MATHEMATICS HIGHER LEVEL

Introduction

This subject report is written by the principal examiners. Each of the authors provides general comments on performance, taking into account the comments of the assistant examiners and the team leaders. This report is the only means of communication between the senior examiners and the classroom teachers and therefore should be read by all teachers of mathematics HL.

The grade award team studied the responses in the G2 forms, the assistant examiners' reports and the grade descriptors (a description of the criteria to be satisfied for each of the individual grade levels) before determining the grade boundaries.

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-16	17-30	31-42	43-54	55-64	65-76	77-100

Internal assessment – the portfolio

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-4	5-6	7-9	10-12	13-14	15-17	18-20

The May 2002 examination session is the third year for portfolio work. The experience of the previous years has resulted in fewer problems, but unfortunately there are still many issues such as:

1. assigning portfolio items correctly
2. applying the assessment criteria correctly
3. correctly completing the necessary forms to go with sample work
4. ensuring that the student's work is authentic

It should also be noted that while these problems do exist there are many schools that have embraced portfolio work and are producing excellent items.

1. Some Mathematics HL candidates are being penalized in schools that would appear to have combined Mathematical Methods and Mathematics HL classes. It is a mistake for teachers to allow students who will be taking the Mathematics HL examination to have included in their portfolios, items that do not meet the requirements of the HL program. The most common example of this was the use of items from the Mathematical Methods SL Teacher Support Material (TSM) documents. Items not to the level of the Mathematics HL program are being moderated down.

   Teachers should not take items from the TSM and assess them in criteria that were not included in the original item. For example, it was not intended that the portfolio item on population growth would be assessed against criteria E (conjectures). It is also not possible to take a TSM item and reclassify it as a different type.

In addition it should be noted once again that all portfolio items should:

- be relevant to the syllabus
- be of appropriate length and difficulty
- allow students to address the criteria as set out in the subject guide

In some schools there is a reluctance for teachers to assign portfolio items from outside the examples provided in the TSM issued by the IBO. Though it is easy to understand teachers sticking to a formula that works it was never the intention that these same TSM items would be repeated year after year.

2. It is still a problem that some teachers are not applying the assessment criteria correctly. To find the correct level, one should start with the lowest descriptor and work upwards until you reach a level which has clearly not been achieved.

   It is still a problem that teachers are completing their assessment without making any marks or comments on the student work. Work should be marked for correct answers; conjectures, proofs etc. tick or cross numerical and verbal answers as appropriate. Comments should be added as feedback to students and moderators about the way in which the item has been answered. Indicate where any omission has occurred. It is the aim of the moderation process, where possible, to support and confirm the mark awarded by the teacher, but this becomes increasingly difficult when no supporting comments are provided by the teacher.

3. Errors are still being made in the calculation of the final mark and in the completion of the necessary forms 5/IA and 5/PFCS which go with samples for moderation. Check the current *Vade Mecum* for the current version of the form and follow instructions for completing the forms correctly.

4. Teachers need to be concerned about the authenticity of the work submitted by their students. There is plenty of evidence in samples submitted for moderation that the work is not being checked carefully enough for authenticity.

Finally and perhaps most importantly teachers should remember to share with their students what is required in their portfolio work and the criteria on which it will be assessed. Only when students fully understand the process and appreciate that this means of assessment gives them opportunities to explore and do mathematics in many different ways, will the full benefits be achieved.

Paper 1

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-21	22-43	44-57	58-71	72-84	85-98	99-120

Summary of the G2 forms

Comparison with last year's paper:

much easier	a little easier	of a similar standard	a little more difficult	much more difficult
2	33	64	13	1

Suitability of question paper:

	too easy	appropriate	too difficult
Level of difficulty	6	140	4

	poor	satisfactory	good
Syllabus coverage	5	85	62

	poor	satisfactory	good
Clarity of wording	4	77	77

	poor	satisfactory	good
Presentation of paper	1	59	93

General comments

Many candidates appear to be unfamiliar with the rules on accuracy so that marks are being lost unnecessarily through a failure to give solutions to the appropriate degree of accuracy.

