MATHEMATICS HL (IBNA & IBLA)

To improve the security of IB examinations, a selection of examination papers now have regional variants, including mathematics HL papers 1 and 2. The following report is for *mathematics HL* taken by candidates in the IB regions of North America and Latin America.

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 17	18 – 32	33 – 46	47 – 57	58 – 68	69 – 80	81 – 100

Portfolio

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 4	5 – 6	7 – 8	9 – 11	12 – 13	14 – 16	17 – 20

Many candidates produced excellent portfolios, and exemplary pieces of work were noted this session. Whereas there were still a number of portfolio tasks not adequate for HL in terms of difficulty or length, there were also a number of pieces of work that were very extensive and more demanding than required. In their Examiner's Reports, moderators have made a number of observations in three areas – the tasks, suggestions to teachers, and the candidates' performance against the criteria.

The tasks:

The majority of the tasks were taken from the Teacher Support Material (TSM) for Mathematics HL. A few excellent teacher-designed tasks were noted, as were poorly designed tasks of questionable content and standard. It is worth noting that with the introduction of the new syllabus, none of the familiar but oversubscribed tasks presently in the TSM will be permitted for portfolio use. Teachers are encouraged to develop their own tasks with shorter "shelf lives".

Some teacher-designed investigation tasks involved mathematical proof, but did not fully meet the requirements of criterion E, as the opportunity to formulate a conjecture was missing. For full marks, there must be suitable components to direct Candidates to generate and observe patterns, formulate a conjecture, then produce an inductive generalisation with proof.

Suggestions to teachers:

It is critical that teachers provide more feedback to Candidates on their work. Very few samples contained actual teacher comments to Candidates. As well, originals must be sent in the sample, as teacher comments on photocopies are often illegible. Moderation was extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks.

Only a few teachers explained the background to the portfolio tasks which moderators need and appreciate.

If a teacher-designed task is submitted, it is recommended that the solution key accompany the portfolios for moderators to justify the accuracy and appropriateness of the work.

Candidates' performance:

Candidates generally performed well against criterion A (Use of Notation and Terminology). The use of computer notation, such as "*x*^3 – 5x +2" or "2.3E6", seemed only sporadic. Correct terminology should include the use of correct mathematical vocabulary, such as "substitute" instead of "plug in".

Criterion B (Communication) was often assessed with little attention being paid to the expected care and detail in the presentation of the work. It does not seem that all candidates were given sufficient direction in meeting the expectations under this criterion.

Some samples contained very poorly presented work. Candidates must be directed to acquire some skills in technical writing. Many of them have merely shown the steps to the solutions of problems and their work was found to be severely lacking in introduction, explanation, annotation, or justification.

Some candidates misinterpret a "conclusion" to be a comprehensive summary or a statement of the personal value of the work undertaken.

Some candidates were generously rewarded by their teacher with the highest level of achievement under criterion C (Mathematical Content) or criterion D (Results and Conclusions) for adequate work, which, although complete, did not manifest much insight or sophistication.

Criterion E (Making Conjectures) was treated inconsistently by teachers and Candidates. Some candidates were not given the task of engaging in formulating a conjecture or in presenting an inductive generalisation with formal argument, as noted above.

Success in meeting criterion F (Use of Technology) varied considerably. The full capabilities of a GDC were often not realised in the limited scope of some of the tasks set. Full marks were given incorrectly for an *appropriate* but not necessarily a *resourceful* use of technology. The inclusion of a single graph produced on a calculator should merit even less – just one mark out of three.

Paper 1

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 21	22 – 42	43 – 57	58 – 72	73 – 86	87 – 101	102 – 120

Summary of the G2 forms

- Comparison with last year's paper:

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
0	1	10	10	2

- Suitability of question paper:

	Too easy	Appropriate	Too Difficult
Level of difficulty	0	28	4
	Poor	Satisfactory	Good
Syllabus coverage	1	17	14
Clarity of wording	0	13	17
Presentation of paper	0	9	21

Based on the general comments of the teachers it was felt that this was a fair paper with an appropriate degree of difficulty. Questions were judged to be appropriate and well presented. Limited concern was raised about the difficulty of questions 18 and 20 yet at the same time some teachers stated that these questions were excellent problems for the better candidates.

The results of the candidates did not seem to indicate that the examination was any more difficult than in the past. There was little evidence that the candidates did not have enough time to attempt all the questions. Certainly Question 20 proved to be the most difficult with very few correct solutions. However, there were many who attempted it. This question, as well as Questions 5, 15 and 18 presented the most difficulty for the candidates.

