MATHEMATICS

This subject report is written by the principal examiners. Each of the authors provides general comments on performance, taking into account the comments of the assistant examiners and the team leaders. This report is the only means of communication between the senior examiners and the classroom teachers and therefore should be read by all teachers of mathematics HL.

The grade award team studied the responses in the G2 forms, the assistant examiners' reports and the grade descriptors (a description of the criteria to be satisfied for each of the individual grade levels) before determining the grade boundaries.

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 33	34 - 43	44 - 55	56 - 67	68 - 79	80 - 100

General remarks

With minor variations the remarks made in the report for the May 2002 session apply equally well for this session.

- Candidates should be encouraged to look at their answers with some critical common sense. Mathematics and common sense should go together. For instance, when the proposed answer is a negative probability, or the sine of an angle is given as 3, candidates should immediately realize that something is wrong. Quite often the error is easy to correct and candidates lose many marks that they could have easily obtained had they spent a minute looking critically at their answers.
- Candidates should be more careful with their graphs which were often drawn without scales (and therefore meaningless).
- Many candidates do not seem to know how to deal with accuracy problems: for instance, if an answer is required to 3 decimal places this usually implies working out the problem with 4 or 5 decimals, since errors may accumulate during the computation. Too many candidates err on the side of inaccuracy (better to give more decimals than required than fewer). Generally speaking the candidate should only consider the degree of accuracy required when giving the final answer and use maximum accuracy throughout the computation (when the computation has more than one step). The use of calculators makes this painless.
- Angles should in general be measured in radians. This is essential when the trigonometric functions are differentiated or integrated (otherwise the standard formulae do not hold!). Candidates should always be encouraged to use radians. If this had been done, many candidates would have avoided costly mistakes in the examination.
- Candidates should be strongly discouraged from writing their examinations in pencil which makes for very careless, messy and sometimes unreadable scripts.
- Clearly some schools do not prepare for any option. Schools ought to realize that this is a definite disservice to their candidates who thus waste 30% of their marks on paper 2.
- In this session, on the whole, candidates used their calculators well. The only relatively common error was overuse of calculators: when exact answers or proofs are required, candidates should realize that they cannot succeed by using their calculators.
- Some candidates lost a mark for failing to indicate the brand and model of the calculator they were using.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 4	5 - 6	7 - 8	9 - 11	12 - 13	14 - 16	17 - 20

In general, candidates did well in their tasks, the majority of which were taken from the Teacher Support Materials (TSM) for mathematics HL. There is also increasing evidence of the use of technology to enhance the work.

Shortcomings, however, were still evident under Criterion A in the misuse of calculator notation, and under Criterion B in the lack of presentation skills in technical writing.

In several instances, the problems were due to the poor selection of topics or assignments by the teacher. Particularly regrettable was the misuse of tasks taken from the mathematical methods SL TSM as each occurrence was penalised by moderators. Such tasks do not contain the rigour expected of mathematics HL assignments and are inappropriate for inclusion in portfolios. The proliferation of such tasks will likely be penalised more heavily in subsequent sessions.

A predictable set of tasks was chosen by teachers from the TSM. Of the tasks not taken from the TSM, those "teacher-designed" type II assignments consisting merely of unrelated questions from past papers or revision questions taken directly from the end of textbook chapters were the most inappropriate.

Some teacher-designed type III tasks missed the mark, due in large measure to a lack of understanding of the concept of mathematical modelling. A type III task should require candidates to start with given data and find a mathematical model (equation) that best describes the data. When a task starts with a given model to have candidates confirm its appropriateness with data, the assignment is not considered a type III task. Deductions of 2 marks were made from the total for non-compliance.

Type I and type III tasks have in common the search for mathematical patterns. They differ in that a type I task deals with patterns that can be precisely determined through conjecture and proof, whereas a type III task deals with patterns that may be approximated by curve fitting.

Paper 1

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 20	21 - 40	41 - 54	55 - 68	69 - 82	83 - 96	97 - 120

Results from G2 forms

- **Comparison with last year's paper:**

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
2	21	88	14	1

- **Suitability of question paper:**

	Too easy	Appropriate	Too Difficult
Level of difficulty	4	149	3
	Poor	**Satisfactory**	**Good**
Syllabus coverage	2	47	75
Clarity of wording	3	63	91
Presentation of paper	1	51	100

Areas of difficulty

Candidates were, in general, least comfortable in the areas of vectors, 3-D geometry, probability and statistics.