Most candidates were confident in their use of a graphical display calculator.

Many candidates seemed unfamiliar with the basic manipulation of complex numbers, suggesting perhaps that some centres are not covering this topic.

The question on basic statistics (Question 14) was poorly answered by many candidates with some candidates clearly unfamiliar with the terms 'median' and 'interquartile range'.

Candidates who use a tree diagram in probability questions are usually more successful than whose who attempt a purely algebraic solution.

Many candidates are unable to use implicit differentiation correctly.

Candidates sometimes give their answers in radians when degrees are requested and vice versa.

Performance on individual questions

QUESTION 1 Arithmetic series

Answers: (a) $\frac{n}{2}(3n+1)$

(b) $n = 30$

This was well answered by almost all the candidates. The most common errors in part (b), not seen very often, were either algebraic errors in obtaining and solving the quadratic equation or evaluating S_{1365}.

QUESTION 2 Calculus, distance, velocity, acceleration

Answers: (a) 0.435

(b) $\dfrac{-2t}{(2+t^2)^2}$

A minority of candidates failed to solve part (a) correctly. Errors made included failure to put limits in the integral, stating that $\int v\,\mathrm{d}t = \ln(2 + t^2)$ and even assuming that the velocity is constant throughout the first second. In part (b), it was not uncommon to see the differentiation done incorrectly.

QUESTION 3 Complex numbers

Answers: (a) $8\mathrm{i} = 8\left(\cos\dfrac{\pi}{2} + \mathrm{i}\sin\dfrac{\pi}{2}\right)$

(b) (i) $z = 2\left(\cos\dfrac{\pi}{6} + \mathrm{i}\sin\dfrac{\pi}{6}\right)$

(ii) $z = \sqrt{3} + \mathrm{i}$

A sizeable minority of candidates appeared to be unfamiliar with the basic manipulation of complex numbers and made little or no progress with this question.

QUESTION 4 Matrices

Answer: $k = 3, -\dfrac{1}{3}$

This was well answered by most candidates. A few made algebraic errors in setting up and solving the quadratic equation for k and some thought that a singular matrix was one whose determinant was equal to 1.

QUESTION 5 Vectors

Answer: 1.24 radians.

This was well answered by most candidates. A few candidates used the result that $\sin\theta = \dfrac{|\boldsymbol{u}\times\boldsymbol{v}|}{|\boldsymbol{u}||\boldsymbol{v}|}$ and usually obtained the correct answer. This method is not to be recommended, however, because of the ambiguity in the value of θ.

QUESTION 6 Integration

Answers: (a) $\dfrac{x^3}{3}\ln x - \dfrac{x^3}{9}$

(b) $\int_1^2 x^2 \ln x\,\mathrm{d}x = 1.07 \quad \left(\text{or } \dfrac{8}{3}\ln 2 - \dfrac{7}{9}\right)$

Most candidates were able to use integration by parts correctly in part (a) to find the integral. Most candidates solved part (b) correctly with those using substitution and those using their calculators being equally successful.

QUESTION 7 Probability

Answer: $P(R \,|\, > 25^\circ) = \frac{0.06}{0.54} = \frac{1}{9}$ (or 0.111)

Many candidates failed to solve this question, with those who drew a tree diagram being the most successful. Candidates who tried to solve the question algebraically often ended up combining the wrong probabilities.

QUESTION 8 Vector equations of lines

Answer: P has position vector $3\boldsymbol{i} + 5\boldsymbol{j} + 7\boldsymbol{k}$.

Most candidates realised that the ***r*** equations for L_1 and L_2 should be equated but some were unable to solve the resulting equations.

QUESTION 9 Probability

Answers: (a) 0.138
(b) $(0.6)^2 \times 0.4 = 0.144$

This was well answered by many candidates. In part (a), the most common error was to omit the combinatorial term.