General comments about the strengths and weaknesses of the candidates

It was noted that many candidates were able to attempt almost all questions, giving evidence of their relatively sound knowledge of most topics in the programme, but also of their good time management skills during the examination. Apart from the topics listed above, the overall standard of scripts was satisfactory. Most candidates used the calculators effectively with the exception that some failed to use radians when necessary. Examiners expressed some concern about the shortcoming of many candidates in the areas of statistics, related rates, transformations and three-dimensional trigonometry.

Question 1

Answer: $a = -5,\ b = 6$

Where candidates used the factor and remainder theorems, these were well applied. The many candidates who used division algorithms were not so successful.

Question 2

Answer: $y = x^2 + \cos x + 1$

This was a very well attempted question. The common mistakes were either to substitute without integration or to fail to introduce a constant of integration.

Question 3

Answer: $x = 0.524,\ y = 0.238$
$x = 1.69,\ y = -0.346$
$x = 4.63,\ y = -1.81$

The most common error was to have a calculator in degree mode instead of radians. Also a large number of candidates lost the accuracy mark on this question.

Question 4

Answer: $r = -\frac{1}{2},\ u_1 = 12$

This question was generally well attempted. However, some candidates were unable to put the information together to get a system of equations. The most common error was to take the positive square root for the value of *r*, despite the information given in the question.

Question 5

Answer: $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

Although this question was attempted by the majority of candidates, a common error was to use an anticlockwise rotation. Candidates also multiplied the matrices in the wrong order.

Question 6

Answer: $x = 27.5$

Whilst this question was answered well by many candidates there were many who gained no marks on the question. Confusion between the z-value and the probability was very common. Some candidates did draw a sketch but often the sketches did not really refer to the question.

Question 7

Answer: (a) $p = 3$

(b) Gradient $= \frac{9}{11}$ $(= 0.818)$

The initial part of this question was completed well by most candidates. The common errors were in the differentiation of the constant or in the sign of the terms.

Question 8

Answer: 23

This was one of the best-answered questions on the paper. The most common error was giving a vector answer for a scalar product.

Question 9

Answer: (a) Range is $[0.5, 1]$

(b) $f^{-1}(x) = -\sqrt{\frac{1-x}{x}}$

This question was fairly well attempted with most candidates showing that they understood how to find the range and the inverse function. Common errors were writing the range as an open interval and the sign of the inverse function.

Question 10

Answer: P is (2, 3, 1)

Many candidates did not really know how to start this question. Those who put the line in parametric form were usually able to solve the question unless they then made a computational error.

Question 11

Answer: (a) $\frac{1}{2}\arctan\left[\frac{m}{2}\right]$

(b) $m = 2\tan\left(\frac{2}{3}\right)$ $(= 1.57)$

The majority of candidates recognised the result as an arctan function but some incorrectly found the integral as a logarithmic function. In part (b) there seemed to be some confusion between the use of the tangent or the use of the arctangent functions.

Question 12

Answer: (a) Mean $= 4$
Standard deviation $= 1.55$

(b) $\mathrm{P}(X \geq 2) = 0.954$

The majority of candidates found the mean correctly but were unable to find the standard deviation. Only a small number incorrectly gave the variance. In part (b) many candidates did not recognise that it was a binomial distribution and that subtracting from 1 was the best option. Those who attempted to evaluate the probability for the exact values of X often failed to use the binomial coefficient.

Question 13

Answer: (a) $\frac{2}{3}$

(b) $\frac{1}{6}$

A large number of candidates answered part (a) correctly but a smaller number were able to produce meaningful work for the conditional probability. The weaker candidates did not seem to have a systematic approach to part (a). Clear probability tree diagrams were a rare sight.

Question 14

Answer: $x^2\mathrm{e}^x - 2x\mathrm{e}^x + 2\mathrm{e}^x + C$

The majority of candidates were completely successful with this question. Minor algebraic errors produced the main loss of marks. Very few candidates tried the integration by parts the wrong way round.

Question 15

Answer: (a) $\text{Median} = 1.04$

(b) $\text{Mean} = 1.05$

Some candidates were able to find the median from the cumulative frequency curve but some confused it with the mean. However, the majority were unable to use the cumulative frequency curve to find the mean. Those who attempted to find the mean often used the incorrect mid-values, some even used the cumulative frequencies.

Question 16

Answer: $]-\infty, 6] \cup [24, \infty[$

Solutions to this question were usually disappointing. Candidates who used their GDCs often missed the critical value of $x = 24$; those who used algebra usually obtained the critical values 6 and 24 but often failed to progress beyond that stage. Far too many candidates tried to multiply the inequality by a variable without paying attention to the domain. Candidates who gave the answer as $x \le 6, x \ge 24$ or $x \le 6 \text{ and } x \ge 24$ were penalised; only $x \le 6 \cup x \ge 24$ (or equivalent) was accepted for full credit.

Question 17

Answer: 0.607

Very few candidates obtained the full derivative correctly. With the GDC, solving the equation involving the derivative presented little problem.