Levels of knowledge, understanding and skill

Apart from the topics listed above, the overall standard of scripts was satisfactory. Most candidates used their calculators effectively. The most common error was carrying out the integration in Question 16 in degree mode; candidates would be well advised to check that they are in radian mode whenever they carry out a process involving calculus.

Performance on individual questions

Question 1 Geometric series

Answer: (a) $r = \frac{2}{3}$

(b) $a = 9$

This was well done by many candidates. Some candidates, however, found it difficult to solve the simultaneous equations for *a* and *r,* especially those who wrote $15 = \frac{a\left(1-r^2\right)}{(1-r)}$ instead of $15 = a(1+r)$. Candidates who tried to find an equation in *a* by eliminating *r* were generally less successful than those who tried to find an equation in *r* by eliminating *a.*

Question 2 Trigonometry

Answer: $\theta = 0.615, 2.53$ (accept 0.196π, 0.804π)

Not all candidates realised that, since roots in the interval [0, π] were asked for, the answers should be given in radians and not degrees. Some candidates gave only the root in the first quadrant.

Question 3 Vectors

Answer: 41

Many candidates solved this question correctly although some bad mistakes were seen, eg

$(\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}) \times (-3\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}) = -3\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}$ and $(6\boldsymbol{i} + \boldsymbol{j} + 8\boldsymbol{k}) \cdot (2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}) = 12\boldsymbol{i} - 3\boldsymbol{j} + 32\boldsymbol{k}$.

Question 4 Remainder and factor theorems

Answer: $a = -2, b = 6$

This question was well done by many candidates. Those who used long division instead of the factor and remainder theorems were usually unsuccessful.

Question 5 Singularity of matrices

Answer: $\lambda = 1$ or 6

Solutions were often disappointing with some candidates using trial and error and sometimes spotting $\lambda = 1$ but not $\lambda = 6$.

Question 6 Probability

Answer: (a) $\mathrm{E}(X) = 2.5$
(b) $\mathrm{P}(X \le 2) = 0.526$

Most candidates solved part (a) correctly. In part (b), candidates who used their calculators were the most successful; those summing individual binomial probabilities often omitted one of them.

Question 7 Functions: maximum values and roots

Answer: (a) $f_{\max} = 1.17$
(b) Roots are $-1.32, 0.537$

This question was well done by most candidates using their calculators. A fairly common error was to give the *x*-coordinate of the maximum point instead of the *y*-coordinate. Some candidates tried to solve the problem analytically; it should be realised at this level that such an approach cannot be used with these particular functions.

Question 8 The sine rule

Answer: $\hat{\mathrm{B}} = 93.6^\circ$ or 26.4°

Many candidates gave only the acute value of $\hat{\mathrm{B}}$. Those candidates who started the question by finding the value of *b* using the cosine rule usually obtained only one correct value of $\hat{\mathrm{B}}$. Some candidates rounded their obtuse value of $\hat{\mathrm{C}}$ to 124°, resulting in a value of 26° for $\hat{\mathrm{B}}$ which incurred an accuracy penalty. Some candidates thought, incorrectly, that the triangle was right-angled.

Question 9 Probability

Answer: (a) $\mathrm{P}(B) = 0.8$
(b) 0.56

A fairly common error was to confuse 'independent' with 'mutually exclusive' and therefore to use $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B})$ to find $\mathrm{P}(\mathrm{A})$. We were, however, able to follow through in part (b) where the two methods using $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B})$ and $\mathrm{P}(\mathrm{A} \cap \mathrm{B}') - \mathrm{P}(\mathrm{A}' \cap \mathrm{B})$ were seen in roughly equal numbers.

Question 10 Functions and equations of normals

Answer: Equation of normal is $y-1=\frac{4}{3}(x-2)$

Solutions were often disappointing with algebraic and arithmetic errors made using implicit differentiation; some candidates incorrectly left the '8' on the right-hand side of the equation. Candidates starting from $y=\sqrt{8}x^{\frac{-3}{2}}$ were often the most successful.