QUESTION 10 Circular functions

Answer: $\theta = 20.9^\circ, 69.1^\circ$

Most successful candidates obtained a quadratic equation in $\tan\theta$ and then found the two roots. Some used an alternative method leading to $\sin 2\theta = 2/3$ although this sometimes led to only one value of θ.

QUESTION 11 Normal distribution

Answer: 0.586

This was well answered by most candidates. Those using the normal distribution function on their calculators appeared to be less likely to make arithmetic errors than those using the statistical tables provided.

QUESTION 12 Compound formula

Answers: (a) $f(\theta) = 5\cos(\theta - 0.644)$
(b) $\theta = 0.644$ radians

Most candidates used a correct method in part (a) but algebraic errors were not uncommon in the evaluation of R and α. Some candidates either failed to evaluate α or gave it in degrees. A surprising number of candidates used their calculators to solve part (b) instead of the easier method of deduction from their solution to part (a).

QUESTION 13 Areas under curves

Answers: (a) At A, $x = 0.753$
At B, $x = 2.45$

(b) Area $\int_{0.753}^{2.45} y\,dx = 1.78$

Almost all the candidates realised that this was a question to be solved using their calculators and most obtained the correct answers. Candidates who tried to evaluate the integral in part (c) algebraically were usually less successful than those who used their calculators.

QUESTION 14 Cumulative frequency

Answers: (a) Median $= 135$
(b) IQ Range $= 141 - 130 = 11$

Many candidates were unable to solve this question correctly with some making no attempt. Some candidates thought that the median corresponded to a cumulative frequency of 50; others even thought that the value of the median was 40. Many candidates thought that the interquartile range was [130, 141] instead of 141 – 130.

QUESTION 15 Linear functions

Answers: (a) $A = \left(-\frac{1}{2}, 2\right)$

(b) $f^{-1}(x) = \frac{1+2x}{2-x}$

A sizeable minority of the candidates obtained the wrong answer to part (a) with $\mathbb{R}$, $x \neq 2$ and [-0.5, 2] often seen. Many candidates solved part (b) correctly although a few thought that $f^{-1}(x) = 1/f(x)$ and others thought that $f^{-1}(x)$ denoted the derivative of $f(x)$.

QUESTION 16 Exponential functions

Answer: $0 \leq x \leq \ln 6$ (or 1.79)

Many candidates made a reasonable attempt at this question. The most common incorrect solutions were $0 < x < \ln 6$ and $x \leq 0$ or $x \geq \ln 6$.

QUESTION 17 Functions and tangents

Answer: Equation of tangent is $y - 1 = -1(x - 1)$ or $x + y = 2$

Not all candidates realised that implicit differentiation was the best approach here with some trying, unsuccessfully of course, to express y explicitly in terms of x. Some candidates who used implicit differentiation on the left-hand side kept the 3 on the right-hand side instead of differentiating it to zero.

QUESTION 18 Matrix transformations

Answers: (a) P is (1, 2)
(b) The only invariant point is (0, 0)

Part (a) was well solved by many candidates although some found the image of (8,5) instead of the inverse image. Some candidates treated the point as a row vector and post-multiplied by the transformation matrix instead of treating the point as a column vector and pre-multiplying by the matrix. In part (b), candidates who showed that invariant points satisfied the equations $2x + 3y = x$ and $x + 2y = y$ were often unable to solve these equations.

