer: $\frac{5}{4}-\frac{\sqrt{3}}{4}\text{i}$

Candidates often made algebraic errors in the process of multiplying through by the complex conjugate. Those who used their calculators in complex mode usually gained only the 3 marks for *a*.

Question 7

Answer: 0.645

Most candidates solved this problem correctly but a sizeable minority made no attempt.

Question 8

Answer: (a) $p=2$

(b) gradient $=-\frac{4}{7}(=-0.571)$

Many candidates obtained the value of p correctly but errors were often seen in the subsequent implicit differentiation. A fairly common error was to state that the derivative of 16 is 16.

Question 9

Answer: P is $(0,\ 1,\ -1)$

This was correctly solved by many candidates, the most common errors involving the substitution for x, y and z in the equation of the plane and the subsequent solution for m.

Question 10

Most candidates drew the graphs correctly. However, the horizontal asymptote in part (a) was often not labelled and intercepts were often labelled incorrectly or not at all.

Question 11

Answer: (a) $g(x)=\sqrt[3]{x+1}$

(b) $g(x)=\sqrt[3]{x}+1$

Most candidates solved this problem correctly. Some candidates gave the solution the wrong way round; others gave the same solution for parts (a) and (b).

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Question 12

Answer: (a) $\frac{1}{2}\ln\left|\frac{2m+3}{3}\right|$

(b) $m=\frac{3}{2}(\mathrm{e}^2-1)$ (=9.58)

Many candidates solved this problem successfully. The most common errors were to state, incorrectly, that $\int\frac{\mathrm{d}x}{2x+3}=\ln(2x+3)$ or to substitute the limits incorrectly. Algebraic errors were often seen in part (b).

Question 13

Answer: (a) $k=\frac{12}{25}$ $(=0.48)$

(b) $\frac{48}{25}$ (=1.92)

Many candidates solved this problem correctly. Some, however, used integration in their solutions showing that the distinction between discrete and continuous variables is not clear to all candidates.

Question 14

Answer: (a) 0.732

(b) 0.180 $\left(=\frac{11}{61}\right)$

Part (a) was solved correctly by many candidates although some assumed that the working week contained 7 days instead of 5. Many candidates were unable to make any progress in part (b).

Question 15

Answer: (a) $\boldsymbol{a}=6\boldsymbol{i}-5\boldsymbol{j}+4\boldsymbol{k}$

(b) $-\frac{28}{5}\boldsymbol{j}++\frac{14}{5}\boldsymbol{k}$

Most candidates found *a* correctly but very few appeared to be familiar with vector projections.

Question 16

Answer: $]-\infty,3]\cup[27,\infty[$

Solutions to this question were usually disappointing. Candidates who used their GDCs often missed the critical value $x = 27$; those who used algebra usually obtained the critical values 3 and 27 but often failed to progress correctly beyond that stage. Candidates who gave the

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answer as $x \leq 3$, $x \geq 27$ or $x \leq 3$ and $x \geq 27$ were penalised; only $x \leq 3 \cup x \geq 27$ (or equivalent) was accepted for full credit.

Question 17

Answer: 0.460

Many candidates showed correctly that $f'(x) = 3^x \ln 3$ but many then used the product rule to find $f''(x)$ in the mistaken belief that ln3 is a variable, usually with derivative 1/3. Most candidates attempted to solve the equation $f''(x) = 2$ algebraically with only a minority using the Solver facility on their calculators.

Question 18

Answer: $2x^{\frac{1}{2}} \ln x - 4x^2 + C$

Solutions to this question were often disappointing with candidates unable to identify suitable u and v functions. Candidates who omitted the constant of integration were penalised.

Question 19

Answer: $\frac{\mathrm{d}q}{\mathrm{d}t} = \frac{1}{10}$ rad per sec

Very few candidates made a successful start to this problem. Candidates who started with the expression (using an obvious notation) $h = 5\tan q$ were generally more successful than those who started with $q = \arctan\left(\frac{h}{5}\right)$, presumably finding the differentiation easier.