Question 18

Answer: 5.16 cm^2 per sec

Candidates in general found this question difficult. They had difficulty writing the equation of the area. Then many candidates were unable to apply the chain rule and the product rule without error. Finally there were many who failed to give the appropriate units in the answer.

Question 19

Answer: (a) $f'(x) = 3x^2 \cos x - x^3 \sin x$

(b) $x = 1.19$

(c) $x = 0.823$

Candidates were very successful in answering this question. A few had a little difficulty finding the *x*-coordinate of the point of inflection. Some simply stated the answer as $x = 0$ without trying to find the inflexion at $x = 0.823$.

Question 20

Answer: 13.6

Very few candidates were completely successful with this question. The majority made no valid attempt. A common error was to assume that triangle ABC was a right-angled triangle and/or isosceles. Also a failure to define the required length in two ways meant that the candidates could not set up an appropriate equation using the cosine rule.

Paper 2

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 16	17 – 32	33 – 47	48 – 57	58 – 67	68 – 77	78 – 100

Summary of the G2 forms

- Comparison with last year's paper:

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
0	5	26	8	0

- Suitability of question paper:

	Too easy	Appropriate	Too Difficult
Level of difficulty	0	37	3
	Poor	Satisfactory	Good
Syllabus coverage	1	19	20
Clarity of wording	1	20	19
Presentation of paper	0	19	21

Areas of Difficulty

Questions involving complex numbers and probability distributions continue to present difficulties for candidates with some showing little understanding of these subjects.

Levels of Knowledge, Understanding and Skill

The general standard of scripts was satisfactory with a better response to the Section B questions than in the past. However, more candidates than usual seem to have suffered accuracy penalties this time.

Question 1

Answer: (a) $3x - 4y + z = 6$

(b) (ii) $\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ 11 \end{pmatrix} + \text{l} \begin{pmatrix} 1 \\ 4 \\ 13 \end{pmatrix}$

(c) $\text{q} = 0.938$ radians (or $\text{q} = 53.7^{\circ}$)

Most candidates solved parts (a) and (b)(i) correctly. Candidates who used vector methods to solve part (b)(ii) were generally successful; those who tried to manipulate the Cartesian equations of the planes often made algebraic errors and lost marks for giving the wrong form of equation for the line. In part (c), some candidates gave an obtuse, instead of acute, angle.

Question 2

Answer: (i) (a) $\frac{2}{3}$

(b) $\frac{2}{9}$

(c) $\frac{3}{4}$

(ii) (b) (i) $\mathrm{E}(X) = \frac{3}{2}$

(ii) $m = \sqrt[3]{4}$ (=1.59 to 3 s.f.)

In part (i), most candidates solved parts (a) and (b) correctly. However, some candidates failed to realise that part (c) required the summation of an infinite geometric series. In part (ii), many candidates showed successfully that $k = \frac{3}{8}$ but there was often confusion between the mean and the median.

Question 3

Answer: (i) (b) (iii) $\arg z_2 = \frac{5}{6}\pi$

(c) $k = 4$

(ii) $\ln \frac{x^{70}}{y^{595}}$

Many candidates were unable to manipulate the modulus signs in part (i)(a) successfully. The same errors were often seen, including $z = -(z - 3\mathrm{i})$ etc and $z^2 = -(z - 3\mathrm{i})^2$ etc.

In part (b), most candidates failed to spot that the simplest way to evaluate $\arg(z_1)$ was using arcsin. Some candidates using arctan evaluated the real part of z_1 as 2.598 (or equivalent) and therefore $\arg(z_1)$ as $\tan^{-1}(0.577)$ which was then stated to be $\frac{\pi}{6}$. This inexact analysis was not accepted. Few correct solutions were seen to part (i)(c) with most candidates unable even to start. Many candidates were successful in part (ii) although some thought that the given series was geometric.

Question 4

Answer: (b) (i) $A = 78$

$k = \frac{1}{15}\ln\frac{48}{78}$ $(= -0.0324)$

(ii) $t = \frac{1}{0.0324}\ln\frac{18}{78}$ $(= 45.3)$

This question was successfully solved by many candidates. The most common error was an inability to change the additive constant of integration into a multiplicative constant after converting to exponentials. It was pleasing to note that candidates who were unable to solve part (a) often solved part (b) successfully.

Question 5

Answer: (b) (i) $T_1(x) = x$

Most candidates solved part (a) correctly and many were successful in part (b). In part (b), those who 'proved' that $\cos(2\arccos x) = 2x^2 - 1$ by plotting the graphs of both functions and showing that they were coincident gained no credit. In part (c), few candidates realised that in order to demonstrate the truth of the proposition for $n = k + 1$, it was necessary to assume true for both $n = k$ and $k - 1$ and also to show it true for $n = 1$ and $n = 2$.