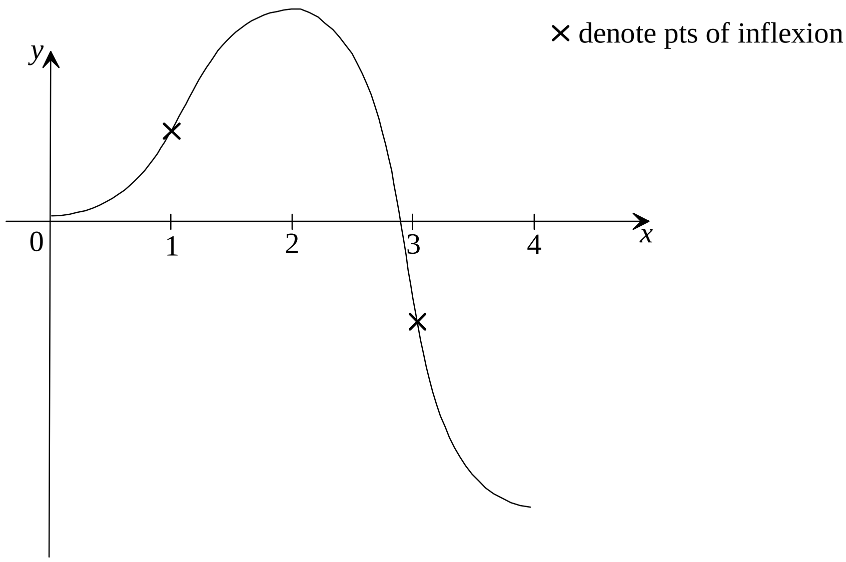
QUESTION 19 Exponential function

Answers: (a) $A = 2x \times e^{-x^2} = 2xe^{-x^2}$
(b) $A_{max} = \sqrt{2}e^{-\frac{1}{2}}$ (or 0.858)

Some candidates were unable to draw the diagram in part (a) and therefore unable to find an expression for *A*. Those candidates who obtained an expression for *A* were usually able to find its maximum value, either using calculus or their calculators.

QUESTION 20 Integrals

Answer:



The majority of candidates failed to realise that points of inflexion on the graph of y_2 correspond to stationary points on the graph of y_1. Most of the graphs of y_2 drawn were incorrect in several respects.

Paper 2

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-13	14-27	28-37	38-48	49-58	59-69	70-100

Summary of the G2 forms

Comparison with last year's paper:

much easier	a little easier	of a similar standard	a little more difficult	much more difficult
2	11	55	39	14

Suitability of question paper:

	too easy	appropriate	too difficult
Level of difficulty	1	128	41

	poor	satisfactory	good
Syllabus coverage	19	93	46

	poor	satisfactory	good
Clarity of wording	7	81	66

	poor	satisfactory	good
Presentation of paper	3	65	88

Many teachers commented on the option questions, with particular concerns being expressed about Questions 7 (Sets, Relations and Groups) and 9 (Analysis and Approximations). A statistical analysis was done, comparing marks on Section A with those on each of the options in Section B, and this seemed to confirm teacher's concerns. The senior examining team considered these comments very carefully at the grade award meeting. It was agreed to adjust the Section B marks for different options, to ensure that candidates who answered the more difficult questions were not unduly disadvantaged. This information was also taken into account when setting the grade boundaries.

The results of all schools were carefully looked at, especially those schools whose candidates had answered questions 7 and 9. The scripts of candidates who had performed poorly only on Paper 2 were then looked at again. All possible steps were taken to ensure that candidates were not disadvantaged. The senior examiners have also been asked to pay particular attention to this on future papers. However, some topics are abstract in nature, whereas others are not.

General comments

They are essentially the same as last year's:

1. Candidates should be encouraged to look and consider whether their answers make sense. Mathematics and common sense should go together. For instance, when the answer obtained is a negative probability, or the sine of an angle is found to be 3, candidates should immediately realize that something is wrong. Quite often the error is easy to correct and the candidates lose many marks that they could have easily obtained had they spent one minute looking critically at their answer.

2. Candidates on the whole used their calculators better this year but progress is still needed. I suspect that many teachers let their candidates fend for themselves when it comes to calculators with the idea that candidates are inherently better at it than their teachers. The candidates may be good at pushing buttons, but they do not always understand the underlying mathematics, and only their teachers can supply that guidance. Some time should be set aside for exercises on matrix algebra, graphing (using the appropriate windows) and locating asymptotes), solving equations (by any method) to the desired accuracy, and computing mean and standard deviation from samples. Further appropriate exercises should be worked out for the options (especially Questions 6 and 9).