Question 20

Answer: $\frac{q}{1-\cos q}$

Some of the better candidates realised that the solution involved an infinite geometric series but only a few were able to apply the appropriate trigonometry to find the first term and common ratio.

Guidance for Future Candidates

The topics mentioned in the first paragraph above have been singled out in previous years as areas of difficulty and more time needs to be spent in teaching them in some centres. Many candidates have incurred accuracy penalties this time and all candidates need to be aware of the accuracy rules. Marks are being lost every year by candidates using their calculators in degree mode instead of radian mode.

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Paper 2

Component grade boundaries

Grade:	1	2	3	4	5	6	7
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Comparison with last year's paper:

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
1	10	38	15	5

Suitability of question paper:

	Too easy	Appropriate	Too Difficult
Level of difficulty	5	70	17
	Poor	Satisfactory	Good
Syllabus coverage	12	45	35
Clarity of wording	1	38	52
Presentation of paper	0	32	60

General remarks

Concerns were expressed about the equity of the option questions. The grade award team looked closely at these and agreed that question 9 was more difficult. This was confirmed by statistical analysis. To ensure that no candidate was disadvantaged, the marks for question 9 were adjusted for all candidates, in line with the information provided by the analysis.

With minor variations the remarks made in the report for the May 2003 session apply equally well for this session:

1) Candidates should be encouraged to look at their answers with some critical common sense. For instance, when the answer is a negative probability, or the sine of an angle is given as 3, candidates should immediately realize that something is wrong. Quite often the error would be easy to correct, but candidates lose many marks that they could have easily obtained had they spent a little time looking critically at their answers.

2) Candidates should be more careful with their graphs, often drawn without scales (and therefore meaningless).

3) Many candidates do not seem to know how to deal with accuracy problems : for instance, if an answer is required to 3 decimal places this usually implies working out the problem with 4 or 5 decimals, since errors may accumulate during the computation. Too many candidates err on the side of inaccuracy (better give more figures than required than fewer). Generally speaking the candidate should only consider the degree of accuracy required when giving the final answer and use maximum accuracy throughout the computation – the use of calculators makes this fairly easy.

4) It should be noted that on examination papers, unless otherwise stated, angles should be assumed to be in radians. Candidates should always work in radians when the trigonometric functions are differentiated or integrated (otherwise the standard formula do not apply!)

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5) Candidates should be strongly discouraged from writing their working in pencil which makes for very careless, messy and sometimes unreadable scripts.

6) Clearly some schools do not prepare for any option. Schools ought to realize that this is a definite disservice to their candidates who thus risk losing 30% of their marks on this paper.

7) In this session, on the whole, candidates used their calculators well. The only relatively common error was overuse of calculators: when exact answers or proofs are required, candidates should realize that they should use analytical methods, rather than use the GDC.

Question 1: Matrices and plane transformations

Answer: (a) (i) $\boldsymbol{M} = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$

(ii) Image of A′ is $(3, 4)$

(b) $\boldsymbol{T} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

(c) (i) Image of D is $(6, 6)$

(ii) Image of D is $(8, 8)$

(a) (i) and (ii) A minority of candidates chose to write the vectors in row form and have the matrices act on them by multiplication on the right. This approach, per se, is absolutely correct and exceptionally some candidates used this method successfully. Teachers should realize, however, that since the IBO uses columns vectors and multiplication by matrices on the left, most candidates using the first approach get confused and end up losing many marks by switching back and forth between the two approaches. Hence, teachers who use the first approach should strongly warn the candidates about the great risks of confusion between the two methods.

(b) A large number of candidates thought that translations are linear transformations and therefore can be described by multiplication by a matrix.

(c) On the whole answered correctly by the candidates who had done parts (a) and (b) satisfactorily, except for occasionally disregarding the order of the operations.