Question 6

Answer: (i) (a) 33.18 (accept 33.2)
3.22 (3 s.f)

(b) $[32.1,\ 34,2]$

(ii) (a) H_0 : There is no association between classification in exams and gender

(b)

	Distinction	Pass	Fail
Male	31.6	68.5	12.9
Female	22.4	48.5	9.12

(c) 4.03 (accept 4.04)

(d) Degrees of freedom = 2
Accept H_0

(iii) (a) $\text{H}_0 : \mu_A = \mu_B$

(b) 5.12

(c) (i) significant at 5 % level

(ii) not significant at 1 % level

(iv) (a) $p = \lambda e^{-\lambda} + \frac{\lambda^2}{2} e^{-\lambda}$

(c) $\lambda = \sqrt{2}$

Most candidates solved part (i)(a) correctly although some used *n* instead of *n* – 1 for the variance and some gave a standard deviation. Part (ii) was generally well done although the null hypothesis was not always clearly stated. In part (iii), a fairly common error was to use an incorrect formula to calculate the pooled variance estimate. In part (iv), most candidates wrote down the correct expression for *p*. Since the exact value of λ which maximises *p* was asked for, those candidates who used their GDC to do this were penalised even if they deduced that the value given was equal to $\sqrt{2}$.

Question 7

Answer: (ii) (b) 0, 4, 8
1, 5, 9
2, 6, 10
3, 7, 11

(c) 3

(iii) (a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

(b) (i) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ has order 2

so $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ has order 3

so $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ has order 4

(ii) $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\}$

In part (i), most candidates solved part (a) correctly and many reasonable attempts at part (b) were seen. In part (ii), most candidates were able to show that *R* was reflexive and symmetric but the transitivity caused problems for many. Identifying the equivalence classes proved a problem for many candidates although most candidates solved part (c) correctly. In part (iii), most candidates wrote down the other four members of *S* correctly but many were unable to find the orders of those elements with negative determinants. Most candidates were unable to solve part (iv) correctly. Those candidates who simply stated that there are, effectively, only two groups of order 4 and then showed commutativity by considering their combination tables gained only limited credit.

Question 8

Answer: (i) $x_n = 5(-4)^n + 2(7)^n$

(ii) (c) (i) The graph is bipartite

(ii) The graphs are isomorphic

(iii) The graphs are not isomorphic

(iii) (b) Total weight $= 139$

Few chose this option. Those who did performed satisfactorily on most parts.
(ii) (a) and (b) Many candidates showed that they understood the questions but had difficulties in putting their answers into words.
(iv) (a) and (b) Very few candidates could state correctly the well ordering principle and none could use it to prove the statement in part (b).

Question 9

Answer: (i) $f(x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \ldots$

(ii) (a) 0.269435

(b) 0.00595

(iii) (a) 3 real roots

(b) $a = 1.8954942670$

(c) $x_1 = 1.900995594$
$x_2 = 1.895511645$

(d) $N = 2$
order of convergence is 2

Most candidates knew what had to be done in part (i) but many algebraic errors were made in finding the successive derivatives of *f*. Those candidates who tried to combine the known series for $\ln(1+x)$ and $\sin x$ were generally unsuccessful. In part (ii), most candidates used 3 intervals and therefore 4 ordinates instead of 3 ordinates as required in the question. Most candidates knew that the error bound was linked in some way to a second derivative but few candidates were able to find the error bound correctly. In part (iii), candidates should be aware that the value of a root can usually be found more accurately using Solver than in graphical mode. In part (iii)(c), some candidates used the numerical derivative facility on their GDC (nderive on the TI 83) as part of the Newton-Raphson procedure. Candidates should be aware that this gives only an approximate value and on this occasion gave an incorrect value for x_1. Most candidates knew what to do in part (iii)(d) but many made algebraic errors in trying to solve the equations.

Question 10

Answer: (i) (a) (i) $(y-b)^2 = (x+a)^2 - (x-a)^2 = 4ax$

(c) $S\left(0, \frac{a\sqrt{6}}{2} + b\right)$

(ii) $r = \frac{\sqrt{10}a}{2}$

This option seems to be chosen at random by candidates whose classmates have chosen another option, giving the impression that most candidates who chose this option do so on their own, without having been prepared for it in class, under the misaprehension that it is an easy option. The result is therefore not surprising : most candidates performed poorly, with some remarkable exceptions).
(i) (a) (i) and (ii) This is what most candidates attempted with some success.
(i) (b) and (c) This proved too difficult for most.
(ii) very few candidates attempted this part (and generally without success).

Guidance for Future Candidates

Complex numbers and probability distributions continue to cause problems for many candidates and schools should perhaps devote more time to these topics. Many candidates have incurred accuracy penalties this time and all candidates need to be aware of the accuracy rules.