3. Candidates should be more careful with their graphs, often drawn without scales (and therefore meaningless) and/or so small that they are illegible.

4. Many candidates do not seem to know how to deal with accuracy problems: for instance, if an answer is required to 3 decimal places this usually implies working out the problem with 4 or 5 decimals, since errors may accumulate during the computation. Too many candidates err on the side of inaccuracy yet it is better to use more decimals than required than fewer. Generally speaking the candidate should only look at the degree of accuracy required when giving the final answer and use maximum available accuracy available on their calculator throughout the computation when the computation has more than one step. The use of calculators makes this painless. Also quite often the problem of decimals would be eliminated altogether by using simple fractions instead (ie 1/3 instead of 0.333...)

5. Angles should in general be measured in radians. This is obligatory when the trigonometric functions are differentiated or integrated. If this had been the case, many candidates would have avoided costly mistakes in the examination.

6. Candidates should be strongly discouraged from writing their examinations in pencil. It makes for very careless, messy and sometimes unreadable scripts. They should also–as they are told to–use a new page for each question.

7. Clearly some schools do not prepare for any option. Schools ought to realize that this is a definite disservice to their candidates who thus waste 30% of their marks on this paper.

8. Correct use of notation is important. Writing "the equation of a plane is $\pi = 3x - 2y + z = 1$" is unacceptable and the writing of an integral sign without the dx (or dt or whatever) is also unacceptable and leads to confusion and errors.

Finally, candidates should be told that the treasury tags are designed to keep their different booklets together. As a help to examiners, they should not tie them too tightly as this makes it difficult for examiners to open the booklets.

Section A

QUESTION 1 Vector Geometry

Answers: (a) (i) $\vec{AB} \times \vec{AC} == -5\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

(ii) $\text{Area} = \frac{\sqrt{35}}{2}$

(b) (i) The equation of the plane Π is $-5x + 3y + z = 5$

(ii) Equations of L are $\frac{x-5}{-5} = \frac{y+2}{3} = z - 1$

(c) Point of intersection is (0, 1, 2).

(d) Perpendicular distance is $= \sqrt{35}$.

This is the question on which the candidates did best by far. Nevertheless some candidates made some mistakes. More precisely:

(a) A substantial minority of candidates did not distinguish between vector $\overrightarrow{AB}$ and vector $\overrightarrow{BA}$ (and the same for the other vectors). This did not have too serious consequences for what followed but nevertheless is a significant error. More serious consequences followed from the fact that about 25% of the candidates did not seem to know that the magnitude of the cross product of two vectors is twice the area of the triangle they determine. As a result these candidates wasted many minutes computing the area by other methods using, for instance, the cosine law and therefore almost certainly getting only an approximate answer.

(b) Many candidates lost points because they failed to give the equation of the line in a **cartesian** form as asked in the question.

(c) This was usually done correctly but many candidates made numerical mistakes.

(d) Many candidates failed to see that what was asked was in fact the distance between P and D and used other methods to find the distance of D from Π. In most cases the other methods were almost as fast but again provided more opportunities for numerical mistakes.

QUESTION 2 Calculus

Answers: (a) (iii) $c = -2$

(b) (i) $x = e^2$

(ii) $x = e^4$

(iii) $\text{Area} = 46.7$

Clearly this area of the syllabus has been neglected since many candidates had great difficulty in dealing with part (a) of this question. Many treated v as a constant and many others just skipped part (a)(i).

In part (a)(ii) as happens frequently when the answer is given, candidates were often careless and failed to distinguish between $\text{In}\left(\frac{x}{2}\right) + c$ and $(\frac{\text{In}x + c}{2})$ but nevertheless claimed the desired answer. That cost them one mark and (sometimes 2) if it led them in part (iii) to find the wrong value for c (in spite of having "found" the correct expression for f).

(b)(i) Many candidates lost a mark here because instead of writing an equation as asked they simply wrote a number, e^2.