Question 2: Probability

Answer: (i) (a) $\frac{2}{3}(\text{or } 0.667)$

(b) $\frac{2}{9}(\text{or } 0.222)$

(c) $\frac{3}{4}$

(ii) (b) (i) $\mathrm{E}(X)=\frac{3}{2}$

(ii) $m=\sqrt[3]{4}$ $(=1.59 \text{ to 3 s.f.})$

(i) (a) and (b) Most candidates answered these questions correctly.
(i) (c) A substantial number of candidates answered this question correctly.
(ii)(a) and (b) A number of candidates tried to use a discrete sum, apparently unaware of the distinction between discrete and continuous distributions.
(c) Many candidates did not know what is the median of a distribution .

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Question 3: Vector geometry

Answer: (b) $\boldsymbol{r} = (\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}) + l\left(\boldsymbol{i} + \boldsymbol{j} + \frac{4}{3}\boldsymbol{k}\right)$

(c) The coordinates of D are $\left(3,\ 5,\ \frac{14}{3}\right)$

(a) This question was by far the most difficult for the candidates. Most candidates ignored completely the fact that the vectors were unit vectors and therefore could not find the proof they were asked for. Surprisingly very few candidates considered the angles between each vector and their sums (which would seem to be the natural approach) but chose instead to look at the angle beween the two vectors, getting nowhere.

(b) Here again most ignored the fact that the vectors had to be unit vectors, thereby getting an incorrect answer.

(c) Those candidates who had neglected to use unit vectors in part (b) obtained follow through marks. On the other hand too many candidates found a non-existent point of intersection of two skew lines because they neglected to verify that the third equation was also satisfied by the solution they had obtained from the first two equations.

Question 4: Study of functions and integration

Answer: (b) (i) $a = 0.860$

(ii) $b = 2.29$

(c) $x = \sin x + \cos x + C$

(d) $\frac{\pi}{2} - 1$

(b) (i) and (ii) This question was generally well answered although some candidates had their calculator in the degree mode, leading to errors. Some candidates lost a mark here for failing to give the answer to three significant figures.

(c) Also well answered on the whole. Some candidates had great difficulties with integration by parts.

(d) One or two marks were lost by candidates who failed to realize that the question asked for an exact answer.

Question 5: Trigonometric identities and proof by induction

Answer: (b) (i) $T_1(x) = x$

(a) Well done on the whole.
(b) (i) Too many candidates did not seem to know that cos(arccos*x*)=*x* but the question was correctly answered by most, as was part (ii).
(c) (i) Correctly answered on the whole except by those candidates who failed to use part (a) and embarked (prematurely) into (an impossible) proof by induction.
(c) (ii) Very few candidates gave a completely satisfactory answer to this question. Most of the difficulties arose not so much from the two tiered induction but from the fact that candidates did not know how to formulate the induction hypothesis (" *Tn(x)* is a polynomial of degree *n*") writing instead things like " $Tn(x) = x^n$ ". Too many candidates write "assume *n*=*k*" instead of

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"assume the property is true for $n=k$" thereby showing that they do not entirely understand what they are doing.

Question 6: Statistics

Answer: (i) (a) 0.224
(b) 0.236
(ii) $[17.1, 19.3]$
(iii) (b) reject H_0
(iv)
conclude that the mean height of men does not exceed the mean height of women by more than 10 cm.

Part (i) (a) was done correctly by most candidates. Quite a few failed completely to obtain the conditional probability in part (b).

(ii) Well done by most (often using the calculator). Some candidates failed to use the *t*-distribution. The usual confusion between the population variance and the sample variance (unbiased estimate of the population variance) appeared in many scripts.

(iii) (a) No candidate gave a satisfactory alternative hypothesis.

(b) Most candidates realized that the question required a c^2-test but very few were able to find the correct expected frequencies.