Question 11 Complex numbers

Answer: $z=-5-12\text{i}$

The most successful candidates were those who solved the problem completely using the complex mode in their calculators. Of those who used algebraic methods, the most successful were those who simplified the $\frac{2}{(1-\text{i})}$ term first. Candidates who attempted to square the whole expression at the outset usually went down in a mass of algebra with the cross product terms often omitted.

Question 12 Exponents and logarithms

Answer: $\frac{\ln 9}{\ln 8}$

Many candidates solved this problem successfully. The most elegant solution, seen several times, started by expressing both sides in terms of powers of 2 and 3. The most common error was to assume that the log of a product is the product of the logs.

Question 13 Satisfying inequalities

Answer: $x\in\left[-3,\frac{1}{3}\right]$

Candidates graphing $|x-2|$ and $|2x+1|$ were the most successful. Those using algebraic methods often made algebraic errors and sometimes only found one of the critical values.

Question 14 Normal distribution

Answer: $\text{E}(X)=9.19$

Solutions were often non-existent with many candidates not knowing where to start.

Question 15 Perpendicular lines and planes

Answer: Foot of perpendicular is (5, 3, 7)

Solutions to this question were very disappointing. This is a topic that has been well answered in paper 2 questions in recent years but relatively few candidates seemed to know what to do here.

Question 16 Kinematics

Answer: Distance travelled $= 0.852$

Many candidates failed to realise that the velocity changed sign half-way through so that the answer 0.387, obtained by simply integrating the velocity between 0 and 2π, was often seen. Some candidates used their calculators in degree mode with disastrous consequences.

Question 17 Transformations of graphs

Answer: $f^{-1}(x) = -\sqrt{\dfrac{1+x}{1-x}}$

This question was reasonably well answered although it was fairly common to see the negative sign omitted in the expression for $f^{-1}(x)$. There are still some candidates who think that f^{-1} means the derivative and others who think that $f^{-1}(x)$ means $\left[f(x)\right]^{-1}$.

Question 18 Integration by substitution

Answer: $\dfrac{4}{2-x} + 4\ln\left|2-x\right| - (2-x) + c$

Solutions to this question were often disappointing with many candidates seemingly unaware of the mechanics of changing variables, although candidates who tried to solve the problem 'otherwise' were even less successful. Candidates should be aware that an 'otherwise' method of solution is usually more difficult than the method recommended in the question.

Question 19 Unbiased estimates

Answer: (a) $\bar{x} = 31.3$
(b) Unbiased esimate = 9.84

Most candidates solved part (a) correctly although some divided by 19. In part (b), only a few candidates appeared to know that an unbiased estimate of the variance is obtained by dividing by $n-1$, 19 here, with most candidates dividing by 20. A fairly common, correct method was to evaluate $\dfrac{\sum x^2}{n - \bar{x}^2}$ and then multiply the answer by $\dfrac{n}{n-1}$. A quicker, and more direct method, is to evaluate $\dfrac{\sum x^2}{n-1} - \dfrac{\left(\sum x\right)^2}{n\left(n-1\right)}$.

Question 20 Graphing functions

It was pleasing to see many reasonably good solutions to this question.

Paper 2

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 34	35 - 44	45 - 55	56 - 67	68 - 78	79 - 100

Results from G2 forms

- **Comparison with last year's paper:**

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
1	29	63	12	0

- **Suitability of question paper:**

	Too easy	Appropriate	Too Difficult
Level of difficulty	3	129	10
	Poor	**Satisfactory**	**Good**
Syllabus coverage	12	73	54
Clarity of wording	1	57	84
Presentation of paper	1	44	96

Section A

Question 1 Differentiation, maximum points, points of inflection

Answer: (a) (i) $f'(x)=\dfrac{2x-x^2\ln 2}{2^{2x}}$

(ii) $f''(x)=\dfrac{x^2(\ln 2)^2-4x\ln 2+2}{2^x}$

(b) (i) $x=\dfrac{2}{\ln 2}$

(c) $x=\dfrac{2\pm\sqrt{2}}{\ln 2}(=0.845, 4.93)$

Most candidates - perhaps because the answer was given - answered part (a) (i) satisfactorily.

In part (ii) a distressingly large number of candidates thought that the derivative of ln2 is $\dfrac{1}{2}$ when calculating the second derivative.