(b)(ii) Many candidates lost a mark here for giving an approximate value for A when an exact answer was specifically required.

(b)(iii) Most candidates were successful with this question.

QUESTION 3 Matrices

Answers: (i) (a) Determinant = 0
(b) $\lambda = 5$
(c) $z = \mu,\ y = 1 + \mu,\ x = 2 - 3\mu$
(Several alternate answers are possible)
(ii) (b) The result is true.

(i)(a) Nearly all candidates found the value of the determinant, often using their calculators.
(i)(b) Most candidates found the value of λ although quite often after pages of random procedures which must have been costly in terms of time.
(i)(c) Almost half of the candidates stated that since the determinant was 0 the system had no solution and moved on, losing three marks. Others were content to give just one solution rather than the general solution. On the other hand some candidates displayed a good understanding of the situation which they described as three planes intersecting on a common line.

(ii)(a) Candidates still have a lot of trouble with proof by induction, in part because for the induction step they start by writing what they have to prove, make some random changes on the left and right hand side and end up with their original equality which they emphatically declare "proven". The practice of starting from the equation that is to be established is questionable (it may be helpful heuristically but should be used only on the scratch paper and once a proof has been found, it should be written in a correct form).
This remark is particularly valid for part (ii)(b): many candidates simply said the result was true for $n = -1$ and wrote twice the same matrix with an equal sign between them without any explanation. That cost them one mark.

QUESTION 4 Probability

Answers: (a) (i) P(Alan scores 9) $= \frac{1}{9}$ $(= 0.111)$
(ii) P(Alan scores 9 and Belle scores 9) $\left(\frac{1}{81}\right)$ $(= 0.0123)$
(b) (i) P(Same score) $= \frac{73}{648}$ $(= 0.113)$
(ii) $\mathrm{P}(A > B) = \frac{575}{1296}$ $(= 0.444)$
(c) (ii)

x	1	2	3	4	5	6
$\mathrm{P}(X = x)$	$\frac{1}{1296}$	$\frac{15}{1296}$	$\mathbf{\frac{65}{1296}}$	$\mathbf{\frac{175}{1296}}$	$\mathbf{\frac{369}{1296}}$	$\frac{671}{1296}$

(iii) $\mathrm{E}(X) = \frac{6797}{1296}$ $(= 5.24)$

In part (a)(i) most candidates got the correct answer.
In part (a)(ii) too many candidates multiplied their answer of part (a)(i) by 2 instead of squaring it.
(b)(i) Roughly one third of the candidates found the correct answer.
(b)(ii) Most of those who had worked out part (b)(i) used the symmetry between Alan and Belle but some worked out the probability very laboriously by direct computation, getting the correct answer but wasting considerable time.

(c)(i) Very few candidates gave a satisfactory answer to this question. Here again, since the answer was given, many candidates wrote seemingly random sentences after which they claimed that they had proven the formula.
(c)(ii) Many candidates who had failed in part (c)(i) used the formula to complete the table correctly, but on the other hand many candidates interpreted the $\leq$ as an = sign while others provided incorrect and unexplained answers.
(c)(iii) It was distressing to see that many candidates had no idea of what the mathematical expectation of a discrete random variable is.

QUESTION 5 Functions

Answers: (a) (i) $f'(x) = \dfrac{2(x^2-1)}{(x^2+x+1)^2}$

(ii) $A\left(1, \dfrac{1}{3}\right)$ $B(-1, 3)$

(b) (ii) $x = -1.53, -0.347, 1.88$

(c) (i) Range is $\left[\dfrac{1}{3}, 3\right]$.

(ii) Range is $\left[\dfrac{1}{3}, \dfrac{7}{13}\right]$.

(a)(i) Most candidates were able to find the simplified expression for $f'(x)$ while most of those who did not failed to do so because of algebraic mistakes.
(a)(ii) Many candidates lost a mark here because they did not read carefully the instructions and gave only the *x*-coordinates of A and B.