(iv) Two sources of errors were predominant here : failure to compute correctly the pooled estimate of the variance and failure to take into account the difference of 10 cm in the calculation of the *t*-value.

Question 7: Sets, Relations and Groups

Answer: (i) (b)
The equivalence classes are lines with equations
$y = x + \text{Constant}$
(ii) (a) $e = 2$
(b) $I = \frac{3}{2}$
(iii) (a) (i) 15
(ii) 5
(iii) 3
(c) (i) Elements of order 2 are 7, 9, 15.
(ii) Elements of order 4 are 3, 5, 11, 13.
(d)
Sub-group of order 4 is $\{1, 3, 9, 11\}$.
Another possibility is $\{1, 5, 9, 13\}$.

(i) (a) Most candidates performed well here (save for the transitivity part which was missed by a small minority).

(i) (b) Very few candidates answered this question satisfactorily.

(ii) (a), (b) and (c) Well done by most.

(iii) (a) and (b)(i). Done correctly by almost every candidate.

(iii) (b) (ii) Many candidates had difficulties in establishing correctly that the operation was closed and the existence of an inverse for every element.

(iii) (c) and (d) On the whole done correctly.

Question 8: Discrete Mathematics

Answer: (i) $x_n = 5(-4)^n + 2(7)^n$

(ii) (c) (i) The graph is bipartite.

(ii) The graphs are isomorphic.

(iii) The graphs are not isomorphic

(iii) (b) Total weight = 139

Few chose this option. Those who did performed satisfactorily on most parts.

(ii) (a) and (b) Many candidates showed that they understood the questions but had difficulties in putting their answers into words.

(iv) (a) and (b) Very few candidates could state correctly the well ordering principle and none could use it to prove the statement in part (b).

Question 9: Analysis and Approximation

Answer: (i) divergent

(ii) (a) $A = 1$

(b) $A = 1.000026312$

(iv) (c) limit zero

Few chose this option which turned out to be the most difficult for candidates. Appropriate measures were taken at the grade award meeting to compensate for this.

(i) Very few candidates observed that $\cos\left(\frac{\pi}{2} - \frac{\pi}{2n}\right) = \sin\left(\frac{\pi}{2n}\right)$. Those who did generally answered the question satisfactorily.

(ii) (a) All but one or two candidates got these two marks.

(ii) (b) Few candidates were able to use the Simpson error term to determine the number of intervals. Some failed to realize that this number must be even. A disappointingly small number of candidates were able to use the Simpson method correctly (for any number of intervals) because of great lack of precision in the computation (one cannot expect an answer to be accurate to 10^{-5} if one uses only two decimals in the intermediate computations!).

(ii) (c) Many candidates did not understand this question.

(iii) (a) Candidates showed that they do not know how to state or how to use the mean value theorem.

(iii) (b) This was more difficult and only those few who could use successfully the McLaurin series could give a correct answer.

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(iv) (a), (b) and (c) Few candidates dealt with this question in an even partially satisfactory way. Those who did had the good sense to compute the first few terms of the sum to get an idea of its behaviour. Many confused the sum with the terms of the sum.

Question 10: Euclidean Geometry and Conic Sections

Answer: (i) (a) (i) $(\text{y-b})^2 = (x+a)^2 - (x+a)^2 = 4ax$

(c) $S\left(0, \frac{\sqrt{6}}{2}a + b\right)$

(ii) $r = \frac{\sqrt{10}}{2}a$

This option seems to be chosen at random by candidates whose classmates have chosen another option, giving the impression that most candidates who chose this option do so on their own, without having been prepared for it in class, under the misaprehension that it is an easy option. The result is therefore not surprising : most candidates performed poorly, with some remarkable exceptions).

(i) (a) (i) and (ii) This is what most candidates attempted with some success.

(i) (b) and (c) This proved too difficult for most.

(ii) Very few candidates attempted this part (generally without success).

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