Far too many candidates ignored the fact (indicated in bold!) that an exact answer was required for part (b) (i) (and therefore could not be produced by a calculator).

In part (ii) again a calculator or a sketch cannot suffice to "show" something (even if they might do the job for "finding" something). The overuse of calculators in this question had the consequence that relatively few candidates thought of evaluating the second derivative and often went into rather contorted and insufficient arguments (occasionally correct) about the values of the first derivative on the right and on the left of the stationary point or indeed even of the function, most of the time with a loss of one or two marks.

For part (c), a substantial minority of the candidates chose - correctly - to look for the maximum and minimum of the first derivative while others found the roots of the second. This was all that was required since the question stated that there were two points of inflexion and therefore there was no need to check that the second derivative actually changed sign. Here calculators were used to good effect.

Question 2 Transformation of matrices, solution of linear equations

Answer: (i) (a) $\boldsymbol{T}_1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}; \boldsymbol{T}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \boldsymbol{T}_3 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(b) (i) $\boldsymbol{T} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(ii) This is a reflection in the x-axis.

(ii) (a) (ii) $k = 1$.

(b) The general solution is $z = \lambda,\ y = \dfrac{(2\lambda - 4)}{3},\ x = \dfrac{(11 - 7\lambda)}{3}$.

In part (i) about half the candidates multiplied the matrices in the wrong order. Also too many lost marks because they wrote the matrices in terms of cosine and sine, without giving their numerical values.

In part (ii) many candidates seemed to be unable to adopt a precise strategy and went around in circles hoping (mostly in vain since in the process many algebraic mistakes were made) that eventually something would come out that made sense. Many candidates missed the logical point that "not a unique solution" meant "either no solution OR infinitely many solutions". Also, surprisingly, some candidates after stating that the system had an infinity of solutions found only one in part (b).

Question 3 Complex numbers, proof by induction, binomial expansion

Answer: (c) (i) $(z + z^{-1})^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$

(ii) $a = 1, b = 5,$ and $c = 10$

Mathematical induction is still largely misunderstood: quite a few candidates stated "assume *n*=*k*" (!) instead of "assume the statement is true for *n*=*k*" in part (a). Very few bothered to make a concluding statement, yet proof by induction requires a high level of formal correctness. Some candidates worked backwards "assuming the statement is correct for *n* + 1 let us show that it is true for *n*". Finally a distressingly large number of candidates did not even attempt to answer this question or made a very limited attempt.

Some candidates thought that part (b) (i) followed on from part (a) thus confirming that they did not understand proof by induction (-1 is not larger than or equal to 1!). There was a lot of fudging here and in part (ii) as is often the case when the answer is given.

In part (c) most candidates who went beyond the binomial theorem part got it right.

Question 4 Probability density functions, median, mode

Answer: (a) (i) $\frac{1}{12}\int_0^2 x(8x - x^3)\,\mathrm{dx} = \mathrm{E}(X)$

(ii) $\mathrm{E}(X) = 1.24$

(b) (ii) $m = 1.29$

(c) $x = 1.63$

In part (a) more than half the candidates neglected to multiply the density function by x when computing $\mathrm{E}(X)$.

In part (b) about half the candidates were able to find the equation for the median. Most of them (as well as those who simply used the given equation) found the median. The most common error here was giving four answers without explaining which one was correct and why.

Part (c) was worked out by about half the candidates but many thought that the mode was the value of $f(x)$ rather than x.

This was an easy straightforward question and the relatively disappointing results are due to the fact that probability is a part of the curriculum which is apparently often neglected.

Question 5 Functions, integration

Answer: (a) $R = 2, \alpha = \frac{\pi}{3}$

(b) (i) Range is $[1, 2]$

(ii) Inverse does not exist because f is not 1:1.

(c) $x = \frac{\pi}{12}$

In this question many candidates showed signs of confusion.

In part (a) some candidates lost marks because they used degrees instead of radians (as a consequence the function was almost constant on the interval [0, 3.14]) and this was encouraged by the fact that some were using their calculators in the degrees mode. Since the answer required was exact, they should not have been using a calculator in the first place.

In part (b) (i), many candidates did not take into account the domain and therefore found the range to be [-2, 2]. In part (ii) most answers seem to have been uneducated guesses with many nonsensical reasons given and again the problem was often that the domain was not taken into accounts.