(b)(i) Sketches (thanks to the calculators) were usually adequate but a distressingly large number of candidates failed to write the scales and/or drew miniscule sketches, almost unreadable, instead of using the available graph paper.
(b)(ii) Most candidates realised that they had to locate the maxima and minima of $f'(x)$ and they generally used their calculators for this purpose with mixed results: quite a few found the right answers but many others gave answers which could not even obtain some follow-through marks since they were totally unexplained.

(c)(i) Many candidates did not seem to know what the range of a function is. Those who did usually managed to give at least approximate answers using their calculators.
(c)(ii) Very few candidates answered correctly (or at all) this question.

Section B

General comments

Perhaps more explanations should be given to the candidates as to what constitutes a proof (as opposed to an emphatic statement). Admittedly this is difficult but it must be considered as an integral part of a mathematics HL course. Proofs should be assigned and marked carefully, drawing the attention of the candidates to faulty or sloppy reasoning or explanation.

QUESTION 6 Statistics

Answers: (i) (a) $\mu \simeq 2.8473$
(b) $\mathrm{P}(2 \le X \le 4) = 0.617$
(ii) $p = 0.164$
(iii) H_0 both varieties have the same yield, H_1 varieties have a different yield.
there is ground to reject the hypothesis H_0.
(iv) Let H_0 be the hypothesis that all coins are fair, and let H_1 be the hypothesis that not all coins are fair.
H_0 cannot be rejected.

(i)(a) Most candidates found the correct equation for the parameter of the Poisson distribution but a number of them failed to find μ (misuse of calculator).

(i)(b) Many candidates wrote mistakenly that $\mathrm{P}(2 \le X \le 4) = \mathrm{P}(X \le 4) - \mathrm{P}(X \le 2)$.

(ii) About half the candidates did not use the proper variance for the distribution of the mean, using instead the variance of the original random variable.

(iii) Many candidates failed to use the pooled estimate and/or the Student distribution. The problem for marking this question was that many candidates used their calculators (which was perfectly all right and even advisable) without giving any explanation so that if their answers were incorrect they earned no credit. We are here reaching the point where candidates may get the right answer by following mechanically some recipe, without having the slightest idea about what they are doing, so that some explanations on their part are necessary.

(iv) Most candidates failed to realise that a binomial distribution was required. Most candidates saw that a x^2 test was not required. Practically all candidates failed to realize (a logical error) that H_1 should read “not all coins are fair” (or “at least one coin is unfair”) instead of “the coins are unfair”!

QUESTION 7 Sets, relations and groups

Part (i)(a) was often dealt with correctly although a certain amount of “waffling” was present.

(b) About half the candidates stated de Morgan’s Laws and left it at that. Here again the procedure of writing down the equality to be proved and dealing randomly with the left and right hand side of the equation leads nowhere and to an unjustified emphatic statement by the candidate that the equality has been proved.

(ii) Many candidates had no idea what an equivalence relation was, and many more seemed to confuse “symmetry”, “reflexivity” and “commutativity”. Many candidates did not see the difference between a binary law (an operation) and a relation. Notation was often totally incorrect (for instance $xRy = yRx$ or $xRy = (x = y)/m$). It should be noted that while some textbooks use the notation $p|m$ to indicate that *p* is a divisor of *m*, the notation $\frac{m}{p}$ only denotes the quotient, and in no way implied that this quotient is an integer.

Many candidates did not seem to be at all familiar with the concept of partition. Some candidates wrote sensible things about cyclic groups while others seemed to be wholly unaware of the concept. As said in the general remarks above this part of Question 7 proved to be too difficult for most candidates.

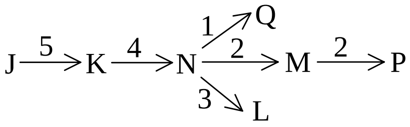
Part (iii) was meant to be the difficult part of the question although the “if” part of the question was easy enough and many candidates groped with the answer to that. At least one candidate gave a correct proof of the “only if” part. It is curious to note, that just as last year for Question 7, and in an equally inappropriate way, some candidates tried to invoke Lagrange’s theorem which seems to be considered by some as a panacea.

QUESTION 8 Discrete mathematics

Answers: (i) (a) 8 is the greatest common divisor of 568 and 208.
(b) $m = 11$ and $n = 30$

(iv) (b) In this case they cannot be isomorphic because *G* has a vertex, {B}, of degree 3 while *H* has no vertex of degree 3.
(c) $\mathrm{B} \to \mathrm{A} \to \mathrm{E} \to \mathrm{B} \to \mathrm{C} \to \mathrm{E} \to \mathrm{F} \to \mathrm{C} \to \mathrm{D} \to \mathrm{F}$
(d) *H* has an Eulerian circuit because all the vertices of *H* have an even degree.

(v)



Total weight = 17

(i)(a) Most candidates found the greatest common divisor although a surprisingly large number without using Euclid's algorithm (which was, of course, unacceptable) and handicapped them for answering part (b).
(ii) and (iii) The results here were mixed essentially because the "proofs" were incomplete, confused and sloppy (even though the candidates often had the correct ideas they simply did not know how to express them).
(iv)(a) Few candidates were able to define correctly an isomorphism. Several said that two graphs are isomorphic if they have the same adjacency matrix, showing that they did not understand the difference between a sufficient, a necessary and an equivalent condition. The suggestion that the difficulty came essentially from faulty logic is supported by the fact that most candidates answered part (b) satisfactorily; in other words they could apply the recipe but could not explain it.
(c) and (d) These parts had mostly correct answers. Some candidates confused Eulerian trail with Eulerian circuit.
(v) In this part practically all candidates answered correctly.

QUESTION 9 Analysis and approximation

Answers: (ii) $\mathrm{e} \approx 2.7182818$
(iii) (b) $\mathrm{e} = 2.7182818$

(i)(a) Candidates still have difficulty stating the mean value theorem (and the distinction between a mean value theorem for functions and a mean value theorem for integrals, when the two are two aspects of the same theorem does not help) and even more difficulty with applying it. Perhaps teachers should insist more on its applications (what is the use of a result that has no application?). In this question they did not have to state it, only to apply it, but nevertheless many candidates did state (or mistate) it. Few did manage this part and not too many were successful with the remainder of part (i). It was particularly distressing on part (b) to see that candidates mulitiplied both sides of an inequality without considering whether they were multiplying by a positive factor (the inequality changing direction). This is an elementary point of algebra. For the rest most candidates did a lot of "waffling" with however some meritorious exceptions. In part (e) one candidate quoted the "sandwich" theorem; this may not be an official theorem but it showed that the candidate knew what he/she was doing. Many missed the point entirely.
In part (ii) many candidates applied the recipes correctly. Some gave the values of the sequence only to three decimal places with seven decimal places appearing miraculously for the final answer.
In part (iii) almost all candidates did part (a) correctly but almost none were able to or even tried to justify their answer, which seems to indicate that as far as they are concerned this is just another recipe.
Still, between parts (ii) and (iii) many candidates got a reasonable amount of marks.

QUESTION 10 Euclidean geometry and conic sections

Answers: (i) (a) centre $= (3, -2)$

eccentricity $= \sqrt{\dfrac{3}{2}}$

foci $= \left(3, -2 \pm \sqrt{3}\right)$

(b) Therefore, the values of *m* are $0, -\dfrac{3}{2}$

The remark of last year's report on this option stands "As often in the past, this option seems to be chosen at random by candidates whose classmates have chosen another option, giving the impression that most candidates who chose this option do so on their own, without having been prepared for it in class, under the misapprehension that it is an easy option. The result is therefore not surprising: most candidates performed miserably".

The problem was as hard as any of the other options but since very few candidates take this option, this fact went largely unnoticed. While part (iii) and especially part (ii) were rather difficult questions, part (i) was quite straightforward and yet candidates performed just as poorly on this part as on the others.