In part (c), once more an exact answer was required, nevertheless candidates often used their calculators (losing one mark).

In part (d) there was a lot of fudging (the answer was given). Some candidates even "found" the answer working in degrees! But other candidates managed well.

Section B

Question 6 Statistics

Answer:	(i)	(a)	(i)	$\bar{v} = 87.13$
			(ii)	$s^2 = 215.58$
		(b)	(i)	$[86.22, 88.04]$
			(ii)	$[86.37, 87.89]$
	(ii)	(a)	(i)	0.245
			(ii)	0.214
			(iii)	0.0524
		(b)	0.464 (or 0.463)	
	(iii)	(b)	The probability of rejecting H_0 when it is true is 0.05	

Parts (i) (a) and (b) were done correctly by a large number of candidates. The usual error about the "unbiased" estimate of the mean was often present (dividing by 999 instead of by 1000). Most candidates rightly used their calculators. Some used them incorrectly and failed to react when getting unbelievable answers (like extremely large confidence intervals, containing all possible reasonable speeds and more).

Part (i) (c) was very poorly done, which shows that statistics is too often taught as recipes without a proper attempt at interpretation.

Part (ii) (a) was done successfully by most candidates while part (b) was done successfully by almost none.

In part (iii) some candidates unaccountably tried to use the binomial law but most candidates successfully tackled part (a) while very few gave even remotely reasonable answers to part (b) (see remark on part (i) (c) above).

Question 7 Sets, relations and groups

Answer: (i) (b) $f^{-1}(z) = \log_3(z)$

In part (i) (a) many candidates failed to be concerned about the injectivity and surjectivity of the mapping. Just about everyone got part (b) right.

Many candidates dealt incompletely with part (ii) (a), ignoring the determinant condition in all their work. In part (a) many candidates stated that the group was abelian because it was commutative! Others made a logical mistake of failing to realize that a non abelian group may have an abelian subgroup and that therefore the fact that the product of matrices, in general, is not commutative did not imply that *G* was not commutative.

In part (c) many candidates made a serious attempt to deal with the question but often floundered when dealing with transitivity (which some candidates confused with associativity!). Finally part (d) was also often incomplete, with right ideas getting bogged down in confusion.

Relatively few candidates successfully tackled part (e).

Question 8 Discrete mathematics

Answer: (i) (a) $x = 11$ and $y = -6$

(ii) $x_n = 2\times 2^n = 2^{n+1}$
$y_n = 2^{n+1} - 3$

(iii) (c) $\mathrm{A} \to \mathrm{E} \to \mathrm{B} \to \mathrm{A} \to \mathrm{C} \to \mathrm{D} \to \mathrm{B} \to \mathrm{D} \to \mathrm{C} \to \mathrm{E} \to \mathrm{D} \to \mathrm{A}$

Candidates who chose this option generally performed well except for parts (i) and (ii).

In part (i) most candidates failed to use Euclid's algorithm fully, resorting instead to a trial and error approach and almost no one worked out part (b).

In part (ii) again many candidates used a trial and error approach while others tried - with no success - to use the formula for second order difference equations.

Question 9 Analysis and approximation

Answer: (i) (b) $x = -2.78913$ (5 d.p.)
$x = -0.60135$ (5 d.p.)

(c) (i) $x_5 = -2.78913$ (5 d.p.)

Most candidates, not surprisingly, did part (i) successfully. They also performed relatively well in part (i) (b).

In parts (c) and (d) it was obvious, however, that many were unfamiliar with the fixed point method and therefore, of course, could not give a reason why it did or did not work. Very few candidates received even partial credit in part (ii).

Question 10 Euclidean geometry and conic sections

Answer: (i) (a) The conic is an ellipse

(b) $y = \frac{x}{2} + 4$ and $y = \frac{x}{2} - 4$

As often in the past, this option seems to be chosen at random by candidates whose classmates have chosen another option, giving the impression that most candidates who chose this option do so on their own, without having been prepared for it in class, under the misapprehension that it is an easy option. The result is therefore not surprising: most candidates performed miserably (with, however, some remarkable exceptions).

Those candidates that did more than write the heading of the question usually managed to get part or all of part (i) correct. Only a handful of candidates went beyond that.

